1 Sets

Lesson at a Glance

1. A **set** is a well-defined collection of distinct elements.
2. A set which does not contain any element is called an **empty set** and is denoted by ϕ.
3. A set which contains only one element is called a **singleton set.**
4. A set which is either empty or contains a definite number of elements is called a **finite set**, otherwise, the set is called **infinite set.**
5. If a finite set A has m elements, we write $n(A) = m$.
6. If $x \in A \Rightarrow x \in B$ for all $x \in A$, i.e. every element of set A is an element of set B, then A is called a subset of B and we write $A \subset B$ and B is called a **superset** of set A.
7. Two sets A and B are said to be **equal** if they have exactly the same elements.
   or $A = B$ iff $A \subset B$ and $B \subset A$.
8. If $n(A) = m$, then A has 2^m subsets.
9. Empty set is a subset of every set and every set is a subset of itself.
10. ϕ and A are called improper subsets of the set A. Every subset of A other than ϕ and A is called a **proper** subset of A.
11. The set of all subsets of set A is called **Power Set** of A and is denoted by P (A). If a set A has m elements, then the set P(A) has 2^m elements. (*i.e.* number of subsets of the set A is 2^m).
12. **Some subsets of set R of real numbers.**
   The set of Natural numbers **N** = {1, 2, 3, 4, 5,..........}
   The set of whole numbers W = {0, 1, 2, 3, 4, 5,......}
   The set of integers Z = { ...... –3, –2, –1, 0, 1, 2, 3, ...}
   The set of rational numbers **Q** is the set of all real numbers of the form $\frac{p}{q}$ where p and q are both integers and $q \neq 0$.

For example, $7 = \frac{7}{1}$, $-3 = -\frac{3}{1}$, $0 = \frac{0}{1}$, $2\frac{1}{2} = \frac{5}{2}$ are some examples of rational numbers. The set of all real numbers which are not rational is called the set of **Irrational numbers** and is denoted by T.

For example, $\sqrt{2}, \sqrt{7}, -\sqrt{11}, \pi$ are some examples of irrational numbers.

$N \subset W \subset Z \subset Q \subset R$, $T \subset R$, $N \not\subset T$.

13. Intervals as subsets of R

Let $a, b \in R$ and $a < b$. Then the set of real numbers $\{y; a < y < b\}$ is called an **open interval** and is denoted by (a, b). All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this open interval (a, b) and the set of all real numbers $\{y: a \le y \le b\}$ is called a **closed interval** and is denoted by $[a, b]$.

14. (*i*) $A \cup B = \{x: x \in A$ **or** $x \in B\}$

(*ii*) $A \cap B = \{x: x \in A$ **and** $x \in B\}$

If $A \cap B = \phi$ *i.e.*, sets A and B have no common element, then sets A and B are called **disjoint sets.**

15. (*i*) $A - B = \{x : x \in A$ and $x \notin B\}$. (*i.e.*) $A - B = A - A \cap B$

(*ii*) $B - A = \{x : x \in B$ and $x \notin A\}$. (*i.e.*) $B - A = B - A \cap B$

16. A' or $A^C = U - A = \{x : x \in U$ and $x \notin A\}$ *i.e.*,

$x \in A' \Rightarrow x \notin A$ and $x \in A \Rightarrow x \notin A'$.

17. (*a*) $A \cup B = B \cup A$, (*b*) $A \cap B = B \cap A$. (Commutative law)

18. (*a*) $(A \cup B) \cup C = A \cup (B \cup C)$,

(*b*) $(A \cap B) \cap C = A \cap (B \cap C)$. (Associative law)

19. (*a*) $A \cup A = A$, (*b*) $A \cap A = A$. (Idempotent law)

20. (*a*) $A \cup \phi = A$, (*b*) $A \cap \phi = A$. (Law of ϕ)

21. (*a*) $A \cup U = U$, (*b*) $A \cap U = A$. (Law of U)

22. (*a*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributive law)

(*b*) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. (Distributive law)

23. (*a*) $(A \cup B)' = A' \cap B'$ (*b*) $(A \cap B)' = A' \cup B'$. (De Morgan's laws)

24. $A \cup A' = U$ and $A \cap A' = \phi$

25. $(A')' = A$.

26. $A - B = A \cap B'$ and $B - A = B \cap A'$

27. (*a*) $\phi' = U$ (*b*) $U' = \phi$

28. $A \cap B \subset A$ and $A \cap B \subset B$.

29. $A \subset A \cup B$ and $B \subset A \cup B$ and $A \cap B \subset A \cup B$.

30. If $A \subset B$, then $A \cup B = B$ and $A \cap B = A$.

31. If A and B are two finite sets, then

(*a*) $A \cap B = \phi$ *i.e.,* A and B are disjoint sets

$\Rightarrow n(A \cup B) = n(A) + n(B)$

(*b*) $A \cap B \neq \phi \Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

32. If A and B are any two finite sets, then

(*i*) number of elements in **A only**

$= n(A - B) = n(A) - n(A \cap B)$

(*ii*) number of elements in **B only**

$= n(B - A) = n(B) - n(A \cap B)$.

33. If A, B and C are three finite sets, then

(*a*) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(*b*) If A, B and C are mutually disjoint sets, then

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$.

34. If A, B and C are three finite sets, then number of elements in **A only** $= n(A) - n(A \cap B) - n(A \cap C)$

$+ n(A \cap B \cap C)$.

35. If A, B and C are three finite sets, then number of elements in

(A and B only) $= n(A \cap B) - n(A \cap B \cap C)$.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 1.1 (*Page No.: 4–5*)

1. Which of the following are sets? Justify your answer.

(*i*) The collection of all the months of a year beginning with the letter J.

(*ii*) The collection of ten most talented writers of India.

(*iii*) A team of eleven best-cricket batsmen of the world.

(*iv*) The collection of all boys in your class.

(*v*) The collection of all natural numbers less than 100.

(*vi*) A collection of novels written by the writer Munshi Prem Chand.

(*vii*) The collection of all even integers.

(*viii*) The collection of questions in this chapter.

(*ix*) A collection of most dangerous animals of the world.

Sol. (*i*) The collection consists of months January, June and July. It is well-defined and therefore, it is a set.

(*ii*) A writer of India may be most talented for one person but not for another person. Opinion varies from person to person. So, the given collection is not well-defined and therefore, not a set.

(*iii*) The term 'best cricket batsman' is vague. The same batsman may be one of the best for one person but not for another. Opinion varies from person to person. So, the given collection is not well-defined and therefore, not a set.

(*iv*) Any boy is either in your class or not in your class. There is no ambiguity. The given collection is well-defined and therefore, it is a set.

(*v*) The collection consists of first 99 natural numbers. It is well-defined and therefore, it is a set.

(*vi*) It is a well-defined collection and therefore, it is a set.

(*vii*) It is a well-defined collection and therefore, it is a set.

(*viii*) It is a well-defined collection and therefore, it is a set.

(*ix*) The criterion for determining an animal as most dangerous varies from person to person. For some people, even a lizard is very dangerous. So, the given collection is not well-defined and therefore, it is not a set.

2. Let A = { 1, 2, 3, 4, 5, 6 }. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

(*i*) 5 ... A **(*ii*) 8 ... A** **(*iii*) 0 ... A**

(*iv*) 4 ... A **(*v*) 2 ... A** **(*vi*) 10 ... A**

Sol. (*i*) $5 \in A$ (*ii*) $8 \notin A$ (*iii*) $0 \notin A$

(*iv*) $4 \in A$ (*v*) $2 \in A$ (*vi*) $10 \notin A$.

3. Write the following sets in roster form:

(i) A = { x : x is an integer and $-3 \le x < 7$ }

(ii) B = { x : x is a natural number less than 6 }

(iii) C = { x : x is a two-digit natural number such that the sum of its digits is 8 }

(iv) D = { x : x is a prime number which is divisor of 60 }

(v) E = The set of all letters in the word TRIGONOMETRY

(vi) F = The set of all letters in the word BETTER.

Sol. (i) The required integers are – 3, – 2, – 1, 0, 1, 2, 3, 4, 5, 6 (not 7).

$\therefore$ In roster form, A = { – 3, – 2, – 1, 0, 1, 2, 3, 4, 5, 6 }

(ii) Natural numbers less than 6 are 1, 2, 3, 4, 5.

$\therefore$ In roster form, B = {1, 2, 3, 4, 5 }

(iii) The required numbers are 17, 26, 35, 44, 53, 62, 71, 80.

$\therefore$ In roster form, C = {17, 26, 35, 44, 53, 62, 71, 80 }

(iv) Divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.

Among them, prime numbers are 2, 3, 5.

$\therefore$ In roster form, D = { 2, 3, 5 }

(v) In the word TRIGONOMETRY, the letters T, R and O are repeated. Dropping the repetitions, in roster form E = { T, R, I, G, O, N, M, E, Y }

(vi) In the word BETTER, the letters E and T are repeated. Dropping the repetitions, in roster form F = { B, E, T, R }

4. Write the following sets in the set-builder form:

(i) { 3, 6, 9, 12 } **(ii) { 2, 4, 8, 16, 32 }**

(iii) { 5, 25, 125, 625 } **(iv) { 2, 4, 6, ...}**

(v) { 1, 4, 9, ..., 100 }.

Sol. (i) {3, 6, 9, 12} $= \{3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4\}$

$= \{x : x = 3n, n \in N \text{ and } 1 \le n \le 4\}$

(ii) { 2, 4, 8, 16, 32 } $= \{2^1, 2^2, 2^3, 2^4, 2^5\}$

$= \{x : x = 2^n, n \in N \text{ and } 1 \le n \le 5\}$

(iii) { 5, 25, 125, 625 } $= \{5^1, 5^2, 5^3, 5^4\}$

$= \{x : x = 5^n, n \in N \text{ and } 1 \le n \le 4\}$

(iv) { 2, 4, 6, ...} $= \{2 \times 1, 2 \times 2, 2 \times 3, \ldots\}$

$= \{x : x = 2n, n \in N\}$

Alternatively, we can write {x : x is an even natural number }.

(v) { 1, 4, 9, ..., 100 } = { $1^2, 2^2, 3^2, ..., 10^2$ }

= { $x : x = n^2, n \in N$ and $1 \le n \le 10$ }.

5. List all the elements of the following sets:

(i) A = {x : x is an odd natural number }

(ii) B = $\left\{x : x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\right\}$

(iii) C = {x : x is an integer, $x^2 \le 4$ }

(iv) D = {x : x is a letter in the word "LOYAL" }

(v) E = {x : x is a month of a year not having 31 days }

(vi) F = {x : x is a consonant in the English alphabet which precedes k }.

Sol. (i) Odd natural numbers are 1, 3, 5, ...

∴ A = { 1, 3, 5, ... }

(ii) Integers greater than $-\frac{1}{2}$ and less than $\frac{9}{2}$ are 0, 1, 2, 3, 4.

∴ B = { 0, 1, 2, 3, 4 }

(iii) Integers whose square is less than or equal to 4 are – 2, – 1, 0, 1, 2.

∴ C = { – 2, – 1, 0, 1, 2 }

(iv) Dropping the repetition

D = { L, O, Y, A }

(v) Months of a year not having 31 days are:

February, April, June, September, November

∴ E = { February, April, June, September, November }

(vi) Consonants in the English alphabet which precede k are:

b, c, d, f, g, h, j

∴ F = { b, c, d, f, g, h, j }.

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) { 1, 2, 3, 6 }	**(a) {x : x is a prime number and a divisor of 6 }**
(ii) { 2, 3 }	**(b) {x : x is an odd natural number less than 10 }**

(*iii*) { M, A, T, H, E, I, C, S }

(*c*) { x : x is a natural number and divisor of 6 }

(*iv*) { 1, 3, 5, 7, 9 }

(*d*) { x : x is a letter of the word MATHEMATICS }.

Sol. (*i*) {1, 2, 3, 6} (*a*) {2, 3}

(*ii*) {2, 3} (*b*) {1, 3, 5, 7, 9}

(*iii*) {M, A, T, H, E, I, C, S} (*c*) {1, 2, 3, 6}

(*iv*) {1, 3, 5, 7, 9} (*d*) {M, A, T, H, E, I, C, S}

We have written all the sets on the right in roster form. Clearly, (*i*) matches (*c*), (*ii*) matches (*a*), (*iii*) matches (*d*) and (*iv*) matches (*b*).

EXERCISE 1.2 (*Page No.: 8–9*)

1. Which of the following are examples of the null set?

(*i*) Set of odd natural numbers divisible by 2.

(*ii*) Set of even prime numbers

(*iii*) { x : x is a natural number, $x < 5$ and $x > 7$ }

(*iv*) { y : y is a point common to any two parallel lines }

Sol. (*i*) There is no odd natural number which is divisible by 2. So, the given set is a null set.

(*ii*) Set of even prime numbers = { 2 } $\neq \phi$. So, the given set is not a null set. It is a singleton set.

Note: A natural number > 1 is said to be **prime** if it has only two divisors 1 and itself. The set of prime numbers is {2, 3, 5, 7, 11,...}.

(*iii*) There is no natural number which is both less than 5 and greater than 7. So, the given set is a null set.

(*iv*) Two parallel have no common point. So, the given set is a null set. Hence (*i*), (*iii*) and (*iv*) are examples of the null set.

2. Which of the following sets are finite or infinite?

(*i*) The set of months of a year

(*ii*) { 1, 2, 3, ... }

(*iii*) { 1, 2, 3, ..., 99, 100 }

(*iv*) The set of positive integers greater than 100

(*v*) The set of prime numbers less than 99.

Sol. (*i*) Since there are 12 months (*i.e.*, a definite number of months) in a year, the given set is finite.

(*ii*) Since the number of elements in the set is infinite, the given set is infinite.

(*iii*) Since the number of elements in the set is 100 (*i.e.*, a definite number), the given set is finite.

(*iv*) Since there are infinitely many numbers greater than 100, the given set is infinite.

(*v*) Since the number of primes less than 99 is a definite number, the given set is finite.

3. State whether each of the following sets is finite or infinite:

(*i*) The set of lines which are parallel to the x-axis

(*ii*) The set of letters in the English alphabet

(*iii*) The set of numbers which are multiple of 5

(*iv*) The set of animals living on the earth

(*v*) The set of circles passing through the origin (0, 0)

Sol. (*i*) Since there are infinite number of lines parallel to the x-axis, the given set is infinite.

(*ii*) Since there are 26 letters, *i.e.*, a definite number of letters, in the English alphabet, the given set is finite.

(*iii*) Since there are infinitely many multiples of 5, the given set is infinite.

(*iv*) The process of counting the animals living on the earth is terminating. Thus, a definite number of animals live on the earth and hence the given set is finite.

(*v*) There is no end to the number of circles passing through the origin (0, 0). Hence, the given set is infinite.

4. In the following, state whether A = B or not:

(*i*) $A = \{a, b, c, d\}$, $B = \{d, c, b, a\}$

(*ii*) $A = \{4, 8, 12, 16\}$, $B = \{8, 4, 16, 18\}$

(*iii*) $A = \{2, 4, 6, 8, 10\}$, $B = \{x : x \text{ is positive even integer and } x \leq 10\}$

(iv) $A = \{x : x \text{ is a multiple of } 10\}$
$B = \{10, 15, 20, 25, 30, ...\}$

Sol. (i) A and B have exactly same elements, though not in the same order.

$\therefore \quad A = B$

(ii) $12 \in A$ but $12 \notin B$

$\therefore \quad A \neq B$

(iii) In roster form $B = \{2, 4, 6, 8, 10\}$
Since A and B have exactly same elements, therefore,
$A = B$.

(iv) In roster form $A = \{10, 20, 30, ...\}$
Since $15 \in B$ but $15 \notin A$, therefore $A \neq B$.

5. Are the following pairs of sets equal? Give reasons.

(i) $A = \{2, 3\}, B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}$

(ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\}$
$B = \{y : y \text{ is a letter in the word WOLF}\}$

Sol. (i) $B = \{x : x \text{ is solution of } (x + 2)(x + 3) = 0\}$

$[\because x^2 + 5x + 6 = x^2 + 2x + 3x + 6$
$= x(x + 2) + 3(x + 2) = (x + 2)(x + 3)]$
$= \{-2, -3\}$

$2 \in A$ but $2 \notin B$ $\quad \therefore \quad A \neq B$

(ii) Dropping repetitions $A = \{F, O, L, W\}$
$B = \{W, O, L, F\}$

Sets A and B have exactly same elements, though not in the same order.

$\therefore \quad A = B$.

6. From the sets given below, select equal sets:

$A = \{2, 4, 8, 12\}, B = \{1, 2, 3, 4\}, C = \{4, 8, 12, 14\},$
$D = \{3, 1, 4, 2\}, E = \{-1, 1\}, F = \{0, a\}, G = \{1, -1\},$
$H = \{0, 1\}$.

Sol. $B = D, E = G$.

EXERCISE 1.3 *(Page No.: 12–13)*

1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:

(i) $\{2, 3, 4\} ... \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} ... \{b, c, d\}$

(iii) { x : x is a student of Class XI of your school }
... { x : x is a student of your school }

(iv) { x : x is a circle in the plane } ... { x : x is a circle in the same plane with radius 1 unit }

(v) { x : x is a triangle in a plane } ... { x : x is a rectangle in the plane }

(vi) { x : x is an equilateral triangle in a plane }
... { x : x is a triangle in the same plane }

(vii) { x : x is an even natural number } ... { x : x is an integer }

Sol. (i) Every element of the set {2, 3, 4} is also an element of the set {1, 2, 3, 4, 5 }

$\therefore$ { 2, 3, 4 } $\subset$ { 1, 2, 3, 4, 5 }.

(ii) $a \in \{a, b, c\}$ but $a \notin \{b, c, d\}$

$\therefore$ $\{a, b, c\} \not\subset \{b, c, d\}$.

(iii) Every student of class XI of your school is a student of your school.

$\therefore$ { x : x is a student of class XI of your school } $\subset$ { x : x is a student of your school }.

(iv) Every circle in a plane is not a circle with radius 1 unit, as it can have any radius r, ($r > 0$).

$\therefore$ { x : x is a circle in the plane } $\not\subset$ { x : x is a circle in the same plane with radius 1 unit }.

(v) Any triangle is never a rectangle.

$\therefore$ { x : x is a triangle in the plane } $\not\subset$ { x : x is a rectangle in the plane }.

(vi) Every equilateral triangle in a plane is a triangle in the plane.

$\therefore$ { x : x is an equilateral triangle in a plane } $\subset$ { x : x is a triangle in the same plane }.

(vii) Every even natural number is an integer.

$\therefore$ { x : x is an even natural number } $\subset$ { x : x is an integer }.

2. Examine whether the following statements are true or false:

(i) $\{a, b\} \not\subset \{b, c, a\}$

(ii) $\{a, e\} \subset$ { x : x is a vowel in the English alphabet }

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$

(iv) **$\{a\} \subset \{a, b, c\}$**

(v) **$\{a\} \in \{a, b, c\}$**

(vi) **$\{x : x$ is an even natural number less than $6\}$ $\subset \{x : x$ is a natural number which divides $36\}$**

Sol. (*i*) **False,** since every element of the set $\{a, b\}$ is also an element of the set $\{b, c, a\}$, therefore, $\{a, b\} \subset \{b, c, a\}$.

(*ii*) **True,** since every element of the set $\{a, e\}$ is also an element of the set of vowels $\{a, e, i, o, u\}$, therefore, $\{a, e\} \subset \{a, e, i, o, u\}$.

(*iii*) **False,** since $2 \in \{1, 2, 3\}$ but $2 \notin \{1, 3, 5\}$.

(*iv*) **True,** since $a \in \{a, b, c\}$.

(*v*) **False,** since $\{a\}$ is subset of the set $\{a, b, c\}$ but not an element of the set $\{a, b, c\}$.

(*vi*) **True,** since $\{x : x$ is an even natural number less than $6\} = \{2, 4\}$ and $\{x : x$ is a natural number which divides $36\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$.

Clearly, $\{2, 4\} \subset \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$.

3. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(i) **$\{3, 4\} \subset A$**

(ii) **$\{3, 4\} \in A$**

(iii) **$\{\{3, 4\}\} \subset A$**

(iv) **$1 \in A$**

(v) **$1 \subset A$**

(vi) **$\{1, 2, 5\} \subset A$**

(vii) **$\{1, 2, 5\} \in A$**

(viii) **$\{1, 2, 3\} \subset A$**

(ix) **$\phi \in A$**

(x) **$\phi \subset A$**

(xi) **$\{\phi\} \in A$.**

Sol. (*i*) **False,** since $3 \notin A$, $4 \notin A$, therefore, $\{3, 4\} \not\subset A$.

(*ii*) **True,** since $\{3, 4\}$ is an element of A.

(*iii*) **True,** since $\{3, 4\} \in A$, therefore, $\{\{3, 4\}\} \subset A$.

(*iv*) **True,** since 1 is an element of A.

(*v*) **False,** since 1 is not a set. Only a set can be a subset of another set.

(*vi*) **True,** since 1, 2, 5 are elements of A.

(*vii*) **False,** since $\{1, 2, 5\}$ is not an element of A.

(*viii*) **False,** since $3 \notin A$.

(*ix*) **False,** since ϕ is not an element of A.

(*x*) **True,** since ϕ is a subset of every set.

(*xi*) **False,** since $\phi \subset A$ and hence $\{\phi\} \in P(A)$, power set of set A.

4. Write down all the subsets of the following sets:

(*i*) { *a* } (*ii*) { *a*, *b* }

(*iii*) { 1, 2, 3 } (*iv*) ϕ.

Sol. (*i*) Let A = { *a* }, then A has one element.

$\therefore$ Number of subsets of the set A = $2^n = 2^1 = 2$.

Subset of A will have either no element or one element.

Subset of A having no element is ϕ

Subset of A having one element is { *a* }

$\therefore$ The subsets of A are ϕ, { *a* }.

(*ii*) Let A = { *a*, *b* }, then A has 2 elements.

$\therefore$ Number of subsets of A = $2^n = 2^2 = 4$.

Subset of A having no element is ϕ

Subsets of A having one element are { *a* }, { *b* }

Subset of A having two elements is { *a*, *b* }

$\therefore$ The subsets of A are ϕ, { *a* }, { *b* }, { *a*, *b* }.

(*iii*) Let A = { 1, 2, 3 }, then *A* has 3 elements.

$\therefore$ Number of subsets of A = $2^n = 2^3 = 8$.

Subsets of A having no element is ϕ

Subsets of A having one element are { 1 }, { 2 }, { 3 }

Subsets of A having two elements are { 1, 2 }, { 1, 3 }, { 2, 3 }

Subset of A having three elements is { 1, 2, 3 }

$\therefore$ The subsets of A are ϕ, { 1 }, { 2 }, { 3 }, { 1, 2 }, { 1, 3 }, { 2, 3 }, { 1, 2, 3 }.

(*iv*) The only subset of the empty set ϕ is ϕ itself.

5. How many elements has P(A), if A = ϕ?

Sol. A = ϕ has no element. Therefore n (A) = 0.

$\therefore$ n[P(A)] $2^0 = 1$

6. Write the following as intervals:

(*i*) $\{x : x \in \mathbf{R}, -4 < x \le 6\}$

(*ii*) $\{x : x \in \mathbf{R}, -12 < x < -10\}$

(*iii*) $\{x : x \in \mathbf{R}, 0 \le x < 7\}$

(*iv*) $\{x : x \in \mathbf{R}, 3 \le x \le 4\}$.

Sol. (*i*) (– 4, 6], since – 4 is not included while 6 is included.

(*ii*) (– 12, – 10), since both end points are excluded.

(*iii*) [0, 7), since 0 is included while 7 is excluded.

(*iv*) [3, 4], since both end points are included.

7. Write the following intervals in set-builder form:

(i) (– 3, 0) (ii) [6, 12] (iii) (6, 12] (iv) [– 23, 5).

Sol. (i) $\{x : x \in R, -3 < x < 0\}$

(ii) $\{x : x \in R, 6 \leq x \leq 12\}$

(iii) $\{x : x \in R, 6 < x \leq 12\}$

(iv) $\{x : x \in R, -23 \leq x < 5\}$.

8. What universal set(s) would you propose for each of the following:

(i) The set of right triangles.

(ii) The set of isosceles triangles.

Sol. The set of all triangles.

9. Given the sets A = { 1, 3, 5 }, B = {2, 4, 6} and C = { 0, 2, 4, 6, 8 }, which of the following may be considered as universal set(s) for all the three sets A, B and C.

(i) { 0, 1, 2, 3, 4, 5, 6 }

(ii) ϕ

(iii) { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

(iv) { 1, 2, 3, 4, 5, 6, 7, 8 }

Sol. All the three sets A, B and C must be subsets of the universal set.

Since $8 \notin \{0, 1, 2, 3, 4, 5, 6\}$

$\therefore \quad C \not\subset \{0, 1, 2, 3, 4, 5, 6\}$

None of the sets A, B and C is a subset of ϕ.

Since $0 \notin \{1, 2, 3, 4, 5, 6, 7, 8\}$

$\therefore \quad C \not\subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

All the three sets A, B and C are subsets of the set { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 } which is, therefore, the required universal set.

EXERCISE 1.4 *(Page No.: 17–18)*

1. Find the union of each of the following pairs of sets:

(i) X = { 1, 3, 5 }   Y = { 1, 2, 3 }

(ii) A = { *a*, *e*, *i*, *o*, *u* }   B = { *a*, *b*, *c* }

(iii) A = { *x* : *x* is a natural number and multiple of 3 }

B = { *x* : *x* is a natural number less than 6 }

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \le 6\}$
$B = \{x : x \text{ is a natural number and } 6 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \phi$.

Sol. (i) $X \cup Y = \{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}$.

(ii) $A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\} = \{a, b, c, e, i, o, u\}$

(iii) $A = \{3, 6, 9, 12, ...\}$
$B = \{1, 2, 3, 4, 5\}$
$A \cup B = \{3, 6, 9, 12, ...\} \cup \{1, 2, 3, 4, 5\}$
$= \{1, 2, 3, 4, 5, 6, 9, 12, ...\}$
$= \{x : x = 1, 2, 4, 5 \text{ or a multiple of } 3\}$

(iv) $A = \{2, 3, 4, 5, 6\}$
$B = \{7, 8, 9\}$
$A \cup B = \{2, 3, 4, 5, 6\} \cup \{7, 8, 9\} = \{2, 3, 4, 5, 6, 7, 8, 9\}$
$= \{x : 1 < x < 10 \text{ and } x \in N\}$.

(v) $A \cup B = \{1, 2, 3\} \cup \phi = \{1, 2, 3\} = A$

Note. The result of this part (v) is true in general also.

$A \cup \phi = A$ for every set A.

2. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Sol. Yes. $A \cup B = \{a, b, c\} = B$.

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Sol. $A \cup B = \{x : x \in A \text{ or } x \in B\}$
$= \{x : x \in B\}$
$(\because A \subset B \quad \therefore x \in A \Rightarrow x \in B)$
$= B$.

4. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$ **(ii)** $A \cup C$ **(iii)** $B \cup C$

(iv) $B \cup D$ **(v)** $A \cup B \cup C$ **(vi)** $A \cup B \cup D$

(vii) $B \cup C \cup D$.

Sol. (i) $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$
$= \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D = \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C = (A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D = (A \cup B) \cup D = \{1, 2, 3, 4, 5, 6\} \cup \{7, 8, 9, 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D = (B \cup C) \cup D = \{3, 4, 5, 6, 7, 8\} \cup \{7, 8, 9, 10\} = \{3, 4, 5, 6, 7, 8, 9, 10\}$

5. Find the intersection of each pair of sets of question 1 above.

Sol. (i) $X \cap Y = \{1, 3, 5\} \cap \{1, 2, 3\} = \{1, 3\}$.

(ii) $A \cap B = \{a, e, i, o, u\} \cap \{a, b, c\} = \{a\}$.

(iii) $A = \{3, 6, 9, 12, ...\}$

$B = \{1, 2, 3, 4, 5\}$

$A \cap B = \{3, 6, 9, 12, ...\} \cap \{1, 2, 3, 4, 5\} = \{3\}$.

(iv) $A = \{2, 3, 4, 5, 6\}$

$B = \{7, 8, 9\}$

$A \cap B = \{2, 3, 4, 5, 6\} \cap \{7, 8, 9\} = \phi$.

(v) $A \cap B = \{1, 2, 3\} \cap \phi = \phi$

Note. The result of this part (v) is true in general also.

$A \cap \phi = \phi$ for every set A.

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$ **(ii) $B \cap C$**
(iii) $A \cap C \cap D$ **(iv) $A \cap C$**
(v) $B \cap D$ **(vi) $A \cap (B \cup C)$**
(vii) $A \cap D$ **(viii) $A \cap (B \cup D)$**
(ix) $(A \cap B) \cap (B \cup C)$ **(x) $(A \cup D) \cap (B \cup C)$.**

Sol. (i) $A \cap B = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\}$
$= \{7, 9, 11\}$

(ii) $B \cap C = \{7, 9, 11, 13\} \cap \{11, 13, 15\} = \{11, 13\}$

(iii) $A \cap C \cap D = (A \cap C) \cap D$
$= (\{3, 5, 7, 9, 11\} \cap \{11, 13, 15\}) \cap \{15, 17\}$
$= \{11\} \cap \{15, 17\} = \phi$.

(*iv*) $A \cap C = \{3, 5, 7, 9, 11\} \cap \{11, 13, 15\} = \{11\}$

(*v*) $B \cap D = \{7, 9, 11, 13\} \cap \{15, 17\} = \phi$

(*vi*) $A \cap (B \cup C) = \{3, 5, 7, 9, 11\} \cap (\{7, 9, 11, 13\} \cup \{11, 13, 15\})$
$= \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$
$= \{7, 9, 11\}$

(*vii*) $A \cap D = \{3, 5, 7, 9, 11\} \cap \{15, 17\} = \phi$

(*viii*) $A \cap (B \cup D) = \{3, 5, 7, 9, 11\} \cap (\{7, 9, 11, 13\} \cup \{15, 17\})$
$= \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15, 17\}$
$= \{7, 9, 11\}$

(*ix*) $(A \cap B) \cap (B \cup C)$
$= (\{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\}) \cap (\{7, 9, 11, 13\} \cup \{11, 13, 15\})$
$= \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$

(*x*) $(A \cup D) \cap (B \cup C)$
$= (\{3, 5, 7, 9, 11\} \cup \{15, 17\}) \cap (\{7, 9, 11, 13\} \cup \{11, 13, 15\})$
$= \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$
$= \{7, 9, 11, 15\}$.

7. If A = { x : x is a natural number }, B = { x : x is an even natural number }, C = { x : x is an odd natural number } and D = { x : x is a prime number}, find

(*i*) $A \cap B$ (*ii*) $A \cap C$ (*iii*) $A \cap D$ (*iv*) $B \cap C$

(*v*) $B \cap D$ (*vi*) $C \cap D$.

Sol. (*i*) $A \cap B = B$ $(\because B \subset A)$

($\because$ Every even natural number is a natural number)

(*ii*) $A \cap C = C$ $(\because C \subset A)$

($\because$ Every odd natural number is a natural number)

(*iii*) $A \cap D = D$ $(\because D \subset A)$

($\because$ Every prime number is a natural number)

(*iv*) $B \cap C = \phi$

($\because$ There is no natural number which is both even and odd)

(*v*) $B \cap D = \{2\}$, 2 is only even prime number.

(*vi*) $C \cap D = \{x : x$ is an odd prime number $\}$.

8. Which of the following pairs of sets are disjoint:

(*i*) { 1, 2, 3, 4 } and { x : x is a natural number and $4 \le x \le 6$ }

(*ii*) { a, e, i, o, u } and { c, d, e, f }

(*iii*) { x : x is an even integer } and { x : x is an odd integer }

Sol. (*i*) Let A = { 1, 2, 3, 4 } and

B = { x: $x \in$ N and $4 \le x \le 6$ } = { 4, 5, 6 }

$A \cap B$ = { 4 } $\neq \phi$ ∴ Sets A and B are not disjoint.

(*ii*) e is a common element of the two sets.

∴ Sets are not disjoint.

(*iii*) Given sets are disjoint sets because there is no natural number which is both even and odd.

9. If A = { 3, 6, 9, 12, 15, 18, 21 }, B = { 4, 8, 12, 16, 20 }, C = { 2, 4, 6, 8, 10, 12, 14, 16 }, D = { 5, 10, 15, 20 }; find

(*i*) A – B (*ii*) A – C (*iii*) A – D (*iv*) B – A

(*v*) C – A (*vi*) D – A (*vii*) B – C (*viii*) B – D

(*ix*) C – B (*x*) D – B (*xi*) C – D (*xii*) D – C.

Sol. (*i*) A – B = { 3, 6, 9, 12, 15, 18, 21 } – { 4, 8, 12, 16, 20 }
= { 3, 6, 9, 15, 18, 21 }

(*ii*) A – C = { 3, 6, 9, 12, 15, 18, 21 } – { 2, 4, 6, 8, 10, 12, 14, 16 }
= { 3, 9, 15, 18, 21 }

(*iii*) A – D = { 3, 6, 9, 12, 15, 18, 21 } – { 5, 10, 15, 20 }
= { 3, 6, 9, 12, 18, 21 }

(*iv*) B – A = { 4, 8, 12, 16, 20 } – { 3, 6, 9, 12, 15, 18, 21 }
= { 4, 8, 16, 20 }

(*v*) C – A = { 2, 4, 6, 8, 10, 12, 14, 16 } – { 3, 6, 9, 12, 15, 18, 21 }
= { 2, 4, 8, 10, 14, 16 }

(*vi*) D – A = { 5, 10, 15, 20} – { 3, 6, 9, 12, 15, 18, 21 }
= { 5, 10, 20 }

(*vii*) B – C = { 4, 8, 12, 16, 20 } – { 2, 4, 6, 8, 10, 12, 14, 16 }
= { 20 }

(*viii*) B – D = { 4, 8, 12, 16, 20} – {5, 10, 15, 20}
= { 4, 8, 12, 16 }

(*ix*) $C - B = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{4, 8, 12, 16, 20\}$
$= \{2, 6, 10, 14\}$

(*x*) $D - B = \{5, 10, 15, 20\} - \{4, 8, 12, 16, 20\}$
$= \{5, 10, 15\}$

(*xi*) $C - D = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{5, 10, 15, 20\}$
$= \{2, 4, 6, 8, 12, 14, 16\}$

(*xii*) $D - C = \{5, 10, 15, 20\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$
$= \{5, 15, 20\}$.

10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

(*i*) $X - Y$ (*ii*) $Y - X$ (*iii*) $X \cap Y$.

Sol. (*i*) $X - Y = \{a, b, c, d\} - \{f, b, d, g\}$
$= \{a, c\}$

(*ii*) $Y - X = \{f, b, d, g\} - \{a, b, c, d\}$
$= \{f, g\}$

(*iii*) $X \cap Y = \{a, b, c, d\} \cap \{f, b, d, g\}$
$= \{b, d\}$.

11. If R is the set of real numbers and Q is the set of rational numbers, then what is R – Q?

Sol. $R - Q = \{x : x \in R \text{ and } x \notin Q\}$
$= \{x : x$ is a real number and x is not a rational number $\}$
$= \{x : x$ is an irrational number $\}$
$= T$

($\because$ Every real number is either rational or irrational but not both)

12. State whether each of the following statements is true or false. Justify your answer.

(*i*) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

(*ii*) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

(*iii*) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.

(*iv*) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

Sol. (*i*) **False,** because $\{2, 3, 4, 5\} \cap \{3, 6\} = \{3\} \neq \phi$

(*ii*) **False,** because $\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\} \neq \phi$

(*iii*) **True,** because $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \phi$

(*iv*) **True,** because $\{2, 6, 10\} \cap \{3, 7, 11\} = \phi$.

EXERCISE 1.5 *(Page No.: 20–21)*

1. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = { 1, 2, 3, 4 }, B = { 2, 4, 6, 8 } and C = { 3, 4, 5, 6 }. Find

(i) A′ (ii) B′ (iii) (A ∪ C)′

(iv) (A ∪ B)′ (v) (A′)′ (vi) (B – C)′.

Sol. (i) $A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$
$= \{5, 6, 7, 8, 9\}$

(ii) $B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$
$= \{1, 3, 5, 7, 9\}$

(iii) $A \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$
$(A \cup C)' = U - (A \cup C)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 5, 6\}$
$= \{7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6, 8\} = \{1, 2, 3, 4, 6, 8\}$
$(A \cup B)' = U - (A \cup B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 6, 8\}$
$= \{5, 7, 9\}$

(v) $A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$
$= \{5, 6, 7, 8, 9\}$
$(A')' = U - A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{5, 6, 7, 8, 9\}$
$= \{1, 2, 3, 4\} = A$

(vi) $B - C = \{2, 4, 6, 8\} - \{3, 4, 5, 6\} = \{2, 8\}$
$(B - C)' = U - (B - C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 8\}$
$= \{1, 3, 4, 5, 6, 7, 9\}.$

2. If U = { *a, b, c, d, e, f, g, h* }, find the complements of the following sets:

(i) A = { *a, b, c* } (ii) B = { *d, e, f, g* }

(iii) C = { *a, c, e, g* } (iv) D = { *f, g, h, a* }

Sol. (i) $A' = U - A = \{a, b, c, d, e, f, g, h\} - \{a, b, c\}$
$= \{d, e, f, g, h\}$

(ii) $B' = U - B = \{a, b, c, d, e, f, g, h\} - \{d, e, f, g\}$
$= \{a, b, c, h\}$

(iii) $C' = U - C = \{a, b, c, d, e, f, g, h\} - \{a, c, e, g\}$
$= \{b, d, f, h\}$

(iv) $D' = U - D = \{a, b, c, d, e, f, g, h\} - \{f, g, h, a\}$
$= \{b, c, d, e\}.$

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) $\{x : x$ **is an even natural number** $\}$

(ii) $\{x : x$ **is an odd natural number** $\}$

(iii) $\{x : x$ **is a positive multiple of 3** $\}$

(iv) $\{x : x$ **is a prime number** $\}$

(v) $\{x : x$ **is a natural number divisible by 3 and 5** $\}$

(vi) $\{x : x$ **is a perfect square** $\}$

(vii) $\{x : x$ **is a perfect cube** $\}$

(viii) $\{x : x + 5 = 8\}$

(ix) $\{x : 2x + 5 = 9\}$

(x) $\{x : x \geq 7\}$

(xi) $\{x : x \in \mathrm{N}$ **and** $2x + 1 > 10\}$

Sol. (i) $\{x : x$ is an odd natural number $\}$

(ii) $\{x : x$ is an even natural number $\}$

(iii) $\{x : x \in \mathrm{N}$ and x is not a multiple of 3 $\}$

(iv) $\{x : x$ is positive composite number and $x = 1\}$

Def: Composite number: A natural number > 1 is said to be composite if it is not prime, *i.e.*, if it has at least one divisor other than 1 and itself. **For example,** 4, 6, 8, 9, ... are composite.

Note. In fact N is the union of (set of primes, set of composites and {1})

(v) $\{x : x \in \mathrm{N}$ and x is not divisible by 3 or not divisible by 5 $\}$

(vi) $\{x : x \in \mathrm{N}$ and x is not a perfect square $\}$

(vii) $\{x : x \in \mathrm{N}$ and x is not a perfect cube $\}$

(viii) $\{x : x = 3\}' = \{x : x \in \mathrm{N}$ and $x \neq 3\}$

(ix) $\{x : x \in \mathrm{N}$ and $x \neq 2\}$ $\quad \{\because 2x + 5 = 9 \Rightarrow 2x = 4 \Rightarrow x = 2)$

(x) $\{x : x \in \mathrm{N}$ and $x < 7\} = \{1, 2, 3, 4, 5, 6\}$

(xi) $\left\{x : x \in \mathrm{N} \text{ and } x > \frac{9}{2}\right\}' = \left\{x : x \in \mathrm{N} \text{ and } x \leq \frac{9}{2}\right\} = \{1, 2, 3, 4\}.$

4. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that

(i) $(\mathrm{A} \cup \mathrm{B})' = \mathrm{A}' \cap \mathrm{B}'$ $\qquad$ **(ii)** $(\mathrm{A} \cap \mathrm{B})' = \mathrm{A}' \cup \mathrm{B}'$

Sol. Here $A' = U - A = \{1, 3, 5, 7, 9\}$

and $B' = U - B = \{1, 4, 6, 8, 9\}$

(*i*) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = U - (A \cup B) = \{1, 9\}$

$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$

$\therefore (A \cup B)' = A' \cap B'$.

(*ii*) $A \cap B = \{2\}$

$(A \cap B)' = U - (A \cap B) = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

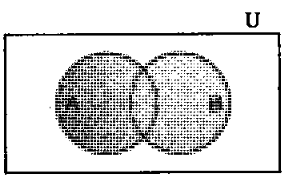
$= \{1, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore (A \cap B)' = A' \cup B'$.

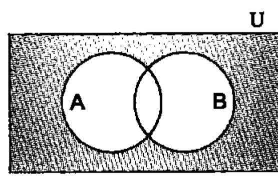
5. Draw appropriate Venn diagram for each of the following:

(*i*) $(A \cup B)'$ (*ii*) $A' \cap B'$ (*iii*) $(A \cap B)'$ (*iv*) $A' \cup B'$.

Sol. (*i*)

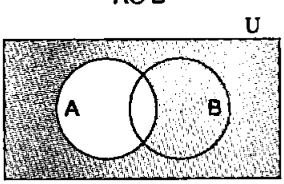


A∪B

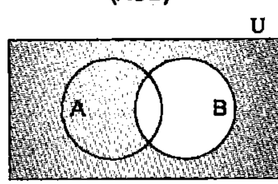


(A∪B)'

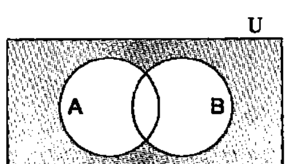
(*ii*)



A'

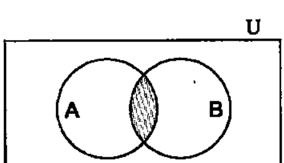


B'

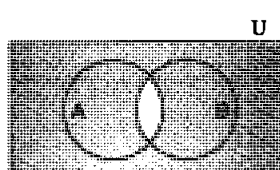


A'∩B'

(*iii*)

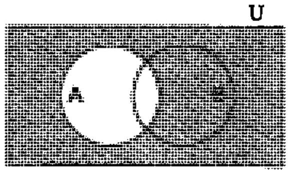


A∩B

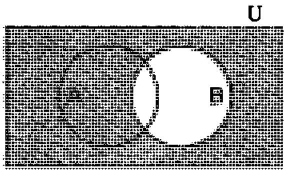


(A∩B)'

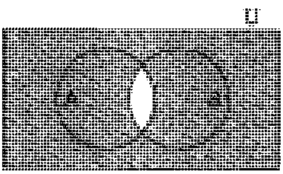
(*iv*)



A′



B′



A′ ∪ B′

6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A′ ?

Sol. $A' = U - A$

= Set of all triangles with no angle different from 60°

= Set of all triangles with each angle 60°

= Set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement:

(*i*) $A \cup A' = ...$ (*ii*) $\phi' \cap A = ...$

(*iii*) $A \cap A' = ...$ (*iv*) $U' \cap A = ...$

Sol. (*i*) $A \cup A' = U$ [Property of complement sets]

(*ii*) $\phi' \cap A = U \cap A = A$

(*iii*) $A \cap A' = \phi$

(*iv*) $U' \cap A = \phi \cap A = \phi$.

EXERCISE 1.6 (*Page No.: 24*)

1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Sol. Using $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we get

$$38 = 17 + 23 - n(X \cap Y)$$

$\Rightarrow \quad n(X \cap Y) = 40 - 38 = 2.$

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements, how many elements does $X \cap Y$ have?

Sol. According to given information,

$$n(X \cup Y) = 18, n(X) = 8 \text{ and } n(Y) = 15$$

Putting values in $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we have

$$18 = 8 + 15 - n(X \cap Y)$$

$\therefore \quad n(X \cap Y) = 23 - 18 = 5.$

3. In a group of 400 people, 250 can speak Hindi, and 200 can speak English. How many people can speak both Hindi and English?

Sol. Let H be the set of people who can speak Hindi and E be the set of people who can speak English. Then,

$$n(H \cup E) = 400, \qquad n(H) = 250, \qquad n(E) = 200$$

We have to find $n(H \cap E)$.

Using $\quad n(H \cup E) = n(H) + n(E) - n(H \cap E)$, we get

$$400 = 250 + 200 - n(H \cap E)$$

$\Rightarrow \quad n(H \cap E) = 450 - 400 = 50.$

Note. It is assumed that each person can speak at least one of the two languages.

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and S ∩ T has 11 elements, how many elements does S ∪ T have?

Sol. According to given information,

$$n(S) = 21, n(T) = 32 \text{ and } n(S \cap T) = 11$$

Using $\quad n(S \cup T) = n(S) + n(T) - n(S \cap T)$, we get

$$n(S \cup T) = 21 + 32 - 11$$
$$= 53 - 11 = 42$$

$\therefore \quad S \cup T$ has 42 elements.

5. If X and Y are two sets such that X has 40 elements, X ∪ Y has 60 elements and X ∩ Y has 10 elements, how many elements does Y have?

Sol. Here, $\quad n(X) = 40, \quad n(X \cup Y) = 60, \quad n(X \cap Y) = 10$

We have to find $n(Y)$.

Using $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we get

$$60 = 40 + n(Y) - 10$$

$\Rightarrow \quad 60 = 30 + n(Y)$

$\therefore \quad n(Y) = 30.$

6. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Sol. Let C be the set of people who like coffee and T be the set of people who like tea.

Given: $n(C \cup T) = 70$, $n(C) = 37$, $n(T) = 52$

Using $n(C \cup T) = n(C) + n(T) - n(C \cap T)$, we get

$$70 = 37 + 52 - n(C \cap T)$$

$\Rightarrow$ $n(C \cap T) = 89 - 70 = 19$

$\therefore$ 19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Sol. Let C denote the set of people who like cricket and T denote the set of people who like tennis.

Given: $n(C \cup T) = 65$, $n(C) = 40$, $n(C \cap T) = 10$

Using $n(C \cup T) = n(C) + n(T) - n(C \cap T)$, we get

$$65 = 40 + n(T) - 10$$

$\Rightarrow$ $65 = 30 + n(T)$ $\Rightarrow$ $n(T) = 35$

$\therefore$ 35 people like tennis.

Now, $n(T - C) = n(T) - n(C \cap T) = 35 - 10 = 25$

$\therefore$ 25 people like tennis only and not cricket.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Sol. Let F denote the set of people who speak French and S denote the set of people who speak Spanish.

Given: $n(F) = 50$, $n(S) = 20$, $n(F \cap S) = 10$

We have to find the number of people who speak at least one of the two languages, *i.e.*, $n(F \cup S)$.

$$n(F \cup S) = n(F) + n(S) - n(F \cap S)$$
$$= 50 + 20 - 10 = 60$$

$\therefore$ The required number of people who speak at least one of the two languages = 60.

MISCELLANEOUS EXERCISE ON CHAPTER 1

(Page No.: 26–27)

1. Decide, among the following sets, which sets are subsets of one and another:

A = $\{x : x \in$ R and x satisfy $x^2 - 8x + 12 = 0\}$
B = $\{2, 4, 6\}$, C = $\{2, 4, 6, 8, ...\}$, D = $\{6\}$.

Sol. A = $\{x : x \in$ R and $(x - 2)(x - 6) = 0\} = \{2, 6\}$
B = $\{2, 4, 6\}$, C = $\{2, 4, 6, 8, ...\}$, D = $\{6\}$
Here, $A \subset B$, $A \subset C$, $B \subset C$,
$D \subset A$, $D \subset B$, $D \subset C$.

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x \in$ A and A $\in$ B, then x $\in$ B
(ii) If A $\subset$ B and B $\in$ C, then A $\in$ C
(iii) If A $\subset$ B and B $\subset$ C, then A $\subset$ C
(iv) If A $\not\subset$ B and B $\not\subset$ C, then A $\not\subset$ C
(v) If $x \in$ A and A $\not\subset$ B, then $x \in$ B
(vi) If A $\subset$ B and x $\notin$ B, then $x \notin$ A.

Sol. (i) **False.** Let A = {1} and B = {{1}, 2}. Clearly, 1 $\in$ A and A $\in$ B but 1 $\notin$ B.
Thus, $x \in$ A and A $\in$ B need not imply $x \in$ B.

(ii) **False.** Let A = {1}, B = {1, 2} and C = {{1, 2}, 3}. Clearly, A $\subset$ B and B $\in$ C but A $\notin$ C.
Thus, A $\subset$ B and B $\in$ C need not imply A $\in$ C.

(iii) **True.** Let x be any element of A. Then
$x \in A \Rightarrow x \in B$ [$\because$ A $\subset$ B]
$\Rightarrow x \in C$ [$\because$ B $\subset$ C]
Thus, $x \in$ A $\Rightarrow$ $x \in$ C for all $x \in$ A, therefore, A $\subset$ C.
Hence, A $\subset$ B and B $\subset$ C $\Rightarrow$ A $\subset$ C.

(iv) **False.** Let A = {1, 2}, B = {2, 3} and C = {1, 2, 5}. Clearly, A $\not\subset$ B since 1 $\in$ A and 1 $\notin$ B. Also B $\not\subset$ C since 3 $\in$ B and 3 $\notin$ C. But A $\subset$ C.
Thus, A $\not\subset$ B and B $\not\subset$ C need not imply A $\not\subset$ C.

(v) **False.** Let A = {1, 2} and B = {2, 3}. Clearly, 1 $\in$ A and A $\not\subset$ B but 1 $\notin$ B.
Thus, $x \in$ A and A $\not\subset$ B need not imply $x \in$ B.

(vi) **True.** Let A $\subset$ B and $x \notin$ B. If possible, suppose $x \in$ A.
Now, $x \in$ A and A $\subset$ B $\Rightarrow x \in$ B which is a contradiction to given.
Therefore, our supposition is wrong. Hence $x \notin$ A.
Thus, A $\subset$ B and $x \notin$ B $\Rightarrow x \notin$ A.

3. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that B = C.

Sol. Let $x \in B$

$\Rightarrow x \in A \cup B$ [$\because$ $B \subset A \cup B$ always]

$\Rightarrow x \in A \cup C$ [$\because$ $A \cup B = A \cup C$ (given)]

$\Rightarrow x \in A$ or $x \in C$

Case I $x \in A$

$\Rightarrow x \in A$ and $x \in B$ [$\because$ we started with $x \in B$]

$\Rightarrow x \in A \cap B$

$\Rightarrow x \in A \cap C$ [$\because$ $A \cap B = A \cap C$ (given)]

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in C$ also

Case II $x \in C$

$\therefore$ In each case $x \in B \Rightarrow x \in C$

$\because$ $B \subset C$ ...(*i*)

Similarly, $C \subset B$ ...(*ii*)

From (*i*) and (*ii*), we have B = C.

4. Show that the following four conditions are equivalent:

(*i*) $A \subset B$ **(*ii*) $A - B = \phi$**

(*iii*) $A \cup B = B$ **(*iv*) $A \cap B = A$.**

Sol. Let us consider condition (*i*), *i.e.*, $A \subset B$ (given)

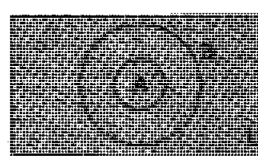
(*ii*) To prove $A - B = \phi$

If possible, let $A - B \neq \phi$

$\therefore$ Let $x \in A - B$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in B$ and $x \notin B$



($\because$ $A \subset B$)

which is absurd. So our supposition is wrong.

Hence, $A - B = \phi$.

(*iii*) Given $A \subset B$. To prove $A \cup B = B$

Let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$ $\Rightarrow x \in B$ or $x \in B$ [$\because$ $A \subset B$ (given)]

$\Rightarrow x \in B \Rightarrow A \cup B \subset B$

Also we know that $B \subset A \cup B$ (always)

$\therefore$ $A \cup B = B$.

(*iv*) To prove $A \cap B = A$,

We know that $A \cap B \subset A$ always.

Now let $x \in A$ $\therefore$ $x \in B$ [$\because A \subset B$ (given)]

$\therefore$ $x \in A$ and $x \in B$ $\Rightarrow$ $x \in A \cap B$

$\therefore$ $A \subset A \cap B$. Therefore, $A \cap B = A$.

Remark. Similarly, if we begin with any of the other three (*ii*), (*iii*) and (*iv*) as given; we can prove the remaining three.

For example, let us start with (*iii*) as given

i.e., $A \cup B = B$

$\therefore$ $A \subset B$ [$\because x \in A \Rightarrow x \in A \cup B \Rightarrow x \in B$]

$\therefore$ $A - B = \phi$

and $A \cap B = A$

Taking (*iv*) or (*ii*) as given and proving the remaining three are being left as an exercise for the reader.

5. Show that if $A \subset B$, then $C - B \subset C - A$.

Sol. Given $A \subset B$

Let $x \in (C - B) \Rightarrow x \in C$ and $x \notin B$

$\Rightarrow$ $x \in C$ and $x \notin A$ [$\because A \subset B$ (given)]

$\Rightarrow$ $x \in (C - A)$

$\Rightarrow$ $(C - B) \subset (C - A)$.

6. Assume that P(A) = P(B). Show that A = B.

Sol. $P(A) = P(B)$, *i.e.*, Power set of A = Power set of B ...(*i*)

To prove $A = B$.

Let $x \in A$.

$\therefore$ $\{x\} \subset A$

$\therefore$ $\{x\} \in P(A)$

$\therefore$ $\{x\} \in P(B)$ [$\because$ $P(A) = P(B)$ (given)]

$\therefore$ $\{x\} \subset B$

$\therefore$ $x \in B$

$\therefore$ $A \subset B$

Similarly, $B \subset A$

$\therefore$ $A = B$.

7. Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Sol. Let $A = \{1\}$, $B = \{2\}$, then

$P(A) = \{\phi, \{1\}\}$ and $P(B) = \{\phi, \{2\}\}$

$P(A) \cup P(B) = \{\phi, \{1\}\} \cup \{\phi, \{2\}\}$
$= \{\phi, \{1\}, \{2\}\}$

Also $A \cup B = \{1, 2\}$, then
$P(A \cup B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Now, $\{1, 2\} \in P(A \cup B)$ but $\{1, 2\} \notin P(A) \cup P(B)$

$\therefore$ $P(A) \cup P(B) \neq P(A \cup B)$.

8. Show that for any sets A and B,
$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = A \cup B$.

Sol. $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$
$= A \cap (B \cup B')$ [Distributive Law]
$= A \cap U = A$

Also $A \cup (B - A) = A \cup (B \cap A')$
$= (A \cup B) \cap (A \cup A')$ [Distributive Law]
$= (A \cup B) \cap U$
$= A \cup B$.

9. Using properties of sets, show that
(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$

Sol. (i) We know that $X \subset Y \Rightarrow X \cup Y = Y$, the super set
Here $A \cap B \subset A$ $\therefore$ $A \cup (A \cap B) = A$

(ii) We know that $X \subset Y \Rightarrow X \cap Y = X$, the subset
Here $A \subset A \cup B$ $\therefore$ $A \cap (A \cup B) = A$.

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Sol. Let $A = \{1, 2, 3\}$, $B = \{2, 4\}$, $C = \{2, 5\}$
Then $A \cap B = \{2\} = A \cap C$
But $B \neq C$.

11. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

Sol. Let $x \in A$
$\Rightarrow x \in A \cup X$ [$\because$ $A \subset A \cup X$ always]
$\Rightarrow x \in B \cup X$ [$\because$ $A \cup X = B \cup X$ (given)]
$\Rightarrow x \in B$ or $x \in X$

Case I. $x \in B$.

Case II. $x \in X$.

Also $x \in A$

$\Rightarrow \quad x \in A \cap X$

$\Rightarrow \quad x \in B \cap X \qquad [\because \ A \cap X = B \cap X \text{ (given)}]$

$\Rightarrow \quad x \in B$ and $x \in X$

$\Rightarrow \quad x \in B$ also

$\therefore$ In each case, $x \in B$.

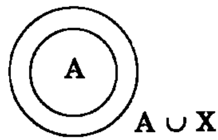
Therefore $A \subset B$.

Similarly $B \subset A$

$\therefore \quad A = B$.

Second solution (Using properties of sets)

We know that $A \subset$ Its superset $A \cup X$



$\therefore \quad A \cap (A \cup X) = A$

$\Rightarrow \quad A = A \cap (A \cup X)$

$\quad = A \cap (B \cup X) \qquad [\because \ A \cup X = B \cup X \text{ (given)}]$

$\Rightarrow \quad A = (A \cap B) \cup (A \cap X) \qquad$ [By Distributive Law]

$\quad = (A \cap B) \cup \phi \qquad [\because \ A \cap X = \phi \text{ (given)}]$

$\Rightarrow \quad A = (A \cap B) \qquad$ ... (*i*) (Law of ϕ)

Interchanging A and B in the above argument,

$B = B \cap A = A \cap B \qquad$ (*ii*) | Commutative Law

From (*i*) and (*ii*), we have $A = B$

12. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.

Sol. Let $A = \{x, y\}$, $B = \{x, z\}$, $C = \{y, z\}$

$A \cap B = \{x\} \neq \phi$, $B \cap C = \{z\} \neq \phi$, $A \cap C = \{y\} \neq \phi$

and $A \cap B \cap C = \phi$.

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Sol. Total students = 600

Let $n(T)$ = Number of students drinking tea = 150

$n(C)$ = Number of students drinking coffee = 225

$n(T \cap C) = n(\text{both tea and coffee}) = 100$

$\therefore \quad n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$= 150 + 225 - 100 = 275$

i.e., number of students who take at least one of the two drinks

$= 275$

$\therefore$ Number of students drinking neither tea nor coffee

$=$ Total number of students $- n(T \cup C)$

$= 600 - 275 = 325$.

14. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Sol. Let H denote the set of students who know Hindi and E denote the set of students who know English.

Then $n(\text{H}) = 100,\ n(\text{E}) = 50,\ n(\text{H} \cap \text{E}) = 25$

Using $n(\text{H} \cup \text{E}) = n(\text{H}) + n(\text{E}) - n(\text{H} \cap \text{E})$, we get

$$n(\text{H} \cup \text{E}) = 100 + 50 - 25 = 125$$

∴ There are 125 students in the group who know at least one of the two languages.

15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers

(ii) the number of people who read exactly one newspaper.

Sol. $n(\text{H})$ = number of people who read newspaper H

= 25 (given)

$n(\text{T}) = 26,\ n(\text{I}) = 26,$ (given)

$n(\text{H} \cap \text{I}) = 9,\ n(\text{H} \cap \text{T}) = 11,\ n(\text{T} \cap \text{I}) = 8$ (given)

$n(\text{H} \cap \text{T} \cap \text{I}) = 3$ (given)

(i) Number of persons who read at least one of the newspapers

$$= n(\text{H} \cup \text{T} \cup \text{I}) = n(\text{H}) + n(\text{T}) + n(\text{I}) - n(\text{H} \cap \text{T}) - n(\text{H} \cap \text{I}) - n(\text{I} \cap \text{T}) + n(\text{H} \cap \text{T} \cap \text{I})$$

$$= 25 + 26 + 26 - 9 - 11 - 8 + 3 = 52.$$

(ii) Number of people who read exactly one newspaper.

$$n[\text{only (H)}] = n(\text{H}) - n(\text{H} \cap \text{T}) - n(\text{H} \cap \text{I}) + n(\text{H} \cap \text{T} \cap \text{I})$$

$$= 25 - 11 - 9 + 3 = 8$$

$$n[\text{only (T)}] = n(\text{T}) - n(\text{T} \cap \text{H}) - n(\text{T} \cap \text{I}) + n(\text{T} \cap \text{H} \cap \text{I})$$

$$= 26 - 11 - 8 + 3 = 10$$

$$n[\text{only (I)}] = n(\text{I}) - n(\text{I} \cap \text{H}) - n(\text{I} \cap \text{T}) + n(\text{T} \cap \text{H} \cap \text{I})$$
$$= 26 - 8 - 9 + 3 = 12$$

$\therefore$ Number of persons who read exactly one newspaper

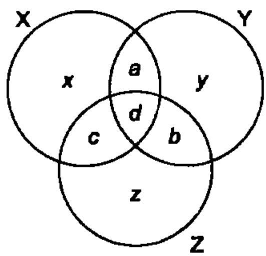
$$= n(\text{only H}) + n(\text{only T}) + n(\text{only I})$$
$$= 8 + 10 + 12 = 30.$$

16. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Sol. Let X = set of people who liked product A

Y = set of people who liked product B

Z = set of people who liked product C



Let a denote the number of people who liked products A and B only, b denote the number of people who liked products B and C only, c denote the number of people who liked products C and A only and d denote the number of people who liked all the three products. Let x, y and z denote the number of people who respectively liked products A, B and C only. Then

$$b + d = 14, \quad c + d = 12 \quad \text{and} \quad d = 8$$

$\Rightarrow \quad b = 6 \quad \text{and} \quad c = 4$

Since $\quad n(Z) = 29$

$\therefore \quad z + b + c + d = 29$

$\Rightarrow \quad z + 6 + 4 + 8 = 29$

$\Rightarrow \quad z = 29 - 18 = 11$

Hence, 11 people liked product C only.