

# 12

# Introduction to Three Dimensional Geometry

## Lesson at a Glance

### 1. Coordinate planes

The three planes determined by the three pairs of axes are the **coordinate planes** called YZ, ZX and XY-planes.

### 2. Octants. The three coordinate planes divide the space into eight parts known as **octants**.

These 8 octants are:

Octant I XOYZ; Octant II X'OYZ; Octant III X'OY'Z;  
Octant IV XOY'Z; Octant V XOYZ; Octant VI X'OYZ;  
Octant VII X'OY'Z; Octant VIII XOYZ'

### 3. Coordinates of a point

The coordinates of a point P in three dimensional geometry are always written in the form of a triplet like  $(x, y, z)$ .

Here  $x, y, z$  are the perpendicular distances of the point P from the three coordinate planes YZ, ZX and XY respectively.

### 4. Equations of coordinate planes

(i) The equation of YZ-plane is  $x = 0$ .

$\therefore$  Any point on YZ-plane is  $(0, y, z)$ .

(ii) The equation of ZX-plane is  $y = 0$ .

$\therefore$  Any point on ZX-plane is  $(x, 0, z)$

(iii) The equation of XY-plane is  $z = 0$ .

$\therefore$  Any point on XY-plane is  $(x, y, 0)$

### 5. Equation of coordinates axes

(i) The equations of  $x$ -axis are  $y = 0, z = 0$

$\therefore$  Any point on  $x$ -axis is  $(x, 0, 0)$ .

(ii) The equations of  $y$ -axis are  $z = 0, x = 0$

$\therefore$  Any point on  $y$ -axis is  $(0, y, 0)$ .

(iii) The equations of  $z$ -axis are  $x = 0, y = 0$ .

$\therefore$  Any point on  $z$ -axis is  $(0, 0, z)$ .

**6. Distances of a point from coordinate axes**

The perpendicular distance of the point  $P(x, y, z)$  from  $x$ -axis is  $\sqrt{y^2 + z^2}$ , from  $y$ -axis is  $\sqrt{x^2 + z^2}$  and from  $z$ -axis is  $\sqrt{x^2 + y^2}$ .

**7. Distance formula**

The distance between two points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  is given by  $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ .

**8. Section formula**

The coordinates of the point which divides the join of the points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  in the ratio:

(i)  $m_1 : m_2$  internally are

$$\left( \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}, \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2} \right).$$

(ii)  $m_1 : m_2$  externally are

$$\left( \frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}, \frac{m_1 z_2 - m_2 z_1}{m_1 - m_2} \right).$$

**9. Mid-point formula**

The coordinates of the mid-point of the line segment joining the

point  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  are  $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ .

**10. Centroid of a triangle**

If  $A(x_1, y_1, z_1)$ ,  $B(x_2, y_2, z_2)$  and  $C(x_3, y_3, z_3)$  are the vertices of a triangle, then coordinates of the centroid of the triangle (i.e., point of intersection of the medians) of a triangle  $ABC$  are

$$\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right).$$

**TEXTBOOK QUESTIONS SOLVED****EXERCISE 12.1 (Page No.: 271)**

1. A point is on the  $x$ -axis. What are its  $y$ -coordinate and  $z$ -coordinate?

**Sol.**  $y$  and  $z$ -coordinates of the point are zero.

[ $\because$  Coordinates of any point on  $x$ -axis are  $(x, 0, 0)$ ]

**2. A point is in the XZ-plane. What can you say about its  $y$ -coordinate?**

**Sol.** Any point on XZ-plane is  $(x, 0, z)$ . Therefore,  $y$ -coordinate of the point is zero.

**3. Name the octants in which the following points lie:**

$(1, 2, 3)$ ,  $(4, -2, 3)$ ,  $(4, -2, -5)$ ,  $(4, 2, -5)$ ,  $(-4, 2, -5)$ ,  
 $(-4, 2, 5)$ ,  $(-3, -1, 6)$ ,  $(2, -4, -7)$ .

**Sol.** (i) For the point  $(1, 2, 3)$

$x = 1 > 0$  is measured along OX

$y = 2 > 0$  is measured along OY

$z = 3 > 0$  is measured along OZ

Hence the point  $(1, 2, 3)$  lies in the octant XOYZ, *i.e.*, in octant I.

(ii) For the point  $(4, -2, 3)$

$x = 4 > 0$  is measured along OX

$y = -2 < 0$  is measured along OY'

$z = 3 > 0$  is measured along OZ

Hence the point  $(4, -2, 3)$  lies in the octant XOY'Z, *i.e.*, in octant IV.

(iii) For the point  $(4, -2, -5)$

$x = 4 > 0$  is measured along OX

$y = -2 < 0$  is measured along OY'

$z = -5 < 0$  is measured along OZ'

Hence the point  $(4, -2, -5)$  lies in the octant XOY'Z', *i.e.*, in octant VIII.

(iv) For the point  $(4, 2, -5)$

$x = 4 > 0$  is measured along OX

$y = 2 > 0$  is measured along OY

$z = -5 < 0$  is measured along OZ'

Hence the point  $(4, 2, -5)$  lies in the octant XOYZ', *i.e.*, in octant V.

(v) For the point  $(-4, 2, -5)$

$x = -4 < 0$  is measured along OX'

$y = 2 > 0$  is measured along OY

$z = -5 < 0$  is measured along OZ'

Hence the point  $(-4, 2, -5)$  lies in the octant  $X'OY'Z'$ ,  
i.e., in octant VI.

(vi) For the point  $(-4, 2, 5)$

$x = -4 < 0$  is measured along  $OX'$

$y = 2 > 0$  is measured along  $OY$

$z = 5 > 0$  is measured along  $OZ$

Hence the point  $(-4, 2, 5)$  lies in the octant  $X'OYZ$ ,  
i.e., in octant II.

(vii) For the point  $(-3, -1, 6)$

$x = -3 < 0$  is measured along  $OX'$

$y = -1 < 0$  is measured along  $OY'$

$z = 6 > 0$  is measured along  $OZ$

Hence the point  $(-3, -1, 6)$  lies in the octant  $X'OY'Z$ ,  
i.e., in octant III.

(viii) For the point  $(2, -4, -7)$

$x = 2 > 0$  is measured along  $OX$

$y = -4 < 0$  is measured along  $OY'$

$z = -7 < 0$  is measured along  $OZ'$

Hence the point  $(2, -4, -7)$  lies in the octant  $XOY'Z'$ ,  
i.e., in octant VIII.

#### 4. Fill in the blanks:

- (i) The  $x$ -axis and  $y$ -axis taken together determine a plane known as .....
- (ii) The coordinates of points in the  $XY$ -plane are of the form .....
- (iii) Coordinate planes divide the space into ..... octants.

Sol. (i)  $XY$ -plane (ii)  $(x, y, 0)$  (iii) 8.

### EXERCISE 12.2 (Page No.: 273)

1. Find the distance between the following pairs of points:

- (i)  $(2, 3, 5)$  and  $(4, 3, 1)$  (ii)  $(-3, 7, 2)$  and  $(2, 4, -1)$   
(iii)  $(-1, 3, -4)$  and  $(1, -3, 4)$  (iv)  $(2, -1, 3)$  and  $(-2, 1, 3)$ .

Sol. (i) The distance between the points  $P(2, 3, 5)$  and  $Q(4, 3, 1)$  by distance formula is,

$$\begin{aligned}PQ &= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} \\ &= \sqrt{4+0+16} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}.\end{aligned}$$

- (ii) The distance between the points P(-3, 7, 2) and Q(2, 4, -1) is

$$\begin{aligned}PQ &= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2} \\ &= \sqrt{25+9+9} = \sqrt{43}.\end{aligned}$$

- (iii) The distance between the points P(-1, 3, -4) and Q(1, -3, 4) is

$$\begin{aligned}PQ &= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2} \\ &= \sqrt{4+36+64} = \sqrt{104} = \sqrt{4 \times 26} = 2\sqrt{26}.\end{aligned}$$

- (iv) The distance between the points P(2, -1, 3) and Q(-2, 1, 3) is

$$\begin{aligned}PQ &= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2} \\ &= \sqrt{16+4+0} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}.\end{aligned}$$

**2. Show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear.**

**Sol.** Let P(-2, 3, 5), Q(1, 2, 3), R(7, 0, -1) be the given points.

$$\begin{aligned}\text{Distance } PQ &= \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} \\ &= \sqrt{14}\end{aligned}$$

$$\begin{aligned}QR &= \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} \\ &\quad \text{(By distance formula)}\end{aligned}$$

$$\begin{aligned}&= \sqrt{36+4+16} \\ &= \sqrt{56} = \sqrt{4 \times 14} = 2\sqrt{14}\end{aligned}$$

$$\begin{aligned}PR &= \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} \\ &= \sqrt{81+9+36}\end{aligned}$$

$$= \sqrt{126} = \sqrt{9 \times 14} = 3\sqrt{14}$$

Thus  $PQ + QR = \sqrt{14} + 2\sqrt{14} = (1 + 2)\sqrt{14} = 3\sqrt{14} = PR$ .

Hence points P, Q, R are collinear.

**3. Verify the following:**

- (i) (0, 7, -10), (1, 6, -6) and (4, 9, -6) are the vertices of an isosceles triangle.  
 (ii) (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled triangle.  
 (iii) (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram.

**Sol.** (i) Let A(0, 7, -10), B(1, 6, -6) and C(4, 9, -6) be the given points.

$$\begin{aligned} AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\ &= \sqrt{1+1+16} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\ &= \sqrt{9+9+0} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(4-0)^2 + (9-7)^2 + (-6+10)^2} \\ &= \sqrt{16+4+16} = \sqrt{36} = 6. \end{aligned}$$

Since  $AB = BC (=3\sqrt{2})$ , triangle ABC is isosceles.

(ii) Let A(0, 7, 10), B(-1, 6, 6) and C(-4, 9, 6) be the given points.

$$\begin{aligned} AB &= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} \\ &= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} \\ &= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} \\ &= \sqrt{16+4+16} = \sqrt{36} = 6 \end{aligned}$$

$$\text{Since } AB^2 + BC^2 = 18 + 18 = 36 = AC^2$$

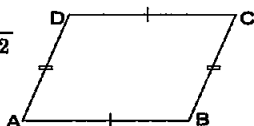
$\therefore \triangle ABC$  is right angled at B.

**Note:** Also,  $AB = BC \Rightarrow \triangle ABC$  is isosceles.

Hence,  $ABC$  is a right-angled isosceles triangle.

- (iii) Let  $A(-1, 2, 1)$ ,  $B(1, -2, 5)$ ,  $C(4, -7, 8)$  and  $D(2, -3, 4)$  be the given points.

$$\begin{aligned} AB &= \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2} \\ &= \sqrt{4+16+16} = \sqrt{36} = 6 \end{aligned}$$



$$\begin{aligned} DC &= \sqrt{(4-2)^2 + (-7+3)^2 + (8-4)^2} \\ &= \sqrt{4+16+16} = \sqrt{36} = 6 \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(2+1)^2 + (-3-2)^2 + (4-1)^2} \\ &= \sqrt{9+25+9} = \sqrt{43} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2} \\ &= \sqrt{9+25+9} = \sqrt{43} \end{aligned}$$

Since  $AB = DC (=6)$  and  $AD = BC (= \sqrt{43})$

$\Rightarrow$  The opposite sides of quadrilateral  $ABCD$  are equal.

$\Rightarrow$  Quadrilateral  $ABCD$  is a parallelogram.

- 4. Find the equation of the set of points which are equidistant from the points  $(1, 2, 3)$  and  $(3, 2, -1)$ .**

**Sol.** Let  $P(x, y, z)$  be equidistant from the points  $A(1, 2, 3)$  and  $B(3, 2, -1)$ .

$$\therefore AP = BP$$

$$\Rightarrow AP^2 = BP^2$$

$$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2$$

$$= (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$\Rightarrow (x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 - 6z + 9)$$

$$= (x^2 - 6x + 9) + (y^2 - 4y + 4) + (z^2 + 2z + 1)$$

$$\Rightarrow -2x + 6x - 6z - 2z + 14 - 14 = 0$$

$$\Rightarrow 4x - 8z = 0$$

Dividing every term by 4,  $x - 2z = 0$

which is the required equation of the set of points  $P(x, y, z)$ .

**5. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(-4, 0, 0) is equal to 10.**

**Sol.** Given points are A(4,0,0) and B(-4,0,0). Let the coordinates of P be (x, y, z).

$$\text{Given: } AP + BP = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2}$$

$$+ \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

Shifting one square root term to R.H.S;

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

Squaring both sides, we have

$$(x-4)^2 + y^2 + z^2 = 100 + (x+4)^2 + y^2 + z^2$$

$$- 20\sqrt{(x+4)^2 + y^2 + z^2}$$

$$\Rightarrow x^2 + 16 - 8x = 100 + x^2 + 16 + 8x$$

$$- 20\sqrt{(x+4)^2 + y^2 + z^2}$$

$$\Rightarrow -16x - 100 = -20\sqrt{(x+4)^2 + y^2 + z^2}$$

Dividing both sides by (-4), we get

$$4x + 25 = 5\sqrt{(x+4)^2 + y^2 + z^2}$$

Squaring both sides again, we have

$$16x^2 + 200x + 625 = 25[x^2 + 8x + 16 + y^2 + z^2]$$

$$\Rightarrow 16x^2 + 200x + 625 = 25x^2 + 200x + 400 + 25y^2 + 25z^2$$

$$\Rightarrow -9x^2 - 25y^2 - 25z^2 = -225$$

$$\text{multiplying by } -1, 9x^2 + 25y^2 + 25z^2 = 225$$

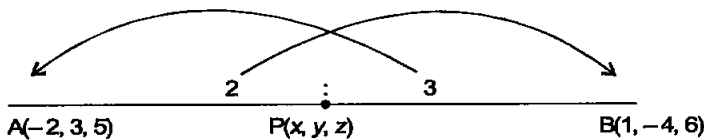
Which is required equation (i.e., locus) of set of points P.

### EXERCISE 12.3 (Page No.: 277)

- 1. Find the coordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) in the ratio (i) 2 : 3 internally, (ii) 2 : 3 externally.**



**Sol.** (i) Let  $P(x, y, z)$  be the point which divides the line segment joining  $A(-2, 3, 5)$  and  $B(1, -4, 6)$  internally in the ratio  $2 : 3$ .



Then, by section formula;

$$x = \frac{2(1) + 3(-2)}{2 + 3} = -\frac{4}{5}, y = \frac{2(-4) + 3(3)}{2 + 3} = \frac{1}{5}$$

$$z = \frac{2(6) + 3(5)}{2 + 3} = \frac{27}{5}$$

Therefore, the required point is  $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)$ .

(ii) Let  $P(x, y, z)$  be the point which divides the line segment joining  $A(-2, 3, 5)$  and  $B(1, -4, 6)$  externally in the ratio  $2 : 3$ . Then, by section formula

$$x = \frac{2(1) - 3(-2)}{2 - 3} = -8, y = \frac{2(-4) - 3(3)}{2 - 3} = 17$$

$$z = \frac{2(6) - 3(5)}{2 - 3} = 3$$

Therefore, the required point is  $(-8, 17, 3)$ .

**2. Given that  $P(3, 2, -4)$ ,  $Q(5, 4, -6)$  and  $R(9, 8, -10)$  are collinear. Find the ratio in which Q divides PR.**

**Sol.** Let Q divide PR in the ratio  $k : 1$ , then the coordinates of

$$Q \text{ are } \left( \frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1} \right).$$

$$\begin{array}{ccc} & k : 1 & \\ \hline P(3, 2, -4) & Q & R(9, 8, -10) \end{array}$$

But the coordinates of Q are given to be  $(5, 4, -6)$

Equating coordinates, we have

$$\therefore \frac{9k + 3}{k + 1} = 5 \text{ and } \frac{8k + 2}{k + 1} = 4 \text{ and } \frac{-10k - 4}{k + 1} = -6$$

Solving any one of these equations for  $k$ , say first, we have

on cross-multiplication,  $9k + 3 = 5k + 5$  or  $4k = 2 \therefore k = \frac{1}{2}$ .

Hence, Q divides PR in the ratio  $\frac{1}{2} : 1 = 1 : 2$ .

3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points  $(-2, 4, 7)$  and  $(3, -5, 8)$ .

Sol. Let the YZ-plane divide the line segment joining  $A(-2, 4, 7)$  and  $B(3, -5, 8)$  at  $P(x, y, z)$  in the ratio  $k : 1$ .

Then the coordinates of P

$$\left( \frac{3k - 2}{k + 1}, \frac{-5k + 4}{k + 1}, \frac{8k + 7}{k + 1} \right)$$

Since P lies on the YZ-plane, its  $x$ -coordinate is zero, i.e.,

$$\frac{3k - 2}{k + 1} = 0$$

cross-multiplying,  $3k - 2 = 0 \Rightarrow 3k = 2$

$$\text{or } k = \frac{2}{3} \text{ (+ve)}$$

Therefore, YZ-plane divides AB internally in the ratio  $\frac{2}{3} : 1$  i.e.,  $2 : 3$ .

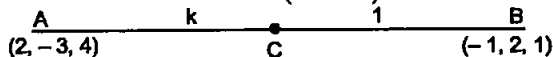
4. Using section formula, show that the points  $A(2, -3, 4)$ ,  $B(-1, 2, 1)$  and  $C\left(0, \frac{1}{3}, 2\right)$  are collinear.

Sol. Let the point C divide the line joining the points A and B in the ratio  $k : 1$ .

Then the coordinates of C are

$$\left( \frac{-k + 2}{k + 1}, \frac{2k - 3}{k + 1}, \frac{k + 4}{k + 1} \right)$$

$$\left( 0, \frac{1}{3}, 2 \right)$$



But coordinates of C are given  $\left(0, \frac{1}{3}, 2\right)$

Equating coordinates of C given and obtained, we have

$$\begin{array}{l} \frac{-k+2}{k+1} = 0, \quad \left| \quad \frac{2k-3}{k+1} = \frac{1}{3} \quad \left| \quad \frac{k+4}{k+1} = 2 \right. \\ \text{cross-multiplying} \quad -k+2=0 \quad \text{or} \quad 6k-9=k+1 \quad \text{or} \quad k+4=2k+2 \\ \text{or} \quad k=2 \quad \text{or} \quad 5k=10 \quad \text{or} \quad 2=k \\ \quad \quad \quad \quad \quad \quad \text{or} \quad k=2 \end{array}$$

Because the value of  $k$  from all the three equations is the same.

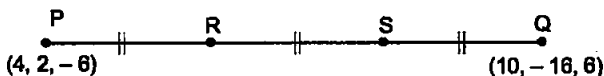
$\therefore$  Points A, B and C are collinear (and the point C divides AB in the ratio 2 : 1).

5. Find the coordinates of the points which trisect the line segment joining the points P(4, 2, -6) and Q(10, -16, 6).

**Sol.** Given: points P(4, 2, -6) and Q(10, -16, 6).

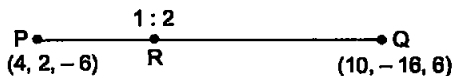
Let us take two points R and S within the segment PQ such that PR = RS = SQ.

Then points R and S are called **points of trisection** of the segment PQ.



$\therefore$  One point of trisection R divides the join of P and Q internally in the ratio 1 : 2 (= 1 + 1).

Point P is (4, 2, -6) and Q is (10, -16, 6).

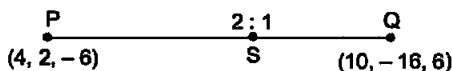


$\therefore$  Coordinates of point R are

$$R \left[ \frac{1(10) + 2(4)}{1+2}, \frac{1(-16) + 2(2)}{1+2}, \frac{1(6) + 2(-6)}{1+2} \right]$$

$$\text{i.e., } R \left( \frac{18}{3}, \frac{-12}{3}, \frac{-6}{3} \right) = (6, -4, -2)$$

Again the second point of trisection S divides the join of P and Q internally in the ratio of 2(= 1 + 1) : 1.



∴ Coordinates of point S are

$$S \left[ \frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)+1(-6)}{2+1} \right]$$

$$\text{i.e., } S \left( \frac{24}{3}, \frac{-30}{3}, \frac{6}{3} \right) = (8, -10, 2)$$

**Note.** We can find coordinates of point S by a second method also.

Point S is the mid-point of RQ

$$\begin{aligned} \therefore S \left( \frac{6+10}{2}, \frac{-4-16}{2}, \frac{-2+6}{2} \right) &= \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right) \\ &= (8, -10, 2). \end{aligned}$$

## MISCELLANEOUS EXERCISE ON CHAPTER 12

(Page No.: 278-279)

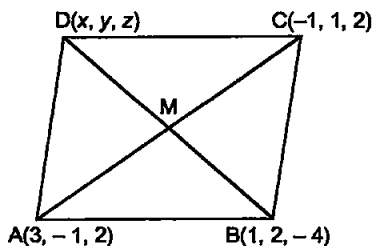
1. Three vertices of a parallelogram ABCD are A(3, -1, 2), B(1, 2, -4) and C(-1, 1, 2). Find the coordinates of the fourth vertex.

**Sol.** Let the coordinates of D be (x, y, z).

Let M be the point of intersection of the diagonals.

We know that diagonals of a  $\parallel^{\text{gm}}$  bisect each other.

(Here at point M)



Since M is the mid-point AC,

∴ Coordinates of M are

$$\left( \frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) = (1, 0, 2) \quad \dots(i)$$

Also M is the mid-point of BD

$$\therefore \text{Coordinates of M are } \left( \frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right) \quad \dots(ii)$$

From (i) and (ii) equating corresponding entries, we have

$$1 = \frac{x+1}{2}, \quad 0 = \frac{y+2}{2} \quad \text{and} \quad 2 = \frac{z-4}{2}$$

cross-multiplying  $x + 1 = 2$ ,  $y + 2 = 0$  and  $z - 4 = 4$

$$\Rightarrow \quad x = 1, \quad y = -2 \quad \text{and} \quad z = 8$$

Hence, vertex D is (1, -2, 8).

- 2. Find the lengths of the medians of the triangle with vertices A(0, 0, 6), B(0, 4, 0) and C(6, 0, 0).**

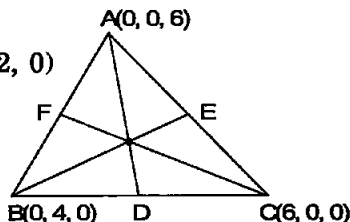
**Sol.** Let D, E, F be the mid-points of sides BC, CA, AB respectively. Then

$$D = \left( \frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) = (3, 2, 0)$$

$\therefore$  Length of median AD

$$= \sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}$$

$$= \sqrt{9+4+36} = \sqrt{49} = 7$$



$$E = \left( \frac{6+0}{2}, \frac{0+0}{2}, \frac{0+6}{2} \right) = (3, 0, 3)$$

$\therefore$  Length of median BE

$$= \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2}$$

$$= \sqrt{9+16+9} = \sqrt{34}$$

$$F = \left( \frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2} \right) = (0, 2, 3)$$

$\therefore$  Length of median CF

$$= \sqrt{(0-6)^2 + (2-0)^2 + (3-0)^2}$$

$$= \sqrt{36+4+9} = \sqrt{49} = 7.$$

- 3. If the origin is the centroid of the triangle PQR with vertices P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c), then find the values of a, b and c.**

**Sol.** By formula; centroid of the triangle PQR with vertices P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c) is

$$\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

$$\text{i.e.,} \quad \left( \frac{2a - 4 + 8}{3}, \frac{2 + 3b + 14}{3}, \frac{6 - 10 + 2c}{3} \right)$$

$$\text{i.e.,} \quad \left( \frac{2a + 4}{3}, \frac{3b + 16}{3}, \frac{2c - 4}{3} \right) = \text{origin} \quad (\text{given})$$

$$= (0, 0, 0)$$

Equating coordinates, we have

$$\begin{array}{l|l|l} \frac{2a + 4}{3} = 0, & \frac{3b + 16}{3} = 0, & \frac{2c - 4}{3} = 0 \\ \therefore 2a + 4 = 0, & 3b + 16 = 0, & 2c - 4 = 0 \\ \text{or } 2a = -4, & 3b = -16, & 2c = 4 \\ \therefore a = -2, & b = -\frac{16}{3}, & c = 2. \end{array}$$

- 4. Find the coordinates of points on y-axis which are at a distance of  $5\sqrt{2}$  from the point P(3, -2, 5).**

**Sol.** Let the required point on y-axis be A(0, y, 0).

Given: Point P (3, -2, 5) and distance AP =  $5\sqrt{2}$

$$\Rightarrow \sqrt{(0 - 3)^2 + (y + 2)^2 + (0 - 5)^2} = 5\sqrt{2}$$

$$\text{Squaring,} \quad 9 + (y + 2)^2 + 25 = 50$$

$$\Rightarrow (y + 2)^2 = 50 - 34 \quad \Rightarrow (y + 2)^2 = 16$$

$$\Rightarrow y + 2 = \pm 4 \Rightarrow y = -2 \pm 4 \Rightarrow y = 2 \text{ or } -6$$

Therefore, the required points are (0, y, 0) = (0, 2, 0) and (0, -6, 0).

- 5. A point R with x-coordinate 4 lies on the line segment joining the points P(2, -3, 4) and Q(8, 0, 10). Find the coordinates of the point R.**

**Sol.** Since R lies on PQ, therefore, R divides PQ in some ratio, say  $k : 1$ .

$$\therefore \text{By section formula, R has coordinates} \left( \frac{8k + 2}{k + 1}, \frac{0 - 3}{k + 1}, \frac{10k + 4}{k + 1} \right)$$

...(i)

$x$ -coordinate of R is given to be 4

$$\therefore \frac{8k+2}{k+1} = 4 \quad \Rightarrow \quad 8k+2 = 4(k+1)$$

$$\Rightarrow \quad 8k+2 = 4k+4 \quad \Rightarrow \quad 4k = 2 \quad \therefore \quad k = \frac{1}{2}$$

Putting this value of  $k$  in (i), the required coordinates of R are

$$\left( 4, \frac{-3}{1/2+1}, \frac{10 \times 1/2 + 4}{1/2+1} \right) = \left( 4, \frac{-3}{\left(\frac{3}{2}\right)}, \frac{9}{\left(\frac{3}{2}\right)} \right) = (4, -2, 6).$$

6. If A and B be the points (3, 4, 5) and (-1, 3, -7) respectively, find the equation of the set of points P such that  $PA^2 + PB^2 = k^2$ , where  $k$  is a constant.

**Sol.** Let the coordinates of P be  $(x, y, z)$ .

Given: Points A(3, 4, 5) and B(-1, 3, -7) and  $PA^2 + PB^2 = k^2$

$$\Rightarrow [(x-3)^2 + (y-4)^2 + (z-5)^2]$$

$$+ [(x+1)^2 + (y-3)^2 + (z+7)^2] = k^2$$

$$\Rightarrow (x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 10z + 25)$$

$$+ (x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 14z + 49) = k^2$$

$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$

$$\Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$$

$$\text{Dividing by 2, } x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}.$$

Which is the required equation of set of points P. □□