

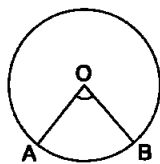
# 3



# Trigonometric Functions

## Lesson at a Glance

1. **Definition: Radian.** Radian is an angle subtended at the centre of the circle by an arc length equal to the radius of the circle. In the adjoining figure,



$\angle AOB = 1$  Radian where arc  $AB =$  Radius of the circle and  $O$  is the centre of the circle.

2. In a circle of radius  $r$ , if an arc of length  $l$  subtends an angle  $\theta$  radians at the centre, then  $l = r\theta$ .

3. (a) Radian measure = Degree measure  $\times \frac{\pi}{180^\circ}$ .

( $\because 180^\circ = \pi$  Radians).

- (b) Degree measure = Radian measure  $\times \frac{180^\circ}{\pi}$ .

( $\because \pi$  Radians =  $180^\circ$ )

- (c)  $1^\circ = 60$  minutes (=  $60'$ ) and 1 minute = 60 seconds (=  $60''$ )

4. (a)  $\operatorname{cosec} x = \frac{1}{\sin x}$  (b)  $\sec x = \frac{1}{\cos x}$  (c)  $\cot x = \frac{1}{\tan x}$ .

5. (a)  $\tan x = \frac{\sin x}{\cos x}$  (b)  $\cot x = \frac{\cos x}{\sin x}$ .

6. (i)  $\cos^2 x + \sin^2 x = 1$ .  
(ii)  $\sec^2 x - \tan^2 x = 1$ .  
(iii)  $\operatorname{cosec}^2 x - \cot^2 x = 1$ . ] square relations

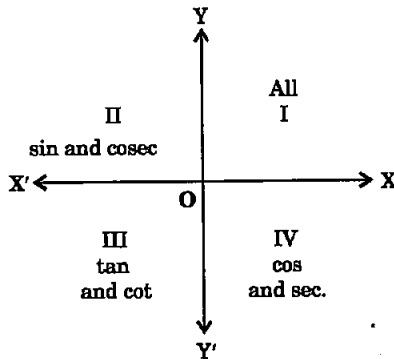
7. (i)  $-1 \leq \sin x \leq 1$  and  $-1 \leq \cos x \leq 1$  i.e.  $\sin x$  and  $\cos x \in [-1, 1]$ .

- (ii)  $\sec x \leq -1$  or  $\geq 1$  and  $\operatorname{cosec} x \leq -1$  or  $\geq 1$   
i.e.  $\sec x$  and  $\operatorname{cosec} x \in \mathbb{R} - (-1, 1)$

- (iii)  $\tan x$  and  $\cot x$  can assume all real values

8. (i) Period of  $\sin x$ ,  $\cos x$ ,  $\sec x$  and  $\operatorname{cosec} x$  is  $2\pi$   
 (ii) Period of  $\tan x$  and  $\cot x$  is  $\pi$ .

9. Rule. ASTC



10. Three important rules for  $t$ -ratios of allied angles:

**Rule 1.** *T-ratios of  $(2n\pi + \theta)$  or  $(n \times 360^\circ + \theta)$  are the same as those of  $\theta$ ,  $n$  being an integer.*

For example,  $\sin(6\pi + \theta) = \sin(3 \times 2\pi + \theta) = \sin \theta$ ,  
 $\cos(8 \times 360^\circ + \theta) = \cos \theta$  and  $\tan(5 \times 360^\circ + 45^\circ) = \tan 45^\circ = 1$ .

Thus, any number of revolutions, i.e.,  $n \times 2\pi$ ,  $n \in \mathbb{Z}$  can be added to or taken away from an angle without affecting its  $t$ -ratios.

**Rule 2.** *To write down the  $t$ -ratios of an angle allied to  $\theta$ .*

(i) Assume  $\theta$  to be a positive acute angle (even if not so, because the form of the result is the same for all values of  $\theta$ ) and determine the quadrant in which the allied angle lies and

**Use the rule 'All-sin-tan-cos' to find the sign of the  $t$ -ratios in this quadrant.**

Thus,

$90^\circ - \theta$ ,  $360^\circ + \theta$  lie in 1st quadrant.

$90^\circ + \theta$ ,  $180^\circ - \theta$  lie in 2nd quadrant.

$180^\circ + \theta$ ,  $270^\circ - \theta$  lie in 3rd quadrant.

$-\theta$ ,  $360^\circ - \theta$ ,  $270^\circ + \theta$  lie in 4th quadrant.

The quadrant of the above angles can easily be learnt from the given figure on next page.

(ii) If the allied angle is  $-\theta$ ,  $180^\circ \pm \theta$ ,  $360^\circ \pm \theta$ , etc. (i.e.,  $n \times 90^\circ \pm \theta$  where  $n$  is an even integer), the  $t$ -ratio is not altered.

(iii) If the allied angle is  $90^\circ \pm \theta$ ,  $270^\circ \pm \theta$ , etc. (i.e.,  $n \times 90^\circ \pm \theta$ ) where  $n$  is an odd integer, the  $t$ -ratio changes.

Thus,  $\sin \rightleftharpoons \cos$ ,

$\tan \rightleftharpoons \cot$ ,  $\sec \rightleftharpoons \operatorname{cosec}$ .

Hence, add 'co' if absent and remove 'co' if present.

**Remark.** While applying Rule 2,

Firstly apply rule 2 (i) namely All-sin-tan-cos and then apply rule 2 (ii) or rule 2 (iii).

**Rule 3.** To express  $t$ -ratios of any negative angle in terms of those of a positive acute angle.

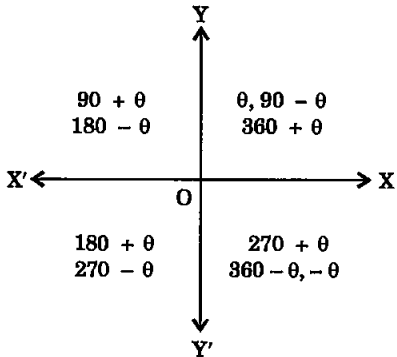
If the angle is negative, make it positive by using the formulae for  $t$ -ratios of  $-\theta$  (of course we know from the above figure that  $-\theta$  lies in 4th quadrant).

$\cos(-\theta) = \cos \theta$ ,  $\sec(-\theta) = \sec \theta$ ,  $\sin(-\theta) = -\sin \theta$ ,

$\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$ ,  $\tan(-\theta) = -\tan \theta$ ,  $\cot(-\theta) = -\cot \theta$ .

Now apply Rules I and II

11. (a)  $\sin(x + y) = \sin x \cos y + \cos x \sin y$   
(b)  $\sin(x - y) = \sin x \cos y - \cos x \sin y$ .
12. (a)  $\cos(x + y) = \cos x \cos y - \sin x \sin y$   
(b)  $\cos(x - y) = \cos x \cos y + \sin x \sin y$ .
13. (a)  $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$   
(b)  $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$ .  
(c)  $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$



$$(d) \cot (A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$14. (a) \tan \left( \frac{\pi}{4} + x \right) = \frac{1 + \tan x}{1 - \tan x}$$

$$(b) \tan \left( \frac{\pi}{4} - x \right) = \frac{1 - \tan x}{1 + \tan x}$$

$$15. \sin (x + y) \sin (x - y) = \sin^2 x - \sin^2 y.$$

$$16. \cos (x + y) \cos (x - y) = \cos^2 x - \sin^2 y.$$

$$17. 2 \sin A \cos B = \sin (A + B) + \sin (A - B).$$

$$18. 2 \cos A \sin B = \sin (A + B) - \sin (A - B).$$

$$19. 2 \cos A \cos B = \cos (A + B) + \cos (A - B).$$

$$20. 2 \sin A \sin B = \cos (A - B) - \cos (A + B).$$

$$21. \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}.$$

$$22. \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}.$$

$$23. \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}.$$

$$24. \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}.$$

$$= 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}.$$

(C - D  
Formulae)

$$25. \sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}.$$

$$26. \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$= \frac{1 - \tan^2 x}{1 + \tan^2 x}.$$

$$27. \text{From result 26; } \sin^2 x = \frac{1 - \cos 2x}{2} \quad \left| \because \cos 2x = 1 - 2 \sin^2 x \right.$$

$$\text{and } \cos^2 x = \frac{1 + \cos 2x}{2} \quad \left| \because \cos 2x = 2 \cos^2 x - 1 \right.$$

$$28. \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$

$$29. \quad \sin 3x = 3 \sin x - 4 \sin^3 x.$$

$$30. \quad \cos 3x = 4 \cos^3 x - 3 \cos x.$$

$$31. \quad \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.$$

$$32. \quad \sin 18^\circ = \frac{\sqrt{5}-1}{4}.$$

$$33. \quad \cos 36^\circ = \frac{\sqrt{5}+1}{4}.$$

$$34. \quad \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}.$$

$$35. \quad \sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}.$$

**Formulae from 36 to 42 are for general solutions (i.e. values of the variable from  $-\infty$  to  $\infty$  satisfying a given T-equation).**

$$36. \quad \sin x = 0 \quad \Rightarrow \quad x = n\pi, \text{ where } n \in \mathbb{Z}.$$

$$37. \quad \cos x = 0 \quad \Rightarrow \quad x = (2n + 1) \frac{\pi}{2}, \text{ where } n \in \mathbb{Z}.$$

$$38. \quad \tan x = 0 \quad \Rightarrow \quad x = n\pi, \text{ where } n \in \mathbb{Z}.$$

$$39. \quad \sin x = \sin y \quad \Rightarrow \quad x = n\pi + (-1)^n y, \text{ where } n \in \mathbb{Z}.$$

$$40. \quad \cos x = \cos y \quad \Rightarrow \quad x = 2n\pi \pm y, \text{ where } n \in \mathbb{Z}.$$

$$41. \quad \tan x = \tan y \quad \Rightarrow \quad x = n\pi + y, \text{ where } n \in \mathbb{Z}.$$

$$42. \quad \left. \begin{array}{l} \sin^2 x = \sin^2 y \\ \cos^2 x = \cos^2 y \\ \tan^2 x = \tan^2 y \end{array} \right\} \Rightarrow x = n\pi \pm y, \text{ where } n \in \mathbb{Z}.$$

43. To solve the classic equation  $a \cos x \pm b \sin x = c$ , divide both sides by  $\sqrt{a^2 + b^2}$ .

**In any triangle ABC,**

$$44. \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}. \quad (\text{Sine formula})$$

$$45. \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}. \quad (\text{Cosine formulae})$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

$$46. \quad a = b \cos C + c \cos B.$$

$$b = c \cos A + a \cos C.$$

$$c = a \cos B + b \cos A.$$

(Projection formulae)

## TEXTBOOK QUESTIONS SOLVED

### EXERCISE 3.1 (Page No.: 54–55)

1. Find the radian measures corresponding to the following degree measures:

(i)  $25^\circ$       (ii)  $-47^\circ 30'$       (iii)  $240^\circ$       (iv)  $520^\circ$ .

Sol. Since  $180^\circ = \pi$  radians  $\therefore 1^\circ = \frac{\pi}{180}$  radians

$$(i) \quad 25^\circ = 25 \times \frac{\pi}{180} \text{ radians} = \frac{5\pi}{36} \text{ radians}$$

$$\therefore \text{Radian measure of } 25^\circ \text{ is } \frac{5\pi}{36}.$$

$$(ii) \quad -47^\circ 30' = -47 \frac{1}{2}^\circ = -\frac{95}{2} \times \frac{\pi}{180} \text{ radians}$$

$$\left( \because 30' = \left( \frac{30}{60} \right)^\circ = \frac{1}{2}^\circ \right)$$

$$= -\frac{19\pi}{72} \text{ radians}$$

$$\therefore \text{Radian measure of } -47^\circ 30' \text{ is } -\frac{19\pi}{72}.$$

$$(iii) \quad 240^\circ = 240 \times \frac{\pi}{180} \text{ radians} = \frac{4\pi}{3} \text{ radians}$$

$$\therefore \text{Radian measure of } 240^\circ \text{ is } \frac{4\pi}{3}.$$

$$(iv) 520^\circ = 520 \times \frac{\pi}{180} \text{ radians} = \frac{26\pi}{9} \text{ radians}$$

$$\therefore \text{Radian measure of } 520^\circ \text{ is } \frac{26\pi}{9}.$$

**2. Find the degree measures corresponding to the following radian measure: (Use  $\pi = \frac{22}{7}$ )**

$$(i) \frac{11}{16}$$

$$(ii) -4$$

$$(iii) \frac{5\pi}{3}$$

$$(iv) \frac{7\pi}{6}$$

**Sol.** Since  $\pi$  radians =  $180^\circ$   $\therefore 1$  radian =  $\frac{180^\circ}{\pi}$

$$(i) \frac{11}{16} \text{ radians} = \frac{11}{16} \times \frac{180^\circ}{\pi}$$

$$= \left( \frac{11 \times 45}{4} \times \frac{7}{22} \right)^\circ = \frac{315^\circ}{8}$$

$$\begin{array}{r} 39^\circ \\ 8 \overline{) 315} \\ \underline{24} \phantom{0} \\ 75 \\ \underline{72} \\ 3^\circ \\ \times 60 \\ \hline 180' \\ 8 \overline{) 180 \cancel{) 22}'} \\ \underline{16} \phantom{0} \\ 20 \\ \underline{16} \\ 4 \times 60 \\ \hline 8 \overline{) 240 \cancel{) 30}''} \\ \underline{240} \\ \times \end{array}$$

**Ans.**  $39^\circ, 22', 30''$

$$(ii) -4 \text{ radians} = -4 \times \frac{180^\circ}{\pi} = \left( -720 \times \frac{7}{22} \right)^\circ$$

$$= -\frac{2520^\circ}{11}$$

$$\begin{array}{r}
 229^{\circ} \\
 11\sqrt{2520} \\
 \underline{22} \\
 32 \\
 \underline{22} \\
 100 \\
 \underline{99} \\
 10 \\
 \times 60 \\
 11\sqrt{60'} \downarrow 5' \\
 \underline{55} \\
 5' \\
 \times 60 \\
 11\sqrt{300'} \downarrow 27'' \\
 \underline{22} \\
 80' \\
 \underline{77} \\
 3
 \end{array}$$

Ans.  $229^{\circ}$ ,  $5'$ ,  $27''$  approximately.

$$(iii) \quad \frac{5\pi}{3} \text{ radians} = \frac{5\pi}{3} \times \frac{180^{\circ}}{\pi} = 300^{\circ}$$

$$(iv) \quad \frac{7\pi}{6} \text{ radians} = \frac{7\pi}{6} \times \frac{180^{\circ}}{\pi} = 210^{\circ}.$$

- 3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?**

**Sol.** Number of revolutions in one minute = 360

$$\Rightarrow \text{Number of revolutions in one second} = \frac{360}{60} = 6$$

Since 1 revolution =  $360^{\circ} = 2\pi$  radians

$$\therefore 6 \text{ revolutions} = 6 \times 2\pi = 12\pi \text{ radians}$$

$$\Rightarrow \text{Number of radians turned in one second} = 12\pi.$$

- 4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of**

**length 22 cm.  $\left( \text{Use } \pi = \frac{22}{7} \right)$**

**Sol.** Here,  $l$  = length of arc = 22 cm

$$r = \text{radius of circle} = 100 \text{ cm}$$

Let  $\theta$  be the angle subtended at the centre, then



$$\begin{aligned}\theta &= \frac{l}{r} \text{ radians} \\ &= \frac{22}{100} \text{ radians} = \frac{11}{50} \times \frac{180^\circ}{\pi} \\ &= \left( \frac{11 \times 18}{5} \times \frac{7}{22} \right)^\circ = \frac{63^\circ}{5} \\ &\quad \frac{12^\circ}{\sqrt[5]{63}} \\ &\quad \frac{60}{3} \\ &\quad \times 60 \\ &\quad \sqrt[5]{180^\circ} 36' \\ &\quad \frac{15}{30} \\ &\quad \frac{30}{30} \\ &\quad \times\end{aligned}$$

Ans.  $12^\circ, 36'$

5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Sol. Radius of circle =  $\frac{1}{2} \times 40 \text{ cm} = 20 \text{ cm}$

Chord AB = 20 cm

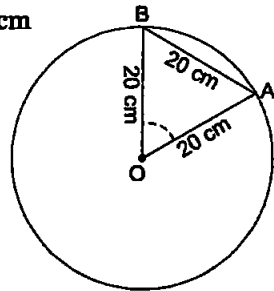
$\therefore$  In triangle OAB,

OA = OB = AB

$\Rightarrow$  Triangle is equilateral

$\Rightarrow$  Each angle =  $60^\circ$

Let arc AB =  $l$  cm.



Now,  $\theta = \angle AOB = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$  radians

$r = 20 \text{ cm}$

$\therefore l = r\theta = 20 \times \frac{\pi}{3} \text{ cm}$  [ $l$  and  $r$  have same units]

$= \frac{20\pi}{3} \text{ cm.}$

6. If in two circles, arcs of the same length subtend angles  $60^\circ$  and  $75^\circ$  at the centre, find the ratio of their radii.

**Sol.** Let the radii of the two circles be  $r_1$  and  $r_2$  respectively.  
Also, let the length of arc in each case be  $l$ .

[ $\therefore$  Arcs are of same length (given)]

$$\text{For the first circle, } \theta = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3} \text{ radians}$$

$$\text{We know that } l = r\theta$$

$$\therefore r = \frac{l}{\theta}$$

$$\Rightarrow r_1 = \frac{l}{\pi/3} = \frac{3l}{\pi} \quad \dots(i)$$

$$\text{For the second circle, } \theta = 75^\circ = 75 \times \frac{\pi}{180} = \frac{5\pi}{12} \text{ radians.}$$

$$\therefore r_2 = \frac{l}{\theta} = \frac{l}{5\pi/12} = \frac{12l}{5\pi} \quad \dots(ii)$$

$$\text{Dividing (i) by (ii), we get } \frac{r_1}{r_2} = \frac{3l}{\pi} \times \frac{5\pi}{12l} = \frac{5}{4}$$

$$\therefore r_1 : r_2 = 5 : 4.$$

**7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length:**

(i) 10 cm

(ii) 15 cm

(iii) 21 cm.

**Sol.** [If one end of an inelastic string is attached to a fixed point and to the other end is attached a heavy particle (called **bob**), then this system is called a pendulum. Length of the string is called the length of the pendulum.]

Here,  $r = 75$  cm.

$$(i) l = 10 \text{ cm}$$

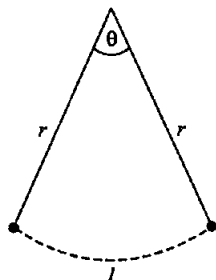
$$\therefore \theta = \frac{l}{r} = \frac{10}{75} = \frac{2}{15} \text{ radians.}$$

$$(ii) l = 15 \text{ cm}$$

$$\therefore \theta = \frac{l}{r} = \frac{15}{75} = \frac{1}{5} \text{ radians.}$$

$$(iii) l = 21 \text{ cm}$$

$$\therefore \theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25} \text{ radians.}$$



**EXERCISE 3.2** (Page No.: 63)

**Find the values of other five trigonometric functions in Exercises 1 to 5.**

1.  $\cos x = -\frac{1}{2}$ ,  $x$  lies in third quadrant.

**Sol.** Given,  $\cos x = -\frac{1}{2}$ ,  $x$  lies in third quadrant.

$$\therefore \sec x = \frac{1}{\cos x} = -2 \quad \dots(i)$$

$$\text{Now, } \sin^2 x = 1 - \cos^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$$

Since  $x$  lies in third quadrant,  $\sin x$  will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2} \quad \dots(ii)$$

$$\text{and } \operatorname{cosec} x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}} \quad \dots(iii)$$

$$\text{Also, } \tan x = \frac{\sin x}{\cos x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3} \quad \dots(iv)$$

$$\text{and } \cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}} \quad \dots(v)$$

2.  $\sin x = \frac{3}{5}$ ,  $x$  lies in second quadrant.

**Sol.** Given,  $\sin x = \frac{3}{5}$ ,  $x$  lies in second quadrant.

$$\therefore \operatorname{cosec} x = \frac{1}{\sin x} = \frac{5}{3} \quad \dots(i)$$

$$\text{Now, } \cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm \frac{4}{5}$$

Since  $x$  lies in second quadrant,  $\cos x$  will be negative.

$$\therefore \cos x = -\frac{4}{5} \quad \dots(ii)$$

$$\text{and } \sec x = \frac{1}{\cos x} = -\frac{5}{4} \quad \dots(iii)$$

$$\text{Also, } \tan x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4} \quad \dots(iv)$$

$$\text{and } \cot x = \frac{1}{\tan x} = -\frac{4}{3} \quad \dots(v)$$

**3.  $\cot x = \frac{3}{4}$ ,  $x$  lies in third quadrant.**

**Sol.** Given,  $\cot x = \frac{3}{4}$ ,  $x$  lies in third quadrant.

$$\therefore \tan x = \frac{1}{\cot x} = \frac{4}{3} \quad \dots(i)$$

$$\text{Now, } \operatorname{cosec}^2 x = 1 + \cot^2 x = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\Rightarrow \operatorname{cosec} x = \pm \frac{5}{4}$$

Since  $x$  lies in third quadrant,  $\operatorname{cosec} x$  will be negative.

$$\therefore \operatorname{cosec} x = -\frac{5}{4} \quad \dots(ii)$$

$$\text{and } \sin x = \frac{1}{\operatorname{cosec} x} = -\frac{4}{5} \quad \dots(iii)$$

$$\text{Also, } \cos x = \frac{\cos x}{\sin x} \cdot \sin x = \cot x \sin x$$

$$= \frac{3}{4} \left( -\frac{4}{5} \right) = -\frac{3}{5} \quad \dots(iv)$$

$$\text{and } \sec x = \frac{1}{\cos x} = -\frac{5}{3} \quad \dots(v)$$

4.  $\sec x = \frac{13}{5}$ ,  $x$  lies in fourth quadrant.

Sol. Given,  $\sec x = \frac{13}{5}$ ,  $x$  lies in fourth quadrant.

$$\therefore \cos x = \frac{1}{\sec x} = \frac{5}{13} \quad \dots(i)$$

$$\text{Now, } \sin^2 x = 1 - \cos^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since  $x$  lies in fourth quadrant,  $\sin x$  will be negative.

$$\therefore \sin x = -\frac{12}{13} \quad \dots(ii)$$

$$\text{and } \operatorname{cosec} x = \frac{1}{\sin x} = -\frac{13}{12} \quad \dots(iii)$$

$$\text{Also, } \tan x = \frac{\sin x}{\cos x} = \frac{-\frac{12}{13}}{\frac{5}{13}} = -\frac{12}{5} \quad \dots(iv)$$

$$\text{and } \cot x = \frac{1}{\tan x} = -\frac{5}{12} \quad \dots(v)$$

5.  $\tan x = -\frac{5}{12}$ ,  $x$  lies in second quadrant.

Sol. Given,  $\tan x = -\frac{5}{12}$ ,  $x$  lies in second quadrant.

$$\therefore \cot x = \frac{1}{\tan x} = -\frac{12}{5} \quad \dots(i)$$

$$\text{Now, } \sec^2 x = 1 + \tan^2 x = 1 + \frac{25}{144} = \frac{169}{144}$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since  $x$  lies in second quadrant,  $\sec x$  will be negative.

$$\therefore \sec x = -\frac{13}{12} \quad \dots(ii)$$

$$\text{and } \cos x = \frac{1}{\sec x} = -\frac{12}{13} \quad \dots(iii)$$

$$\begin{aligned} \text{Also, } \sin x &= \frac{\sin x}{\cos x} \cdot \cos x = \tan x \cos x \\ &= \left(-\frac{5}{12}\right) \left(-\frac{12}{13}\right) = \frac{5}{13} \end{aligned} \quad \dots(iv)$$

$$\text{and cosec } x = \frac{1}{\sin x} = \frac{13}{5}. \quad \dots(v)$$

**Find the values of the trigonometric functions in Exercises 6 to 10.**

**6.  $\sin 765^\circ$ .**

$$\begin{aligned} \text{Sol. } \sin 765^\circ &= \sin (2 \times 360^\circ + 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}} \\ &[\because \sin (n \times 360^\circ + x) = \sin x, n \in \mathbb{Z}] \end{aligned}$$

**7. cosec  $(-1410^\circ)$ .**

$$\begin{aligned} \text{Sol. cosec } (-1410^\circ) &= \text{cosec } (4 \times 360^\circ - 1410^\circ) \\ &= \text{cosec } (1440^\circ - 1410^\circ) = \text{cosec } 30^\circ = 2 \\ &[\because \text{cosec } x = \text{cosec } (n \times 360^\circ + x), n \in \mathbb{Z}] \end{aligned}$$

**8.  $\tan \frac{19\pi}{3}$ .**

$$\begin{aligned} \text{Sol. } \tan \frac{19\pi}{3} &= \tan \left(\frac{18\pi + \pi}{3}\right) = \tan \left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3} \\ &[\because \tan (n\pi + x) = \tan x, n \in \mathbb{Z}] \end{aligned}$$

**9.  $\sin \left(-\frac{11\pi}{3}\right)$ .**

$$\begin{aligned} \text{Sol. } \sin \left(-\frac{11\pi}{3}\right) &= \sin \left(4\pi - \frac{11\pi}{3}\right) \\ &[\because \sin x = \sin (2n\pi + x), n \in \mathbb{Z}] \\ &= \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}. \end{aligned}$$

**10.  $\cot \left(-\frac{15\pi}{4}\right)$ .**

$$\begin{aligned} \text{Sol. } \cot \left(-\frac{15\pi}{4}\right) &= \cot \left(4\pi - \frac{15\pi}{4}\right) \\ &[\because \cot x = \cot (n\pi + x), n \in \mathbb{Z}] \\ &= \cot \frac{\pi}{4} = 1. \end{aligned}$$

**EXERCISE 3.3** (Page No.: 73–74)

**1. Prove that:**  $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$ .

**Sol.** We know that  $\sin \frac{\pi}{6} = \frac{1}{2}$ ,  $\cos \frac{\pi}{3} = \frac{1}{2}$ ,  $\tan \frac{\pi}{4} = 1$

$$\begin{aligned} \therefore \text{L.H.S.} &= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 1^2 = \frac{1}{4} + \frac{1}{4} - 1 \\ &= \frac{1}{2} - 1 = -\frac{1}{2} = \text{R.H.S.} \end{aligned}$$

**2. Prove that:**  $2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$ .

**Sol.** We know that  $\sin \frac{\pi}{6} = \frac{1}{2}$

$$\begin{aligned} \operatorname{cosec} \frac{7\pi}{6} &= \operatorname{cosec} \left(\frac{6\pi + \pi}{6}\right) = \operatorname{cosec} \left(\pi + \frac{\pi}{6}\right) \\ &= -\operatorname{cosec} \frac{\pi}{6} = -2, \quad \cos \frac{\pi}{3} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \therefore \text{L.H.S.} &= 2\left(\frac{1}{2}\right)^2 + (-2)^2 \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{2} + 1 = \frac{3}{2} = \text{R.H.S.} \end{aligned}$$

**3. Prove that:**  $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = 6$ .

**Sol.** We know that  $\cot \frac{\pi}{6} = \sqrt{3}$

$$\begin{aligned} \operatorname{cosec} \frac{5\pi}{6} &= \operatorname{cosec} \left(\frac{6\pi - \pi}{6}\right) = \operatorname{cosec} \left(\pi - \frac{\pi}{6}\right) \\ &= \operatorname{cosec} \frac{\pi}{6} = 2, \quad \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned}\therefore \text{L.H.S.} &= (\sqrt{3})^2 + 2 + 3 \left( \frac{1}{\sqrt{3}} \right)^2 = 3 + 2 + 1 \\ &= 6 = \text{R.H.S.}\end{aligned}$$

**4. Prove that:  $2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} = 10$ .**

**Sol.** We know that  $\sin \frac{3\pi}{4} = \sin \left( \frac{4\pi - \pi}{4} \right) = \sin \left( \pi - \frac{\pi}{4} \right)$

$$= \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \sec \frac{\pi}{3} = 2$$

$$\begin{aligned}\therefore \text{L.H.S.} &= 2 \left( \frac{1}{\sqrt{2}} \right)^2 + 2 \left( \frac{1}{\sqrt{2}} \right)^2 + 2(2)^2 \\ &= 1 + 1 + 8 = 10 = \text{R.H.S.}\end{aligned}$$

**5. Find the value of:**

(i)  $\sin 75^\circ$

(ii)  $\tan 15^\circ$ .

**Sol.** (i)  $\sin 75^\circ = \sin (45^\circ + 30^\circ)$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

[ $\because \sin (x + y) = \sin x \cos y + \cos x \sin y$ ]

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

(ii)  $\tan 15^\circ = \tan (45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

$$\left[ \because \tan (x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

**Rationalising**

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2}$$



$$= \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}.$$

6. Prove that:

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) = \sin(x+y).$$

$$\text{Sol. L.H.S.} = \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$= \text{Put } \frac{\pi}{4}-x = A \text{ and } \frac{\pi}{4}-y = B$$

$$\therefore \text{L.H.S.} = \cos A \cos B - \sin A \sin B = \cos(A+B)$$

$$= \cos\left(\frac{\pi}{4}-x + \frac{\pi}{4}-y\right) = \cos\left(\frac{2\pi}{4} - (x+y)\right)$$

$$= \cos\left[\frac{\pi}{2} - (x+y)\right]$$

$$= \sin(x+y) = \text{R.H.S.} \quad \left[ \because \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \right]$$

7. Prove that: 
$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2.$$

$$\text{Sol. L.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \cdot \tan x}}{\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \cdot \tan x}} = \frac{\frac{1 + \tan x}{1 - \tan x}}{\frac{1 - \tan x}{1 + \tan x}}$$

$$= \frac{1 + \tan x}{1 - \tan x} \times \frac{1 + \tan x}{1 - \tan x} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{R.H.S.}$$

8. Prove that: 
$$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x.$$

$$\begin{aligned} \text{Sol. L.H.S.} &= \frac{\overset{\text{III}}{\cos(\pi+x)} \overset{\text{IV}}{\cos(-x)}}{\underset{\text{II}}{\sin(\pi-x)} \underset{\text{II}}{\cos\left(\frac{\pi}{2}+x\right)}} = \frac{(-\cos x) \cos x}{\sin x (-\sin x)} \\ &= \frac{\cos^2 x}{\sin^2 x} = \cot^2 x = \text{R.H.S.} \end{aligned}$$

9. Prove that:

$$\cos\left(\frac{3\pi}{2}+x\right) \cos(2\pi+x) \left[ \cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x) \right] = 1.$$

$$\begin{aligned} \text{Sol. L.H.S.} &= \underset{\text{IV}}{\cos\left(\frac{3\pi}{2}+x\right)} \underset{\text{I}}{\cos(2\pi+x)} \\ &\quad \left[ \underset{\text{III}}{\cot\left(\frac{3\pi}{2}-x\right)} + \underset{\text{I}}{\cot(2\pi+x)} \right] \\ &= \sin x \cos x [\tan x + \cot x] \\ &= \sin x \cos x \left[ \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right] \\ &= \sin x \cos x \left[ \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \right] = 1 = \text{R.H.S.} \end{aligned}$$

10. Prove that:

$$\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x.$$

$$\begin{aligned} \text{Sol. L.H.S.} &= \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x \\ &= \text{Put } (n+1)x = A \text{ and } (n+2)x = B \\ \therefore \text{L.H.S.} &= \sin A \sin B + \cos A \cos B \\ &= \cos A \cos B + \sin A \sin B = \cos(A-B) \\ &= \cos[(n+1)x - (n+2)x] = \cos(nx+x-nx-2x) \\ &= \cos(-x) = \cos x = \text{R.H.S.} \end{aligned}$$

11. Prove that:

$$\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2} \sin x.$$

$$\begin{aligned}
 \text{Sol. L.H.S.} &= \cos \left( \frac{3\pi}{4} + x \right) - \cos \left( \frac{3\pi}{4} - x \right) \\
 &= \cos \frac{3\pi}{4} \cos x - \sin \frac{3\pi}{4} \sin x - \left( \cos \frac{3\pi}{4} \cos x + \sin \frac{3\pi}{4} \sin x \right) \\
 &[\because \cos(A + B) = \cos A \cos B - \sin A \sin B \text{ and} \\
 &\cos(A - B) = \cos A \cos B + \sin A \sin B] \\
 &= \cos \frac{3\pi}{4} \cos x - \sin \frac{3\pi}{4} \sin x - \cos \frac{3\pi}{4} \cos x - \sin \frac{3\pi}{4} \sin x \\
 &= -2 \sin \frac{3\pi}{4} \sin x = -2 \sin \left( \frac{4\pi - \pi}{4} \right) \sin x \\
 &= -2 \sin \left( \pi - \frac{\pi}{4} \right) \sin x = -2 \sin \frac{\pi}{4} \sin x = -2 \cdot \frac{1}{\sqrt{2}} \sin x \\
 &= -\sqrt{2} \sin x
 \end{aligned}$$

**12. Prove that:  $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$ .**

$$\begin{aligned}
 \text{Sol. L.H.S.} &= \sin^2 6x - \sin^2 4x \\
 &= \sin(6x + 4x) \sin(6x - 4x) \\
 &[\because \sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)] \\
 &= \sin 10x \sin 2x = \text{R.H.S.}
 \end{aligned}$$

**13. Prove that:  $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$ .**

$$\begin{aligned}
 \text{Sol. L.H.S.} &= \cos^2 2x - \cos^2 6x \\
 &[\text{Remember: } \cos^2 A - \cos^2 B = \cos(A + B) \cos(A - B)] \\
 &= (1 - \sin^2 2x) - (1 - \sin^2 6x) \quad [\because \cos^2 \theta = 1 - \sin^2 \theta] \\
 &= \sin^2 6x - \sin^2 2x \\
 &= \sin(6x + 2x) \sin(6x - 2x) \\
 &[\because \sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)] \\
 &= \sin 8x \sin 4x = \text{R.H.S.}
 \end{aligned}$$

**14. Prove that:  $\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$ .**

$$\begin{aligned}
 \text{Sol. L.H.S.} &= (\sin 6x + \sin 2x) + 2 \sin 4x \\
 &= 2 \sin \frac{6x + 2x}{2} \cos \frac{6x - 2x}{2} + 2 \sin 4x \\
 &[\because \sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}] \\
 &= 2 \sin 4x \cos 2x + 2 \sin 4x
 \end{aligned}$$

$$\begin{aligned}
 & \text{Taking } 2 \sin 4x \text{ common,} \\
 & = 2 \sin 4x (\cos 2x + 1) \\
 & = 2 \sin 4x (2 \cos^2 x - 1 + 1) \\
 & = 4 \cos^2 x \sin 4x = \text{R.H.S.}
 \end{aligned}$$

**15. Prove that:  $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$ .**

$$\begin{aligned}
 \text{Sol. L.H.S.} &= \cot 4x (\sin 5x + \sin 3x) \\
 &= \cot 4x \cdot 2 \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2} \\
 & \quad \left[ \because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right] \\
 &= \frac{\cos 4x}{\sin 4x} \cdot 2 \sin 4x \cos x \\
 &= 2 \cos 4x \cos x. \qquad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S.} &= \cot x (\sin 5x - \sin 3x) \\
 &= \cot x \cdot 2 \cos \frac{5x+3x}{2} \sin \frac{5x-3x}{2} \\
 & \quad \left[ \because \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2} \right] \\
 &= \frac{\cos x}{\sin x} \cdot 2 \cos 4x \sin x \\
 &= 2 \cos x \cos 4x. \qquad \dots(ii)
 \end{aligned}$$

From (i) and (ii), we get

$$\text{L.H.S.} = \text{R.H.S.} \quad [\because \text{Each} = 2 \cos 4x \cos x]$$

**16. Prove that:  $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$ .**

$$\begin{aligned}
 \text{Sol. L.H.S.} &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\
 &= \frac{-2 \sin \frac{9x+5x}{2} \sin \frac{9x-5x}{2}}{2 \cos \frac{17x+3x}{2} \sin \frac{17x-3x}{2}} \quad [\text{Using } C - D \text{ formula}] \\
 &= -\frac{\sin 7x \sin 2x}{\cos 10x \sin 7x} = -\frac{\sin 2x}{\cos 10x} = \text{R.H.S.}
 \end{aligned}$$

17. Prove that:  $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$ .

$$\begin{aligned} \text{Sol. L.H.S.} &= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} \\ &= \frac{2 \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2}}{2 \cos \frac{5x+3x}{2} \cos \frac{5x-3x}{2}} \quad [\text{Using C-D formulae}] \\ &= \frac{\sin 4x \cos x}{\cos 4x \cos x} = \frac{\sin 4x}{\cos 4x} = \tan 4x = \text{R.H.S.} \end{aligned}$$

18. Prove that:  $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$ .

$$\begin{aligned} \text{Sol. L.H.S.} &= \frac{\sin x - \sin y}{\cos x + \cos y} \\ &= \frac{2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}}{2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}} \quad [\text{Using C-D formulae}] \\ &\text{Cancelling } 2 \cos \frac{x+y}{2}, \\ &= \frac{\sin \frac{x-y}{2}}{\cos \frac{x-y}{2}} = \tan \frac{x-y}{2} = \text{R.H.S.} \end{aligned}$$

19. Prove that:  $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$ .

$$\begin{aligned} \text{Sol. L.H.S.} &= \frac{\sin x + \sin 3x}{\cos x + \cos 3x} \\ &= \frac{2 \sin \frac{x+3x}{2} \cos \frac{x-3x}{2}}{2 \cos \frac{x+3x}{2} \cos \frac{x-3x}{2}} \quad [\text{Using C-D formulae}] \\ &= \frac{\sin 2x \cos (-x)}{\cos 2x \cos (-x)} = \tan 2x = \text{R.H.S.} \end{aligned}$$

20. Prove that:  $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$ .

$$\begin{aligned} \text{Sol. L.H.S.} &= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{-(\sin 3x - \sin x)}{-(\cos^2 x - \sin^2 x)} \\ &= \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x} \\ &= \frac{2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}{\cos 2x} \quad [\text{Using C - D formulae}] \\ &\quad [\because \cos^2 x - \sin^2 x = \cos 2x] \\ &= \frac{2 \cos 2x \sin x}{\cos 2x} = 2 \sin x = \text{R.H.S.} \end{aligned}$$

21. Prove that:  $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$ .

$$\begin{aligned} \text{Sol. L.H.S.} &= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x} \quad (\because 4x + 2x = 2 \times 3x) \\ &= \frac{2 \cos \frac{4x+2x}{2} \cos \frac{4x-2x}{2} + \cos 3x}{2 \sin \frac{4x+2x}{2} \cos \frac{4x-2x}{2} + \sin 3x} \\ &\quad [\text{Using C - D formulae}] \\ &= \frac{2 \cos 3x \cos x + \cos 3x}{2 \sin 3x \cos x + \sin 3x} \end{aligned}$$

Taking  $\cos 3x$  common both from numerator and denominator.

$$= \frac{\cos 3x (2 \cos x + 1)}{\sin 3x (2 \cos x + 1)} = \cot 3x = \text{R.H.S.}$$

22. Prove that:  $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$ .

Sol. We know that  $3x = 2x + x$

$$\therefore \cot 3x = \cot (2x + x)$$

$$\Rightarrow \cot 3x = \frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}$$

$$\left[ \because \cot (A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A} \right]$$

cross-multiplying

$$\begin{aligned} \cot 3x (\cot x + \cot 2x) &= \cot 2x \cot x - 1 \\ \Rightarrow \cot 3x \cot x + \cot 3x \cot 2x &= \cot 2x \cot x - 1 \\ \Rightarrow \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x &= 1. \end{aligned}$$

**23. Prove that:**  $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$ .

**Sol.** Since  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Put  $A = 2x,$

$$\therefore \tan 4x = \frac{2 \tan 2x}{1 - \tan^2 2x} = \frac{2 \left( \frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left( \frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

$$= \frac{\frac{4 \tan x}{1 - \tan^2 x}}{1 - \frac{4 \tan^2 x}{(1 - \tan^2 x)^2}}$$

$$= \frac{\frac{4 \tan x}{1 - \tan^2 x}}{(1 - \tan^2 x)^2 - 4 \tan^2 x}$$

$$(1 - \tan^2 x)^2$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 - 2 \tan^2 x + \tan^4 x - 4 \tan^2 x}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}.$$

**24. Prove that:**  $\cos 4x = 1 - 8 \sin^2 x \cos^2 x.$

**Sol.** Since  $\cos 2A = 1 - 2 \sin^2 A$

Put  $A = 2x,$

$$\begin{aligned} \therefore \cos 4x &= 1 - 2 \sin^2 2x \\ &= 1 - 2(2 \sin x \cos x)^2 \\ &= 1 - 8 \sin^2 x \cos^2 x. \end{aligned}$$

**25. Prove that:**  $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$ .

**Sol.** Since  $\cos 2A = 2 \cos^2 A - 1$

Put  $A = 3x$ ,

$$\begin{aligned} \therefore \cos 6x &= 2 \cos^2 3x - 1 \\ &= 2(4 \cos^3 x - 3 \cos x)^2 - 1 \\ &= 2(16 \cos^6 x - 24 \cos^4 x + 9 \cos^2 x) - 1 \\ &= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1. \end{aligned}$$

### EXERCISE 3.4 (Page No.: 78)

**Note :** For general solutions, it is customary to use the following relations:

$$\begin{aligned} -\sin \alpha &= \sin(\pi + \alpha), \quad -\cos \alpha = \cos(\pi - \alpha) \text{ and} \\ -\tan \alpha &= \tan(\pi - \alpha) \end{aligned}$$

**Find the principal and general solutions of the following equations:**

1.  $\tan x = \sqrt{3}$

2.  $\sec x = 2$

3.  $\cot x = -\sqrt{3}$

4.  $\operatorname{cosec} x = -2$

**Sol.** 1.  $\tan x = \sqrt{3}$

(i) Since  $\tan x$  is positive, principal solutions (*i.e.* values of  $x$  between  $0^\circ$  and  $360^\circ = 2\pi$ ) are in first and third quadrants.

$$\begin{aligned} \text{Now, } \tan x &= \sqrt{3} && \text{III} \\ &= \tan \frac{\pi}{3} = \tan \left( \pi + \frac{\pi}{3} \right), \text{ i.e., } \tan \frac{4\pi}{3} \end{aligned}$$

$$\therefore \text{Principal solutions are } \frac{\pi}{3}, \frac{4\pi}{3}.$$

(ii) Since  $\tan x = \sqrt{3} = \tan \frac{\pi}{3}$ ,

$$\therefore x = n\pi + \frac{\pi}{3},$$

$n \in \mathbb{Z}$  is the general solution.

2.  $\sec x = 2 \quad \Rightarrow \quad \cos x = \frac{1}{2}$

(i) Since  $\cos x$  is positive, principal solutions are in first and fourth quadrants.



$$\text{Now, } \cos x = \frac{1}{2}$$

IV

$$= \cos \frac{\pi}{3} = \cos \left( 2\pi - \frac{\pi}{3} \right), \text{ i.e., } \cos \frac{5\pi}{3}$$

$$\therefore \text{ Principal solutions are } \frac{\pi}{3}, \frac{5\pi}{3}.$$

$$(ii) \text{ Since } \cos x = \frac{1}{2} = \cos \frac{\pi}{3},$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3},$$

$n \in \mathbb{Z}$  is the general solution.

$$3. \quad \cot x = -\sqrt{3} \Rightarrow \tan x = -\frac{1}{\sqrt{3}}$$

(i) Since  $\tan x$  is negative, principal solutions are in second and fourth quadrants.

$$\text{Now, } \tan x = -\frac{1}{\sqrt{3}} = -\tan \frac{\pi}{6}$$

II

$$= \tan \left( \pi - \frac{\pi}{6} \right), \text{ i.e., } \tan \frac{5\pi}{6}$$

IV

$$\text{and } \tan \left( 2\pi - \frac{\pi}{6} \right), \text{ i.e., } \tan \frac{11\pi}{6}$$

$$\therefore \text{ Principal solutions are } \frac{5\pi}{6}, \frac{11\pi}{6}.$$

$$(ii) \text{ Since } \tan x = -\frac{1}{\sqrt{3}} = -\tan \frac{\pi}{6}$$

$$= \tan \left( \pi - \frac{\pi}{6} \right) = \tan \frac{5\pi}{6}$$

$$\therefore x = n\pi + \frac{5\pi}{6},$$

$n \in \mathbb{Z}$  is the general solution.

$$4. \operatorname{cosec} x = -2 \Rightarrow \sin x = -\frac{1}{2}$$

(i) Since  $\sin x$  is negative, principal solutions are in third and fourth quadrants.

$$\text{Now, } \sin x = -\frac{1}{2} = -\sin \frac{\pi}{6}$$

$$\begin{array}{c} \text{III} \\ = \sin \left( \pi + \frac{\pi}{6} \right), \text{ i.e., } \sin \frac{7\pi}{6} \end{array}$$

$$\text{and} \quad = \sin \left( 2\pi - \frac{\pi}{6} \right), \text{ i.e., } \sin \frac{11\pi}{6}.$$

IV

$$\therefore \text{Principal solutions are } \frac{7\pi}{6}, \frac{11\pi}{6}.$$

$$\begin{aligned} \text{(ii) Since } \sin x = -\frac{1}{2} &= -\sin \frac{\pi}{6} = \sin \left( \pi + \frac{\pi}{6} \right) \\ &= \sin \frac{7\pi}{6} \end{aligned}$$

$$\therefore x = n\pi + (-1)^n \cdot \frac{7\pi}{6},$$

$n \in \mathbb{Z}$  is the general solution.

**Find the general solution for each of the following equations:**

5.  $\cos 4x = \cos 2x$

6.  $\cos 3x + \cos x - \cos 2x = 0$

7.  $\sin 2x + \cos x = 0$

8.  $\sec^2 2x = 1 - \tan 2x$

9.  $\sin x + \sin 3x + \sin 5x = 0.$

**Sol.**

5. Given equation is  $\cos 4x = \cos 2x$

or  $\cos 4x - \cos 2x = 0$

$$\text{or } -2 \sin \frac{4x+2x}{2} \sin \frac{4x-2x}{2} = 0$$

$$\left[ \because \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

or  $\sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0$  or  $\sin x = 0$

$\Rightarrow 3x = n\pi$  or  $x = n\pi, \quad n \in \mathbb{Z}$

$$\therefore x = \frac{n\pi}{3}, n\pi, \quad n \in \mathbb{Z}.$$

6. Given equation is  $\cos 3x + \cos x - \cos 2x = 0$

$$\text{or } 2 \cos \frac{3x+x}{2} \cos \frac{3x-x}{2} - \cos 2x = 0 \quad (3x+x = 2 \times 2x)$$

$$\left[ \because \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$\text{or } 2 \cos 2x \cos x - \cos 2x = 0$$

Taking  $\cos 2x$  common,  $\cos 2x(2 \cos x - 1) = 0$

$$\Rightarrow \text{either } \cos 2x = 0 \quad \text{or } 2 \cos x - 1 = 0$$

$$\Rightarrow 2x = (2n+1) \frac{\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$\Rightarrow x = (2n+1) \frac{\pi}{4}, n \in \mathbb{Z} \quad \text{or} \quad \cos x = \cos \frac{\pi}{3}$$

$$\therefore x = (2n+1) \frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}.$$

7. Given equation is  $\sin 2x + \cos x = 0$

$$\text{or } 2 \sin x \cos x + \cos x = 0$$

$$\text{or } \cos x (2 \sin x + 1) = 0$$

$$\text{or either } \cos x = 0 \quad \text{or } 2 \sin x + 1 = 0$$

$$\begin{aligned} \Rightarrow x = (2n+1) \frac{\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \sin x = -\frac{1}{2} &= -\sin \frac{\pi}{6} \\ &= \sin \left( \pi + \frac{\pi}{6} \right) = \sin \frac{7\pi}{6} \end{aligned}$$

$$\Rightarrow x = (2n+1) \frac{\pi}{2} \quad \text{or} \quad x = n\pi + (-1)^n \cdot \frac{7\pi}{6}, n \in \mathbb{Z}.$$

8. Given equation is  $\sec^2 2x = 1 - \tan 2x$

$$\text{or } 1 + \tan^2 2x = 1 - \tan 2x$$

$$\because \sec^2 \theta = 1 + \tan^2 \theta$$

$$\text{or } \tan^2 2x + \tan 2x = 0$$

$$\text{or } \tan 2x (\tan 2x + 1) = 0$$

$$\Rightarrow \text{either } \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$$

$$\Rightarrow 2x = n\pi, n \in \mathbb{Z} \quad \text{or} \quad \tan 2x = -1 = -\tan \frac{\pi}{4}$$

$$= \tan \left( \pi - \frac{\pi}{4} \right) = \tan \frac{3\pi}{4}$$

$$\Rightarrow x = \frac{n\pi}{2}, \quad \text{or} \quad 2x = n\pi + \frac{3\pi}{4}, \quad n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{2}, \quad \text{or} \quad x = \frac{n\pi}{2} + \frac{3\pi}{8}, \quad n \in \mathbb{Z}.$$

9. Given equation is  $\sin x + \sin 3x + \sin 5x = 0$   
 or  $(\sin 5x + \sin x) + \sin 3x = 0$   
 $(\because 5x + x = 2 \times 3x)$

$$\text{Applying } (\sin C + \sin D) = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\text{or} \quad 2 \sin \frac{5x+x}{2} \cos \frac{5x-x}{2} + \sin 3x = 0$$

$$\text{or} \quad 2 \sin 3x \cos 2x + \sin 3x = 0$$

Taking  $\sin 3x$  common

$$\sin 3x (2 \cos 2x + 1) = 0$$

$$\Rightarrow \text{either } \sin 3x = 0 \quad \text{or} \quad 2 \cos 2x + 1 = 0$$

$$\Rightarrow 3x = n\pi, \quad n \in \mathbb{Z} \quad \text{or} \quad \cos 2x = -\frac{1}{2} = -\cos \frac{\pi}{3}$$

$$= \cos \left( \pi - \frac{\pi}{3} \right) = \cos \frac{2\pi}{3}$$

$$\Rightarrow x = \frac{n\pi}{3}, \quad n \in \mathbb{Z} \quad \text{or} \quad 2x = 2n\pi \pm \frac{2\pi}{3}, \quad n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi \pm \frac{\pi}{3}, \quad n \in \mathbb{Z}.$$

### MISCELLANEOUS EXERCISE ON CHAPTER 3

(Page No.: 81–82)

1. Prove that:  $2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$ .

$$\text{Sol. L.H.S.} = \cos \left( \frac{\pi}{13} + \frac{9\pi}{13} \right) + \cos \left( \frac{\pi}{13} - \frac{9\pi}{13} \right) + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13}$$

$$[\because 2 \cos A \cdot \cos B = \cos (A+B) + \cos (A-B)]$$

$$\begin{aligned}
&= \cos \frac{10\pi}{13} + \cos \left( \frac{-8\pi}{13} \right) + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \\
&= \cos \frac{10\pi}{13} + \cos \frac{8\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \\
&\qquad\qquad\qquad [\because \cos(-\theta) = \cos \theta] \\
&= \cos \frac{13\pi - 3\pi}{13} + \cos \frac{13\pi - 5\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \\
&= \cos \left( \pi - \frac{3\pi}{13} \right) + \cos \left( \pi - \frac{5\pi}{13} \right) + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \\
&\qquad\qquad\qquad \Pi \qquad\qquad\qquad \Pi \\
&= -\cos \frac{3\pi}{13} - \cos \frac{5\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \\
&\qquad\qquad\qquad [\because \cos(\pi - \theta) = -\cos \theta] \\
&= 0 = \text{R.H.S.}
\end{aligned}$$

**2. Prove that:**

$$(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0.$$

$$\text{Sol. L.H.S.} = (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$$

$$\begin{aligned}
&= 2 \sin \frac{3x+x}{2} \cos \frac{3x-x}{2} \sin x \\
&\qquad\qquad\qquad - 2 \sin \frac{3x+x}{2} \sin \frac{3x-x}{2} \cos x
\end{aligned}$$

[Using C - D formulae]

$$\begin{aligned}
&= 2 \sin 2x \cos x \sin x - 2 \sin 2x \sin x \cos x \\
&= 0 = \text{R.H.S.}
\end{aligned}$$

**3. Prove that:**

$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}.$$

$$\text{Sol. L.H.S.} = (\cos x + \cos y)^2 + (\sin x - \sin y)^2$$

$$= \left( 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \right)^2 + \left( 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2} \right)^2$$

[Using C - D formulae]

$$= 4 \cos^2 \frac{x+y}{2} \cos^2 \frac{x-y}{2} + 4 \cos^2 \frac{x+y}{2} \sin^2 \frac{x-y}{2}$$



$$\begin{aligned}
 &= 2 \sin 4x \cdot 2 \cos \frac{3x+x}{2} \cos \frac{3x-x}{2} \\
 &\hspace{15em} [\text{Using C - D formulae}] \\
 &= 4 \sin 4x \cos 2x \cos x \\
 &= 4 \cos x \cos 2x \sin 4x = \text{R.H.S.}
 \end{aligned}$$

6. Prove that:  $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$ .

Sol. L.H.S. =  $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

$$\begin{aligned}
 &= \frac{2 \sin \frac{7x+5x}{2} \cos \frac{7x-5x}{2} + 2 \sin \frac{9x+3x}{2} \cos \frac{9x-3x}{2}}{2 \cos \frac{7x+5x}{2} \cos \frac{7x-5x}{2} + 2 \cos \frac{9x+3x}{2} \cos \frac{9x-3x}{2}} \\
 &\hspace{15em} [\text{Using C - D formulae}] \\
 &= \frac{2 \sin 6x \cos x + 2 \sin 6x \cos 3x}{2 \cos 6x \cos x + 2 \cos 6x \cos 3x} \\
 &\quad \text{Taking } 2 \sin 6x \text{ common from numerator and } 2 \cos 6x \\
 &\quad \text{common from denominator,} \\
 &= \frac{2 \sin 6x (\cos x + \cos 3x)}{2 \cos 6x (\cos x + \cos 3x)} \\
 &= \frac{\sin 6x}{\cos 6x} = \tan 6x = \text{R.H.S.}
 \end{aligned}$$

7. Prove that:

$$\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}.$$

Sol. L.H.S. =  $\sin 3x + \sin 2x - \sin x$

$$\begin{aligned}
 &= (\sin 3x - \sin x) + \sin 2x \\
 &= 2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2} + \sin 2x \quad [\text{Using C - D formulae}] \\
 &= 2 \cos 2x \sin x + 2 \sin x \cos x \\
 &= 2 \sin x (\cos 2x + \cos x) \\
 &= 2 \sin x \cdot 2 \cos \frac{2x+x}{2} \cos \frac{2x-x}{2} \\
 &\hspace{15em} [\text{Using C - D formulae}]
 \end{aligned}$$

$$= 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2} = \text{R.H.S.}$$

Find  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$  in each of the following in Exercises 8 to 10:

8.  $\tan x = -\frac{4}{3}$ ,  $x$  in quadrant II.

Sol.  $\tan x = -\frac{4}{3}$ , ( $x$  is in quadrant II i.e.  $\frac{\pi}{2} < x < \pi$ )

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

i.e.,  $\frac{x}{2}$  lies in the first quadrant, so that all  $t$ -ratios of  $\frac{x}{2}$  are positive.

$$\text{Now } \sec^2 x = 1 + \tan^2 x = 1 + \frac{16}{9} = \frac{9+16}{9} = \frac{25}{9}$$

$$\therefore \sec x = \pm \frac{5}{3}$$

But  $x$  is in II<sup>nd</sup> Quadrant

$$\therefore \sec x \text{ is negative and } = -\frac{5}{3} \Rightarrow \cos x = -\frac{3}{5}$$

$$\text{We know that } \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} \quad \because \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\begin{aligned} \therefore \sin \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{2}} = \pm \sqrt{\frac{1 - \left(-\frac{3}{5}\right)}{2}} \\ &= \pm \sqrt{\frac{4}{5}} = \pm \frac{2}{\sqrt{5}} \end{aligned}$$

But  $\frac{x}{2}$  is in the first quadrant, therefore  $\sin \frac{x}{2}$  is positive

$$\text{and } = \frac{2}{\sqrt{5}}$$

$$\text{We know that } \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} \quad \because \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$



$$\therefore \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}} = \pm \sqrt{\frac{1 - \frac{3}{5}}{2}} = \pm \frac{1}{\sqrt{5}}$$

$$\therefore \cos \frac{x}{2} = \frac{1}{\sqrt{5}} \quad (\because \frac{x}{2} \text{ is in 1st quadrant})$$

$$\therefore \tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = 2.$$

9.  $\cos x = -\frac{1}{3}$ ,  $x$  in quadrant III.

Sol.  $\cos x = -\frac{1}{3}$ , ( $x$  in quadrant III, i.e.,  $\pi < x < \frac{3\pi}{2}$ ) (given)

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow 90^\circ < \frac{x}{2} < 135^\circ$$

i.e.,  $\frac{x}{2}$  lies in the second quadrant, so that  $\sin \frac{x}{2} > 0$ ,

$\cos \frac{x}{2} < 0$  and  $\tan \frac{x}{2} < 0$ .

We know that

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$\therefore \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} = \pm \sqrt{\frac{1 - \left(-\frac{1}{3}\right)}{2}} = \pm \sqrt{\frac{2}{3}}$$

$$\therefore \sin \frac{x}{2} = \sqrt{\frac{2}{3}}. \quad \left[ \because \sin \frac{x}{2} > 0 \right]$$

Again,  $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$

$$\therefore \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\therefore \cos \frac{x}{2} = -\sqrt{\frac{1 + \cos x}{2}} \quad (\because \cos \frac{x}{2} < 0)$$

$$= -\sqrt{\frac{1-\frac{1}{3}}{2}} = -\sqrt{\frac{\frac{2}{3}}{2}} = -\sqrt{\frac{1}{3}} = -\frac{1}{\sqrt{3}}.$$

$$\therefore \tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\frac{\sqrt{2}}{3}}{-\frac{1}{\sqrt{3}}} = -\sqrt{2}.$$

10.  $\sin x = \frac{1}{4}$ ,  $x$  in quadrant II.

Sol.  $\sin x = \frac{1}{4}$ , ( $x$  in quadrant II, i.e.,  $\frac{\pi}{2} < x < \pi$ )

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

i.e.,  $\frac{x}{2}$  lies in the first quadrant, so that all  $t$ -ratios of  $\frac{x}{2}$  are positive.

$$\text{Also, } \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\therefore \cos x = \pm \frac{\sqrt{15}}{4}$$

and  $\cos x$  is negative in the second quadrant. ( $x$  is given to be in the second quadrant)

$$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$$

$$\text{We know that } \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$\begin{aligned} \therefore \sin \frac{x}{2} &= \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}} \\ &= \sqrt{\frac{4 + \sqrt{15}}{8}} \\ &= \sqrt{\frac{8 + 2\sqrt{15}}{16}} = \sqrt{\frac{5 + 3 + 2\sqrt{5}\sqrt{3}}{16}} \end{aligned}$$

$$= \sqrt{\frac{(\sqrt{5})^2 + (\sqrt{3})^2 + 2\sqrt{5}\sqrt{3}}{16}} = \sqrt{\left(\frac{\sqrt{5} + \sqrt{3}}{4}\right)^2} = \frac{\sqrt{5} + \sqrt{3}}{4}.$$

Similarly,  $\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}} = \sqrt{\frac{1 - \frac{\sqrt{15}}{4}}{2}}$

$$= \sqrt{\frac{4 - \sqrt{15}}{8}} = \frac{\sqrt{5} - \sqrt{3}}{4}$$

(For simplification, see the simplification for  $\sin \frac{x}{2}$ )

$$\begin{aligned} \therefore \tan \frac{x}{2} &= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \\ &= \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \\ &\quad \text{(Rationalising)} \\ &= \frac{(\sqrt{5} + \sqrt{3})^2}{5 - 3} = \frac{5 + 3 + 2\sqrt{5}\sqrt{3}}{2} = \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}. \end{aligned}$$

