6 Linear Inequalities

Lesson at a Glance

1. Two real numbers or two algebraic expressions related by the symbols <, >, ≥ or ≤ form an inequality.
2. Equal numbers can be added to both sides of an inequality.
3. Equal numbers can be subtracted from both sides of an inequality.
4. Both sides of an inequality can be multiplied (or divided) by the same **positive** number.
5. When both sides of an inequality are multiplied (or divided) by a **negative number**, then the **sign of inequality** is **reversed.**
6. The values of x, which make an inequality a true statement, are called solutions of the inequality.
7. The set of all solutions of an inequality is called its solution set.
8. The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.
9. If an inequality is of the form $ax + by \le c$ or $ax + by \ge c$, then the points on the line $ax + by = c$ are included in the solution region and the line is drawn thick.
10. If an inequality is of the form $ax + by < c$ or $ax + by > c$, then the points on the line $ax + by = c$ are not included in the solution region and the line is drawn dotted.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 6.1 (*Page No.: 122-123*)

1. Solve $24x < 100$, when

(*i*) x is a natural number (*ii*) x is an integer.

Sol. Given, $24x < 100$

Dividing both sides by 24, which is positive

$$x < \frac{100}{24} \quad i.e., \quad x < \frac{25}{6} = 4.1 \text{ nearly.}$$

(*i*) When $x \in N$, the following values of x make the statement true 1, 2, 3, 4.

$\therefore$ The solution set = {1, 2, 3, 4}.

(*ii*) When $x \in Z$, the following values of x make the statement true ..., – 3, – 2, – 1, 0, 1, 2, 3, 4.

$\therefore$ The solution set = {..., – 3, – 2, – 1, 0, 1, 2, 3, 4}.

2. Solve $-12x > 30$, when

(*i*) x is a natural number (*ii*) x is an integer.

Sol. Given, $-12x > 30$

Dividing both sides by – 12, which is negative (so that the sign of inequality is reversed.)

$$x < \frac{30}{-12}, \quad i.e., \ x < -\frac{5}{2} = -2.5$$

(*i*) When $x \in N = \{1, 2, 3, ...\}$, no value of x makes the statement true. Therefore the given inequality has no solution.

$\therefore$ The solution set = ϕ.

(*ii*) When $x \in Z$, the following values of x make the statement true

..., – 5, – 4, – 3.

$\therefore$ The solution set = {..., – 5, – 4, – 3}.

3. Solve $5x - 3 < 7$, when

(*i*) x is an integer (*ii*) x is a real number.

Sol. Given, $5x - 3 < 7$

Adding 3 to both sides, we get $5x < 10$

Dividing both sides by 5, $x < 2$.

(*i*) When $x \in Z$, the following values of x make the statement true ..., – 2, – 1, 0, 1.

$\therefore$ The solution set = {..., – 2, – 1, 0, 1}.

(*ii*) When $x \in R$, all real numbers x which are less than 2 make the statement true.

$\therefore$ The solution set = $(-\infty, 2)$.

4. Solve $3x + 8 > 2$, when

(i) x is an integer (ii) x is a real number.

Sol. Given, $3x + 8 > 2$

Adding -8 to both sides, we get $3x > -6$

Dividing both sides by 3, $x > -2$.

(i) When $x \in Z$, the following values of x make the statement true $-1, 0, 1, 2, 3, \ldots$.

$\therefore$ The solution set $= \{-1, 0, 1, 2, 3, \ldots\}$.

(ii) When $x \in R$, all real numbers greater than -2 make the statement true.

$\therefore$ The solution set $= (-2, \infty)$.

Solve the inequalities in Exercises 5 to 16 for real x.

5. $4x + 3 < 5x + 7$

Sol. $4x + 3 < 5x + 7$

$\Rightarrow$ $4x - 5x < 7 - 3$ $\Rightarrow$ $-x < 4$

$\Rightarrow$ $x > -4$

$\therefore$ The solution set $= (-4, \infty)$.

6. $3x - 7 > 5x - 1$

Sol. $3x - 7 > 5x - 1$

$\Rightarrow$ $3x - 5x > -1 + 7 \Rightarrow -2x > 6$

$\Rightarrow$ $x < -3$

$\therefore$ The solution set $= (-\infty, -3)$.

7. $3(x - 1) \leq 2(x - 3)$

Sol. $3(x - 1) \leq 2(x - 3)$

$\Rightarrow$ $3x - 3 \leq 2x - 6$ $\Rightarrow$ $3x - 2x \leq -6 + 3$

$\Rightarrow$ $x \leq -3$

$\therefore$ The solution set $= (-\infty, -3]$.

8. $3(2 - x) \geq 2(1 - x)$

Sol. $3(2 - x) \geq 2(1 - x)$

$\Rightarrow$ $6 - 3x \geq 2 - 2x$ $\Rightarrow$ $-3x + 2x \geq 2 - 6$

$\Rightarrow$ $-x \geq -4$ $\Rightarrow$ $x \leq 4$

$\therefore$ The solution set $= (-\infty, 4]$.

9. $x + \dfrac{x}{2} + \dfrac{x}{3} < 11$

Sol. $x + \frac{x}{2} + \frac{x}{3} < 11$

$\Rightarrow \frac{6x+3x+2x}{6} < 11$

Multiplying both sides by 6, we get

$6x + 3x + 2x < 66$

$\Rightarrow \quad 11x < 66 \quad \Rightarrow \quad x < 6$

$\therefore$ The solution set = $(-\infty, 6)$.

10. $\frac{x}{3} > \frac{x}{2} + 1$

Sol. $\frac{x}{3} > \frac{x}{2} + 1 \Rightarrow \frac{x}{3} > \frac{x+2}{2}$

Cross-multiplying, we have

$2x > 3x + 6$

$\Rightarrow \quad 2x - 3x > 6 \quad \Rightarrow \quad -x > 6$

$\Rightarrow \quad x < -6$

$\therefore$ The solution set = $(-\infty, -6)$.

11. $\frac{3(x-2)}{5} \le \frac{5(2-x)}{3}$

Sol. $\frac{3(x-2)}{5} \le \frac{5(2-x)}{3}$

Cross-multiplying, we have

$9(x - 2) \le 25(2 - x)$

$\Rightarrow \quad 9x - 18 \le 50 - 25x \quad \Rightarrow \quad 9x + 25x \le 50 + 18$

$\Rightarrow \quad 34x \le 68 \quad \Rightarrow \quad x \le \frac{68}{34}$

$\Rightarrow \quad x \le 2$

$\therefore$ The solution set = $(-\infty, 2]$.

12. $\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$

Sol. $\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$

$\Rightarrow \quad \frac{1}{2}\left(\frac{3x+20}{5}\right) \ge \frac{x-6}{3} \qquad \Rightarrow \quad \frac{3x+20}{10} \ge \frac{x-6}{3}$

Cross-multiplying, we have

$3(3x + 20) \ge 10(x - 6)$

$\Rightarrow \quad 9x + 60 \ge 10x - 60 \qquad \Rightarrow \quad 9x - 10x \ge -60 - 60$

$\Rightarrow \quad -x \ge -120 \qquad \Rightarrow \quad x \le 120$

$\therefore$ The solution set = $(-\infty, 120]$.

13. $2(2x + 3) - 10 < 6(x - 2)$

Sol. $2(2x + 3) - 10 < 6(x - 2)$

$\Rightarrow \quad 4x + 6 - 10 < 6x - 12 \qquad \Rightarrow \quad 4x - 4 < 6x - 12$

$\Rightarrow \quad 4x - 6x < 4 - 12 \qquad \Rightarrow \quad -2x < -8$

$\Rightarrow \quad x > \frac{-8}{-2} \qquad \Rightarrow \quad x > 4$

$\therefore$ The solution set = $(4, \infty)$.

14. $37 - (3x + 5) \ge 9x - 8(x - 3)$

Sol. $37 - (3x + 5) \ge 9x - 8(x - 3)$

$\Rightarrow \quad 37 - 3x - 5 \ge 9x - 8x + 24$

$\Rightarrow \quad 32 - 3x \ge x + 24 \qquad \Rightarrow \quad -3x - x \ge 24 - 32$

$\Rightarrow \quad -4x \ge -8 \qquad \Rightarrow \quad x \le \frac{-8}{-4}$

$\Rightarrow \quad x \le 2$

$\therefore$ The solution set = $(-\infty, 2]$.

15. $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

Sol. $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

$\Rightarrow \quad \frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$

$\Rightarrow \quad \frac{x}{4} < \frac{25x-10-21x+9}{15} \qquad \Rightarrow \quad \frac{x}{4} < \frac{4x-1}{15}$

Cross-multiplying, we have

$15x < 4(4x - 1)$

$\Rightarrow \quad 15x < 16x - 4 \qquad \Rightarrow \quad 15x - 16x < -4$

$\Rightarrow \quad -x < -4 \qquad \Rightarrow \qquad x > 4$

$\therefore$ The solution set = $(4, \infty)$.

16. $\dfrac{(2x-1)}{3} \geq \dfrac{(3x-2)}{4} - \dfrac{(2-x)}{5}$

Sol. $\dfrac{(2x-1)}{3} \geq \dfrac{(3x-2)}{4} - \dfrac{(2-x)}{5}$

$\Rightarrow \quad \dfrac{2x-1}{3} \geq \dfrac{5(3x-2)-4(2-x)}{20}$

$\Rightarrow \quad \dfrac{2x-1}{3} \geq \dfrac{15x-10-8+4x}{20} \Rightarrow \dfrac{2x-1}{3} \geq \dfrac{19x-18}{20}$

Cross-multiplying, we have

$20(2x - 1) \geq 3(19x - 18)$

$\Rightarrow \quad 40x - 20 \geq 57x - 54 \quad \Rightarrow \quad 40x - 57x \geq 20 - 54$

$\Rightarrow \quad -17x \geq -34 \Rightarrow \quad x \leq \dfrac{-34}{-17}$

$\Rightarrow \quad x \leq 2$

$\therefore$ The solution set = $(-\infty, 2]$.

Solve the inequalities in Exercises 17 to 20 and show the graph of the solution in each case on number line.

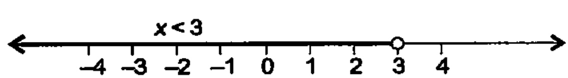
17. $3x - 2 < 2x + 1$

Sol. $3x - 2 < 2x + 1$

$\Rightarrow \quad 3x - 2x < 1 + 2 \qquad \Rightarrow \quad x < 3$

$\therefore$ The solution set = $(-\infty, 3)$.

The graphical representation of the solutions is given in the figure below.



18. $5x - 3 \geq 3x - 5$

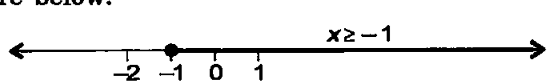
Sol. $5x - 3 \geq 3x - 5$

$\Rightarrow \quad 5x - 3x \geq 3 - 5$

$\Rightarrow \quad 2x \geq -2 \qquad \Rightarrow \quad x \geq -1$

$\therefore$ The solution set = $[-1, \infty)$.

The graphical representation of the solutions is given in the figure below.



19. $3(1 - x) < 2(x + 4)$

Sol. $3(1 - x) < 2(x + 4)$

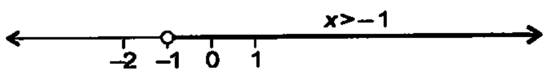
$\Rightarrow \; 3 - 3x < 2x + 8 \qquad \Rightarrow \; -3x - 2x < 8 - 3$

$\Rightarrow \; -5x < 5 \qquad \Rightarrow \; x > \dfrac{5}{-5}$

$\Rightarrow \; x > -1$

$\therefore$ The solution set = $(-1, \infty)$.

The graphical representation of the solutions is given in the figure below.



20. $\dfrac{x}{2} \geq \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

Sol. $\dfrac{x}{2} \geq \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

$\Rightarrow \; \dfrac{x}{2} \geq \dfrac{5(5x-2)-3(7x-3)}{15}$

$\Rightarrow \; \dfrac{x}{2} \geq \dfrac{25x-10-21x+9}{15}$

$\Rightarrow \; \dfrac{x}{2} \geq \dfrac{4x-1}{15}$

Cross-multiplying, we have

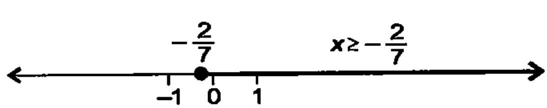
$15x \geq 2(4x - 1)$

$\Rightarrow \; 15x \geq 8x - 2 \qquad \Rightarrow \; 15x - 8x \geq -2$

$\Rightarrow \; 7x \geq -2 \qquad \Rightarrow \; x \geq \dfrac{-2}{7}$

$\therefore$ The solution set = $\left[-\dfrac{2}{7}, \infty\right)$.

The graphical representation of the solutions is given in the figure below.



21. Ravi obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Sol. Let x be the marks obtained by Ravi in the third test.

Then, Average marks $= \dfrac{70+75+x}{3} \geq 60$ [$\because$ At least $\Rightarrow$ $\geq$]

$\Rightarrow \quad 145 + x \geq 180 \quad \Rightarrow \quad x \geq 180 - 145$

$\Rightarrow \quad x \geq 35$

Thus, Ravi must obtain a minimum of 35 marks to get an average of at least 60 marks.

Note. A minimum of 35 marks.

$\Rightarrow$ Marks greater than or equal to 35.

22. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get Grade 'A' in the course.

Sol. Let x be the marks obtained by Sunita in the fifth examination.

Then, Average marks $= \dfrac{87+92+94+95+x}{5} \geq 90$

$\Rightarrow \quad 368 + x \geq 450 \quad \Rightarrow \quad x \geq 450 - 368$

$\Rightarrow \quad x \geq 82$

Thus, Sunita must obtain marks greater than or equal to 82, *i.e.*, a minimum of 82 marks.

23. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Sol. Let x be the smaller of the two consecutive odd positive integers, then the other is $x + 2$. [$\because$ Odd + 1 = Even]

According to the given conditions,

$x < 10, \quad x + 2 < 10$

and $\quad x + (x + 2) > 11$

$\Rightarrow \quad x < 10, \quad x < 8$

and $2x > 9$

$\Rightarrow$ $x < 8$, the smallest of the lesser than.

[$\because$ $x < 8$ automatically $\Rightarrow$ $x < 10$] ...(*i*)

and $x > \frac{9}{2}$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\frac{9}{2} \ (= 4.5) < x < 8$$

Also, x is an odd positive integer.

$\therefore$ x can take values 5 and 7.

So, the required possible pairs will be $(x, x + 2) = (5, 7)$, $(7, 9)$.

24. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

Sol. Let x be the smaller of the two consecutive even positive integers, then the other is $x + 2$. [$\because$ Even + 1 = odd]

According to the given conditions,

$x > 5, \quad x + 2 > 5$

and $x + (x + 2) < 23$

$\Rightarrow$ $x > 5, \quad x > 3$

and $2x < 21$

$\Rightarrow$ $x > 5$, largest of greater than

($\because$ $x > 5$ automatically $\Rightarrow$ $x > 3$), ...(*i*)

and $x < \frac{21}{2}$ ...(*ii*)

From (*i*) and (*ii*), we get

$$5 < x < \frac{21}{2} \ (= 10.5)$$

Also, x is an even positive integer.

$\therefore$ x can take the values 6, 8 and 10.

So, the required possible pairs will be $(x, x + 2) = (6, 8)$, $(8, 10)$, $(10, 12)$.

25. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Sol. Let the length of the shortest side be x cm, then the length of the longest side is $3x$ cm and the length of the third side is $(3x - 2)$ cm.

Perimeter (*i.e.* sum of all three sides) of the triangle

$= x + 3x + (3x - 2)$

$= (7x - 2)$ cm

According to the given condition,

Perimeter ≥ 61 cm ($\because$ At least $\Rightarrow \geq$)

$\Rightarrow \quad 7x - 2 \geq 61 \quad \Rightarrow \quad 7x \geq 63$

$\Rightarrow \quad x \geq 9$

$\therefore$ The minimum length of the shortest side = 9 cm.

26. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

Sol. Let the length of the shortest piece be x cm, then length of second is $(x + 3)$ cm and the length of third is $2x$ cm.

Since total length is 91 cm, we must have

$$x + (x + 3) + 2x \leq 91$$

$\Rightarrow \quad 4x \leq 88 \quad \Rightarrow \quad x \leq 22$ ...(*i*)

Also, length of third piece is at least 5 cm longer than the second i.e. III – II ≥ 5

$\therefore \quad 2x - (x + 3) \geq 5$

$\Rightarrow \quad 2x - x - 3 \geq 5 \quad \Rightarrow \quad x \geq 8$ ...(*ii*)

Combining (*i*) and (*ii*), we have

$$8 \leq x \leq 22$$

$\therefore$ The length of the shortest board must be greater than or equal to 8 cm but less than or equal to 22 cm.

Remark : In the first inequality in the solution of above question, we have taken:

Sum of lengths of three pieces $= x + (x + 3) + 2x \leq 91$ and not only = 91 because even after cutting all the three pieces according to the requirement of the question, a part of the length of the board out of 91 cm may remain unused (left)

EXERCISE 6.2 *(Page No.: 127)*

Solve the following inequalities graphically in two-dimensional plane:

1. $x + y < 5$

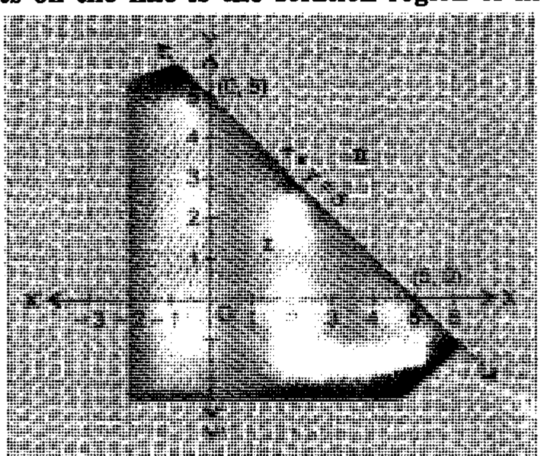
Sol. The given inequality is $x + y < 5$ ...(*i*)

Replacing < by =, the corresponding equation is

$$x + y = 5 \qquad ...(ii)$$

Putting $y = 0$ in (*ii*), we get $x = 5$
Putting $x = 0$ in (*ii*), we get $y = 5$ } Table of values

∴ The graph of straight line (*ii*) passes through the points (5, 0) and (0, 5). The line is drawn dotted because the inequality is strict (Because it is < and not ≤) This line divides the *xy*-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0$, $y = 0$ in (*i*), we get $0 + 0 < 5$ or $0 < 5$, which is true. We shade the half plane I in which the origin lies. Thus, the shaded half plane I excluding the points on the line is the solution region of inequality (*i*).



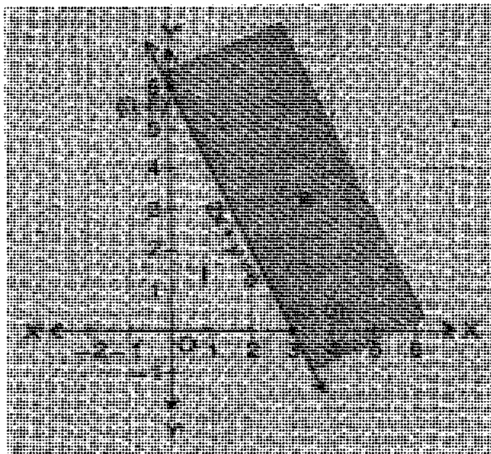
2. $2x + y \geq 6$

Sol. The given inequality is $2x + y \geq 6$ ...(*i*)

Replacing ≥ by =, the corresponding equation is

$$2x + y = 6 \qquad ...(ii)$$

Putting $y = 0$ in (*ii*), we get $x = 3$
Putting $x = 0$ in (*ii*), we get $y = 6$ } Table of values



∴ The graph of straight line (*ii*) passes through the points (3, 0) and (0, 6). The line is drawn thick because the inequality is slack (*i.e.* = is also there in ≤) This line divides the xy-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0$, $y = 0$ in (*i*), we get $0 + 0 \geq 6$ or $0 \geq 6$, which is false. We shade the half plane II in which the origin does not lie. Thus, the shaded half plane II including the points on the line is the solution region of inequality (*i*).

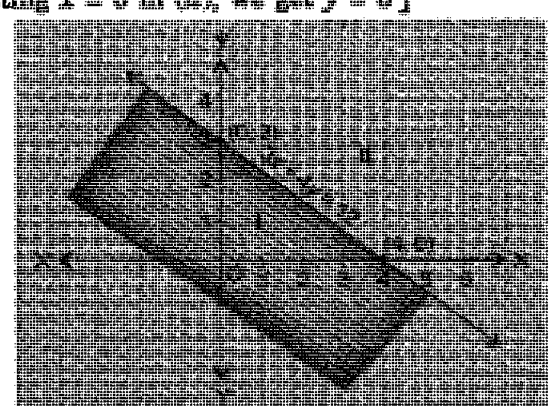
3. $3x + 4y \leq 12$

Sol. The given inequality is $3x + 4y \leq 12$ ...(*i*)

Replacing ≤ by =, the corresponding equation is

$3x + 4y = 12$ ...(*ii*)

Putting $y = 0$ in (*ii*), we get $x = 4$ ⎤
Putting $x = 0$ in (*ii*), we get $y = 3$ ⎦ Table of values



$\therefore$ The graph of straight line (*ii*) passes through the points (4, 0) and (0, 3). The line is drawn thick because the inequality is slack (*i.e.* = is also there in $\le$). This line divides the *xy*-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0$, $y = 0$ in (*i*), we get $0 + 0 \le 12$ or $0 \le 12$, which is true. We shade the half plane I in which the origin lies. Thus, the shaded half plane I including the points on the line is the solution region of inequality (*i*).

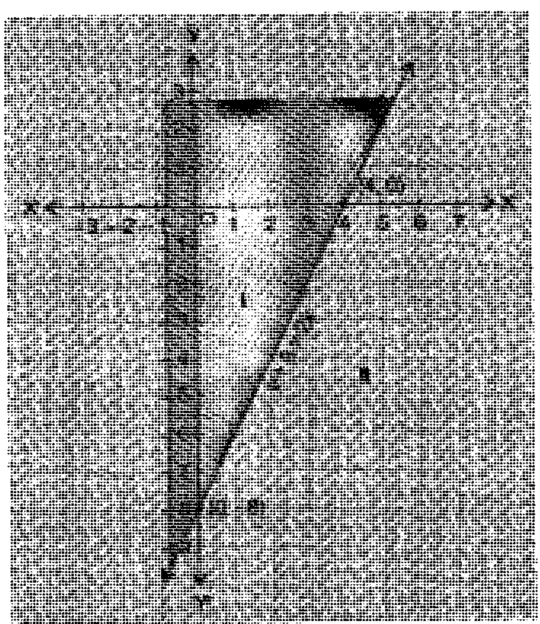
4. $y + 8 \ge 2x$

Sol. The given inequality is $y + 8 \ge 2x$ ...(*i*)

Replacing $\ge$ by =, the corresponding equation is

$$y + 8 = 2x \quad ...(ii)$$

Putting $y = 0$ in (*ii*), we get $x = 4$
Putting $x = 0$ in (*ii*), we get $y = -8$] Table of values



$\therefore$ The graph of straight line (*ii*) passes through the points (4, 0) and (0, – 8). The line is drawn thick because the inequality is slack. This line divides the *xy*-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0$, $y = 0$ in (*i*), we get $0 + 8 \ge 0$ or $8 \ge 0$, which is true. We shade the half plane I in which the origin lies. Thus, the shaded half plane I including the points on the line is the solution region of inequality (*i*).

5. $x - y \leq 2$

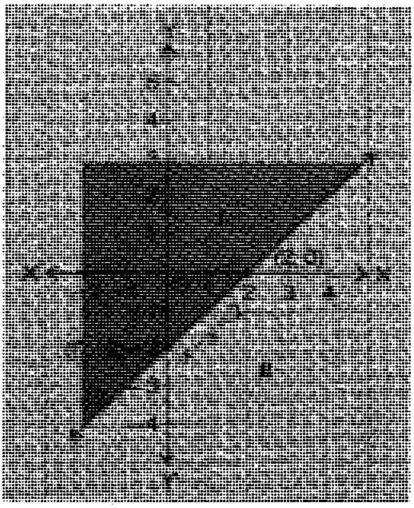
Sol. The given inequality is $x - y \leq 2$ ...(*i*)

Replacing $\leq$ by =, the corresponding equation is

$x - y = 2$ ...(*ii*)

Putting $y = 0$ in (*ii*), we get $x = 2$
Putting $x = 0$ in (*ii*), we get $y = -2$] Table of values

$\therefore$ The graph of straight line (*ii*) passes through the points (2, 0) and (0, – 2). The line is drawn thick because the inequality is slack. This line divides the *xy*-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0, y = 0$ in (*i*), we get $0 \leq 2$, which is true. We shade the half plane I in which the origin lies. Thus, the shaded half plane I including the points on the line is the solution region of inequality (*i*).



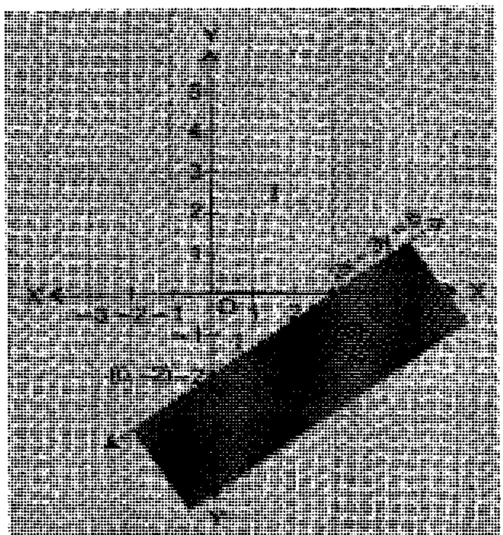
6. $2x - 3y > 6$

Sol. The given inequality is $2x - 3y > 6$ ...(*i*)

Replacing > by =, the corresponding equation is

$2x - 3y = 6$ ...(*ii*)

Putting $y = 0$ in (*ii*), we get $x = 3$
Putting $x = 0$ in (*ii*), we get $y = -2$] Table of values



$\therefore$ The graph of straight line (*ii*) passes through the points (3, 0) and (0, – 2). The line is drawn dotted because the inequality is strict ($\because$ equality sign is absent in >). This line divides the *xy*-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0, y = 0$ in (*i*), we get $0 > 6$, which is false. We shade the half plane II in which the origin does not lie. Thus, the shaded half plane II excluding the points on the line is the solution region of inequality (*i*).

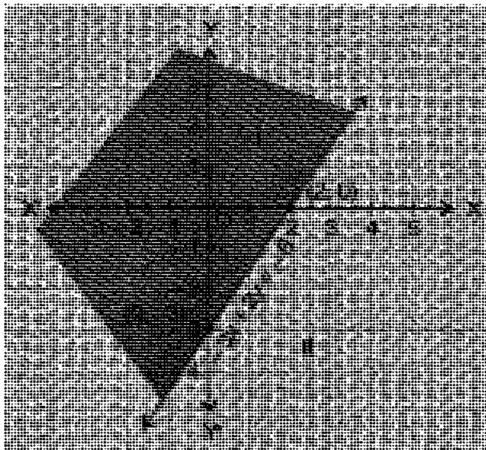
7. $-3x + 2y \geq -6$

Sol. The given inequality is $-3x + 2y \geq -6$ ...(*i*)

Replacing $\geq$ by =, the corresponding equation is

$-3x + 2y = -6$ ...(*ii*)

Putting $y = 0$ in (*ii*), we get $x = 2$
Putting $x = 0$ in (*ii*), we get $y = -3$] Table of values



∴ The graph of straight line (*ii*) passes through the points (2, 0) and (0, – 3). The line is drawn thick because the inequality is slack. This line divides the xy-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0$, $y = 0$ in (*i*), we get $0 \geq -6$, which is true. We shade the half plane I in which the origin lies. Thus, the shaded half plane I including the points on the line is the solution region of inequality (*i*).

8. $3y - 5x < 30$

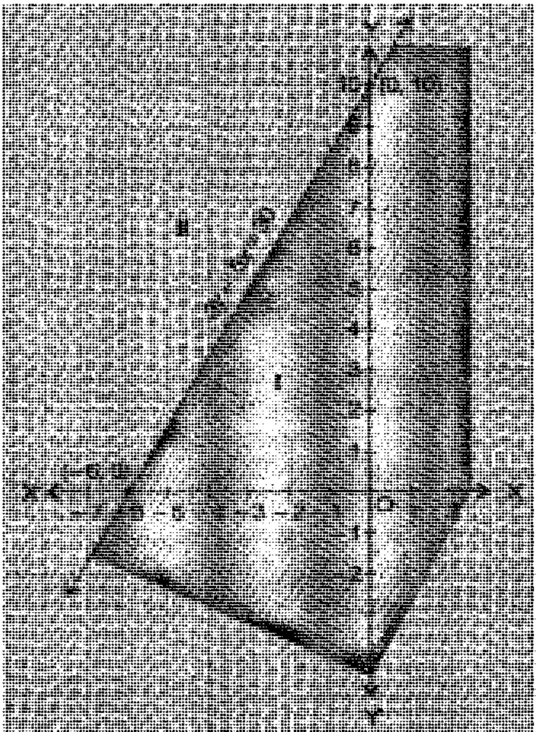
Sol. The given inequality is $3y - 5x < 30$ ...(*i*)

Replacing < by =, the corresponding equation is

$3y - 5x = 30$ ...(*ii*)

Putting $y = 0$ in (*ii*), we get $x = -6$ ⎤ Table of values
Putting $x = 0$ in (*ii*), we get $y = 10$ ⎦

∴ The graph of straight line (*ii*) passes through the points (– 6, 0) and (0, 10). The line is drawn dotted because the inequality is strict. This line divides the xy-plane into two half planes I and II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0$, $y = 0$ in (*i*), we get $0 < 30$, which is true. We shade the half plane I in which the origin lies. Thus, the shaded half plane I excluding the points on the line is the solution region of inequality (*i*).



Note: Graphs of equations of two special type of straight lines

(*i*) **Graph of $y = k$** is a **horizontal** line (*i.e.* a line parallel to x-axis) at a distance k from it.

If $k > 0$, the horizontal line is **above** the x-axis

If $k < 0$, the horizontal line is **below** the x-axis

(*ii*) **Graph of $x = k$** is a **vertical** line (*i.e.* a line parallel to y-axis) at a distance k from it.

If $k > 0$, the vertical line is to the **right** of y-axis

If $k < 0$, the vertical line is to the **left** of y-axis

9. $y < -2$

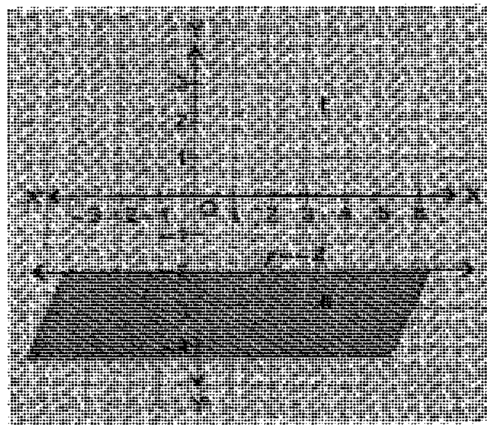
Sol. The given inequality is $y < -2$ ...(*i*)

Replacing < by =, the corresponding equation is

$y = -2$ ...(*ii*)

Its graph is a horizontal line (*i.e.*, a line parallel to x-axis) 2 units below the origin.

The line is drawn dotted because the inequality is strict. This line divides the xy-plane into two half planes; the upper half plane I and the lower half plane II.



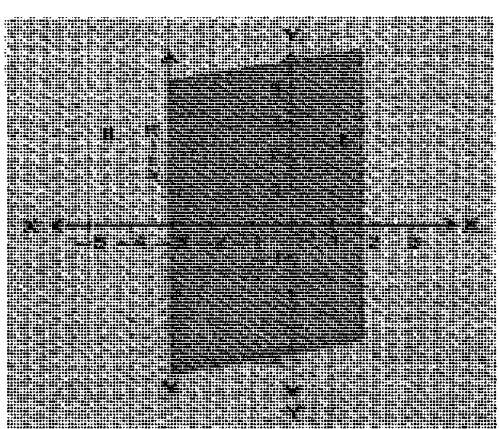
Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $y = 0$ in (*i*), we get $0 < -2$, which is false. We shade the lower half plane II in which the origin does not lie. Thus, the shaded lower plane II excluding the points on the line is the solution region of inequality (*i*).

10. $x > -3$

Sol. The given inequality is $x > -3$ ...(*i*)

Replacing > by =, the corresponding equation is

$x = -3$ ...(*ii*)



Its graph is a vertical line (*i.e.*, a line parallel to y-axis) 3 units to the left of origin. The line is drawn dotted because the inequality is strict. This line divides the

xy-plane into two half planes, the right half plane I and the left half plane II. Now, we select a point not on the line (*ii*). Since the line does not pass through the origin (0, 0), we select the origin. Putting $x = 0$ in (*i*), we get $0 > -3$, which is true. We shade the right half plane I in which the origin lies. Thus, the shaded right half plane I excluding the points on the line is the solution region of inequality (*i*).

EXERCISE 6.3 (*Page No.: 129*)

Solve the following system of inequalities graphically:

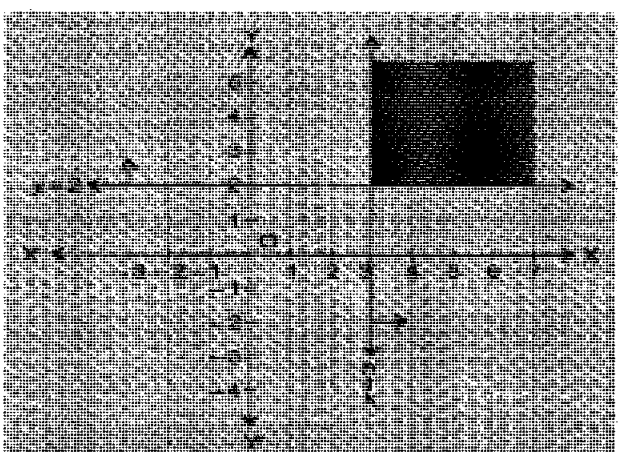
1. $x \geq 3, y \geq 2$

Sol. The given system of inequalities is

$$x \geq 3 \quad ...(i)$$

$$y \geq 2 \quad ...(ii)$$

The inequality $x \geq 3$ represents the region to the right of the vertical line $x = 3$ (indicated by an arrow), including the points on the line. The inequality $y \geq 2$ represents the upper half plane determined by the horizontal line $y = 2$ (indicated by an arrow), including the points on the line.



Hence, the common shaded region including the points on the lines is the required solution region of the given system of inequalities.

2. $3x + 2y \leq 12, x \geq 1, y \geq 2$

Sol. The given system of inequalities is

$$3x + 2y \le 12 \qquad ...(i)$$
$$x \ge 1 \qquad ...(ii)$$
$$y \ge 2 \qquad ...(iii)$$

Replacing $\le$ by $=$ in (i), the corresponding equation is

$3x + 2y = 12$

Table of values

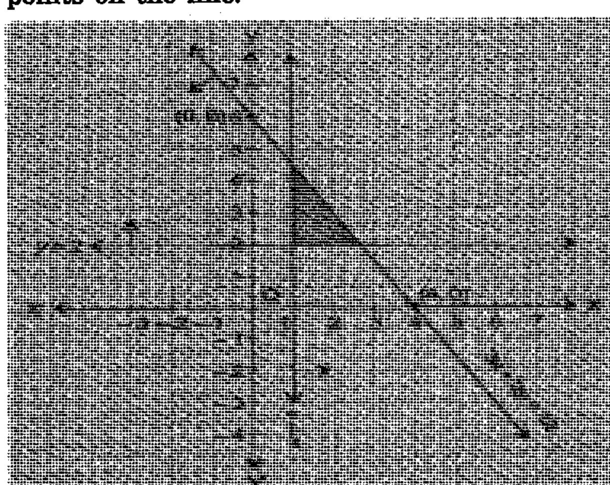
x	0	4
y	6	0

$\therefore$ Graph of straight line $3x + 2y = 12$ is the line joining the point (0, 6) and (4, 0). This line is drawn thick because the inequality includes equality sign also.

Let us test for origin (0, 0) in inequality (i), we have $0 \le 12$ which is true.

$\therefore$ The region for inequality (i) is the region on the origin side of the line (indicated by an arrow).

The inequality $x \ge 1$ represents the right half plane determined by the vertical line $x = 1$ (indicated by an arrow), including the points on the line. The inequality $y \ge 2$ represents the upper half plane determined by the horizontal line $y = 2$ (indicated by an arrow), including the points on the line.



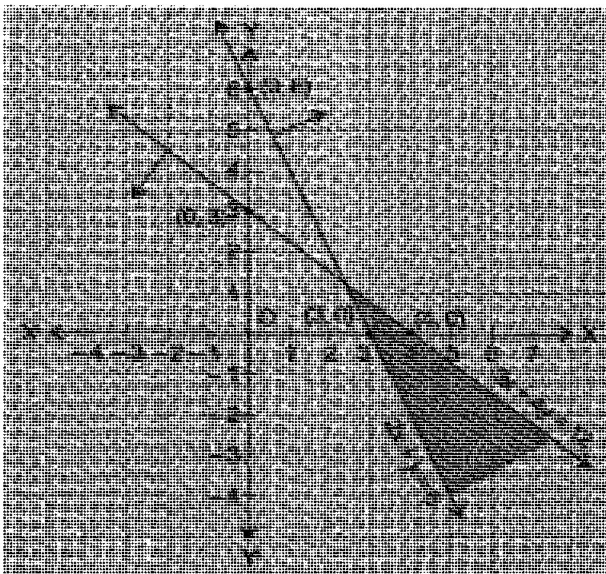
Hence, the common shaded region including the points on the lines is the required solution region of the given system of inequalities.

3. $2x + y \geq 6, 3x + 4y \leq 12$

Sol. The given system of inequalities is

$$2x + y \geq 6 \quad ...(i)$$

$$3x + 4y \leq 12 \quad ...(ii)$$



Replacing $\geq$ by = in (*i*), the corresponding equation is $2x + y = 6$

Table of values

x	0	3
y	6	0

$\therefore$ Graph of straight line $2x + y = 6$ is the line joining the points (0, 6) and (3, 0).

Let us test for origin (0, 0) in inequality (*i*), we have $0 \geq 6$ which is not true.

$\therefore$ The region for inequality (*i*) is the region on the non-origin side of the line (indicated by an arrow) including the points on the line (because the inequality is slack).

Now replacing $\leq$ by = in (*ii*), corresponding equation is $3x + 4y = 12$

Table of values

x	0	4
y	3	0

$\therefore$ Graph of straight line $3x + 4y = 12$ is the line joining the points (0, 3) and (4, 0).

Let us test for origin (0, 0) in inequality (*ii*), we have $0 \le 12$ which is true

$\therefore$ The region for inequality (*ii*) is the region on the origin side of the line (indicated by an arrow) including the points on the line (because the inequality is slack).

Hence, the common shaded region including the points on the lines is the required solution region of the given system of inequalities.

4. $x + y \ge 4,\ 2x - y > 0$

Sol. The given system of inequalities is

$$x + y \ge 4 \qquad ...(i)$$

$$2x - y > 0 \qquad ...(ii)$$

Replacing $\ge$ by = in (*i*), the corresponding equation is $x + y = 4$

Table of values

x	0	4
y	4	0

$\therefore$ Graph of straight line $x + y = 4$ is the line joining the points (0, 4) and (4, 0).

Let us test for origin (0, 0) in inequality (*i*) we have $0 \ge 4$ which is not true.

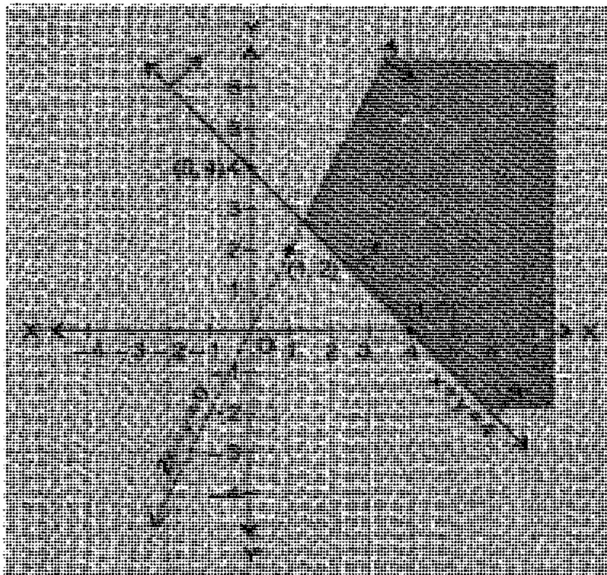
$\therefore$ The region for inequality (*i*) is the region on the non-origin side of the line (indicated by an arrow) including the points on the line.

Now replacing $\ge$ by = in (*ii*), the corresponding equation is $2x - y = 0$ i.e. $y = 2x$

Table of values

x	0	1
y	0	2

∴ Graph of straight line $2x - y = 0$ is the line joining the points (0, 0) and (1, 2)



Now we select a point not on the line $2x - y = 0$. Since this line passes through the origin, we select a point other than origin say (4, 0). Putting $x = 4$ and $y = 0$ in (*ii*), $8 > 0$ which is true. Therefore region represented by (*ii*) is towards the side of the point (4, 0) as indicated by an arrow. (excluding the points on the line)

Hence, the common shaded region including the points on the line $x + y = 4$ and excluding points on the line $2x - y = 0$ is the required solution region of the given system of inequalities.

5. $\quad 2x - y > 1, x - 2y < -1$

Sol. The given system of inequalities is

$2x - y > 1$ ...(*i*)

$x - 2y < -1$ ...(*ii*)

Replacing > by = in (*i*) the corresponding equation is $2x - y = 1$

Table of values

x	0	1	3
y	–1	1	5

∴ Graph of straight line $2x - y = 1$ is the line joining the points (0, –1) and (1, 1).

Let us test for origin (0, 0) in inequality (*i*) we have $0 > 1$ which is not true.

∴ Region for inequality (*i*) is the region on the non-origin side of the line (indicated by an arrow) excluding the points on the line (because the inquality is strict).

Now replacing < by = in (*ii*), the corresponding equation is $x - 2y = -1$.

Table of values

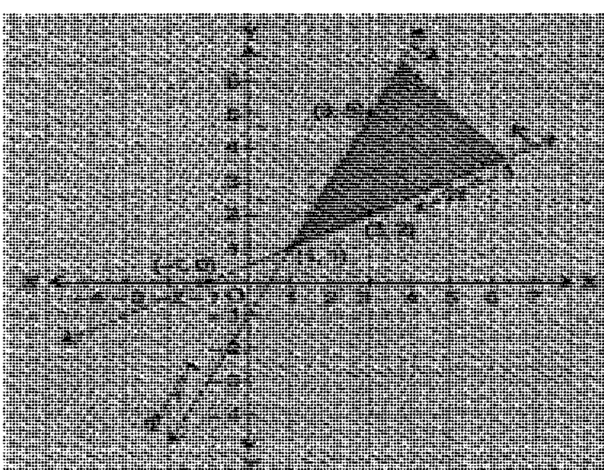
x	–1	1	3
y	0	1	2

∴ Graph of straight line $x - 2y = -1$ is the line joining the points (–1, 0) and (1, 1).

Let us test for origin (0, 0) in (*ii*), we have

$0 < -1$ which is not true.

∴ Region for inequality (*ii*) is the region on the non-origin side of the line (indicated by an arrow) excluding the points on the line.



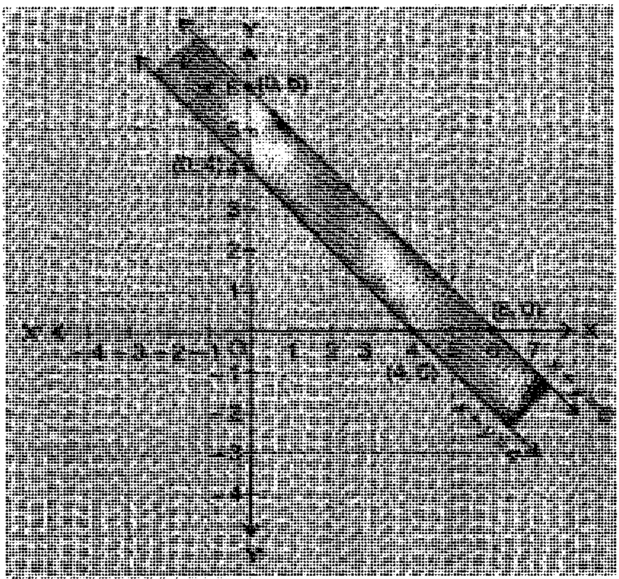
Hence, the common shaded region excluding the points on the lines is the required solution region of the given system of inequalities.

6. $x + y \le 6, x + y \ge 4$

Sol. The given system of inequalities is

$$x + y \le 6 \quad \text{...(i)}$$

$$x + y \ge 4 \quad \text{...(ii)}$$



Replacing ≤ by = in (*i*), the corresponding equation is $x + y = 6$

Table of values

x	0	6
y	6	0

∴ Graph of straight line $x + y = 6$ is the line joining the points (0, 6) and (6, 0).

Let us test for origin (0, 0) in inequality (*i*) we have $0 \le 6$ which is true.

∴ The region for inequality (*i*) is the region on the origin side of the line (indicated by an arrow) including points on the line.

Now replacing ≥ by = in (*ii*), the correspoinding equation is $x + y = 4$

Table of values

x	0	4
y	4	0

∴ Graph of straight line $x + y = 4$ is the line joining the points (0, 4) and (4, 0).

Let us test for origin (0, 0) in inequality (*ii*), we have

$0 \geq 4$ which is not true.

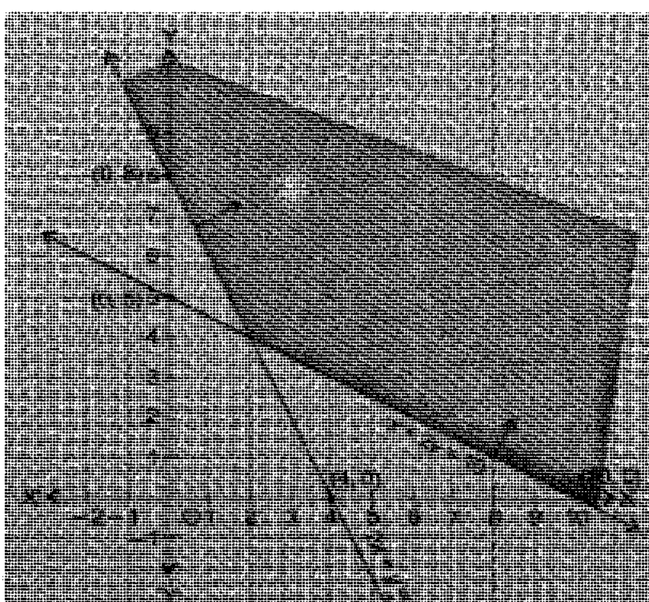
∴ The region for inequality (*ii*) is the region on the non-origin side of the line (indicated by an arrow) including points on the line.

Hence, the common shaded region between the two parallel line including the points on the two lines is the required solution region of the given system of inequalities.

7. $2x + y \geq 8, x + 2y \geq 10$

Sol. The given system of inequalities is

$$2x + y \geq 8 \quad ...(i)$$
$$x + 2y \geq 10 \quad ...(ii)$$



Replacing $\geq$ by = in (*i*), the corresponding equation is $2x + y = 8$

Table of values

x	0	4
y	8	0

$\therefore$ Graph of straight line $2x + y = 8$ is the line joining the points (0, 8) and (4, 0).

Let us test for origin (0, 0) in inequality (*i*) we have $0 \geq 8$ which is not true.

$\therefore$ The region for inequality (*i*) is the region on the non-origin side of the line (indicated by an arrow) including points on the line.

Now replacing $\geq$ by = in (*ii*), the corresponding equation is $x + 2y = 10$

Table of values

x	0	10
y	5	0

$\therefore$ Graph of straight line $x + 2y = 10$ is the line joining the points (0, 5) and (10, 0).

Let us test for origin (0, 0) in inequation (*ii*), we have $0 \geq 10$ which is not true.

$\therefore$ The region for inequality (*ii*) is the region on the non-origin side of the line (indicated by an arrow) including points on the line.

Hence, the common shaded region including all the points on the lines is the required solution region of the given system of the inequalities.

8. $x + y \leq 9, y > x, x \geq 0$

Sol. The given system of inequalities is

$$x + y \leq 9 \quad ...(i)$$

$$y > x \quad ...(ii)$$

$$x \geq 0 \quad ...(iii)$$

Replacing $\leq$ by = in inequality (*i*), the corresponding equation is $x + y = 9$

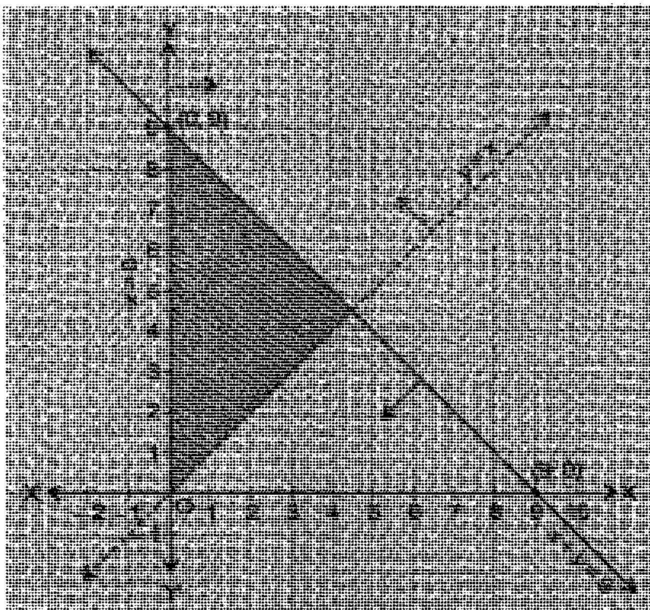


Table of values

x	0	9
y	9	0

∴ Graph of straight line $x + y = 9$ is the line joining the points (0, 9) and (9, 0).

Let us test for origin (0, 0) in inequation (*i*) we have $0 \leq 9$ which is true.

∴ The region for inequation (*i*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

Now replacing > by = in inequality (*ii*), the corresponding equation is $y = x$

Table of values

x	0	2
y	0	2

∴ Graph of straight line $y = x$ is the line joining the points (0, 0) and (2, 2).

Now we select a point not on the line $y = x$. Since this line passes through the origin, we select a point other than origin say (0, 9). Putting $x = 0$ and $y = 9$ in (*ii*),

we have $9 > 0$ which is true.

Therefore region represented by (*ii*) is towards the side of the point (0, 9) (as indicated by an arrow) excluding the points on the line.

The inequality $x \geq 0$ represents the right half plane determined by the y-axis ($x = 0$) [indicated by an arrow] including the points on x-axis.

Hence, the common shaded region including the points on y-axis and the line $x + y = 9$ but excluding the points on the line $y = x$ is the required solution region of the given system of inequalities.

9. $5x + 4y \leq 20,\ x \geq 1,\ y \geq 2$

Sol. The given system of inequalities is

$$5x + 4y \leq 20 \quad ...(i)$$

$$x \geq 1 \quad ...(ii)$$

$$y \geq 2 \quad ...(iii)$$

Replacing $\leq$ by = in (*i*), the corresponding equation is $5x + 4y = 20$

Table of values

x	0	4
y	5	0

$\therefore$ Graph of straight line $5x + 4y = 20$ is the line joining the points (0, 5) and (4, 0).

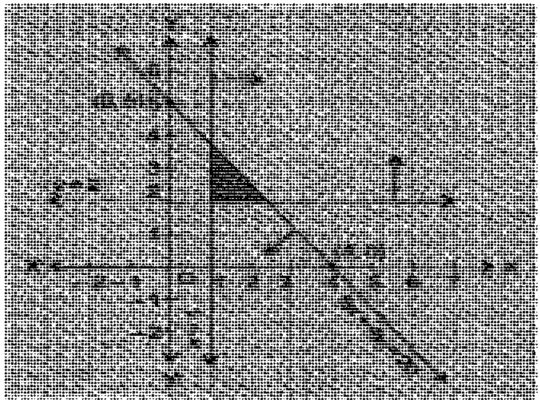
Let us test for origin (0, 0) in (*i*),

(Put $x = 0$, $y = 0$),

we have $0 \leq 20$ which is true.

$\therefore$ The region for inequality (*i*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

The inequality $x \geq 1$ represents the right half plane determined by the vertical line $x = 1$ (indicated by an arrow) including points on the line. The inequality $y \geq 2$ represents the upper half plane determined by the horizontal line $y = 2$ [indicated by an arrow], including the points on the line.



Hence, the common shaded region, a right triangle, including points on the sides is the required solution region of the given system of inequalities.

10. $3x + 4y \le 60,\ x + 3y \le 30,\ x \ge 0,\ y \ge 0$

Sol. The given system of inequalities is

$$3x + 4y \le 60 \qquad ...(i)$$

$$x + 3y \le 30 \qquad ...(ii)$$

$$x \ge 0 \qquad ...(iii)$$

$$y \ge 0 \qquad ...(iv)$$

Replacing $\le$ by $=$ in (*i*), the corresponding equation is $3x + 4y = 60$

Table of values

x	0	20
y	15	0

∴ Graph of straight line $3x + 4y = 60$ is the line joining the points (0, 15) and (20, 0).

Let us test for origin (0, 0) in (*i*) we have $0 \le 60$ which is true.

∴ The region for inequality (*i*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

Now replacing $\le$ by $=$ in (*ii*), the correspoinding equation is $x + 3y = 30$

Table of values

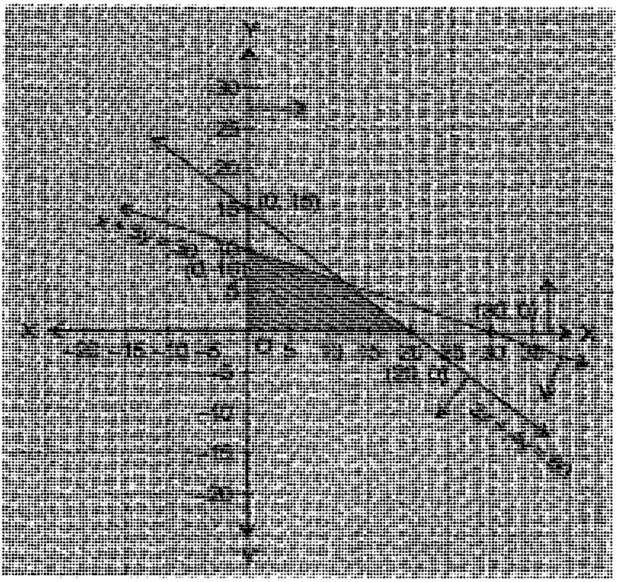
x	0	30
y	10	0

$\therefore$ Graph of straight line $x + 3y = 30$ is the line joining the points (0, 10) and (30, 0).

Let us test for origin (0, 0) in (*i*), we have $0 \le 30$ which is true.

$\therefore$ The region for inequality (*ii*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

The inequalities $x \ge 0$, $y \ge 0$ represents the region in the first quadrant, including the points on the axes.



Hence, the common shaded region, including the points on the lines and the axes, is the required solution region of the given system of the inequalities.

11. $2x + y \ge 4$, $x + y \le 3$, $2x - 3y \le 6$

Sol. The given system of inequalities is

$2x + y \ge 4$ ...(*i*)

$x + y \le 3$ ...(*ii*)

$2x - 3y \le 6$ ...(*iii*)

Replacing $\ge$ by = in (*i*), the corresponding equation is

$2x + y = 4$

Table of values

x	0	2
y	4	0

$\therefore$ Graph of straight line $2x + y = 4$ is the line joining the points (0, 4) and (2, 0).

Let us test for origin (0, 0) in inequation (*i*), we have $0 \geq 4$ which is not true.

$\therefore$ The region for inequality (*i*) is the region on the non-origin side of the line (indicated by an arrow) including the points on the line.

Now replacing $\leq$ by = in (*ii*), the corresponding equation is $x + y = 3$

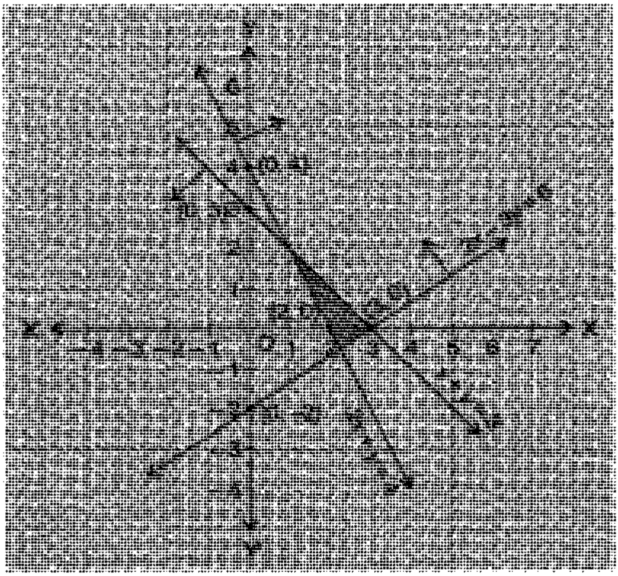
Table of values

x	0	3
y	3	0

$\therefore$ Graph of straight line $x + y = 3$ is the line joining the points (0, 3) and (3, 0).

Let us test for origin (0, 0) in inequality (*ii*), we have $0 \leq 3$ which is true.

$\therefore$ The region for inequality (*ii*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.



Now replacing ≤ by = in (*iii*), the corresponding equation is $2x - 3y = 6$

Table of values

x	0	3
y	–2	0

∴ Graph of straight line $2x - 3y = 6$ is the line joining the points (0, –2) and (3, 0).

Let us test for origin (0, 0) in inequality (*iii*), we have $0 \le 6$ which is true. Therefore, region for inequality (*iii*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

Hence, the common shaded region including the points on the lines, is the required solution region of the given system of the inequalities.

12. $x - 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$

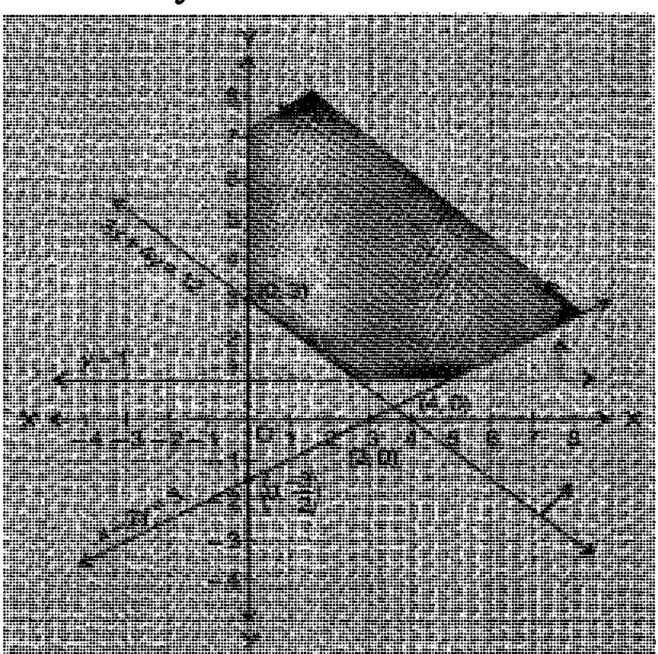
Sol. The given system of inequalities is

$$x - 2y \le 3 \quad ...(i)$$

$$3x + 4y \ge 12 \quad ...(ii)$$

$$x \ge 0 \quad ...(iii)$$

$$y \ge 1 \quad ...(iv)$$



Replacing ≤ by = in (*i*), the corresponding equation is $x - 2y = 3$.

Table of values

x	0	3
y	$-\frac{3}{2}$	0

∴ Graph of straight line $x - 2y = 3$ is the line joining the points $\left(0, -\frac{3}{2}\right)$ and (3, 0).

Let us test for origin (0, 0) in (*i*), we have $0 \leq 3$ which is true.

∴ The region for inequality (*i*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

Now replacing ≥ by = in (*ii*), the corresponding equation is $3x + 4y = 12$

Table of values

x	0	4
y	3	0

∴ Graph of straight line $3x + 4y = 12$ is the straight line joining the points (0, 3) and (4, 0).

Let us test for origin (0, 0) in (*ii*), we have

$0 \geq 12$ which is not true.

∴ The region for inequality (*ii*) is the region on the non-origin side of the line (indicated by an arrow) including the points on the line.

The inequality (*iii*) $x \geq 0$ represents the right half plane determined by the y-axis ($x = 0$), including points on y-axis.

The inequality (*iv*) $y \geq 1$ represents the upper half plane determined by the horizontal line $y = 1$, including points on the line.

Hence, the common shaded region, including the points on the lines, is the required solution region of the given system of inequalities.

13. $4x + 3y \leq 60$, $y \geq 2x$, $x \geq 3$, $x, y \geq 0$

Sol. The given system of inequalities is

$$4x + 3y \leq 60 \quad ...(i)$$

$$y \geq 2x \quad ...(ii)$$

$$x \geq 3 \quad ...(iii)$$

$$x, y \geq 0 \quad ...(iv)$$

Replacing ≤ by = in (*i*), the corresponding equation is $4x + 3y = 60$

Table of values

x	0	15
y	20	0

$\therefore$ Graph of straight line $4x + 3y = 60$ is the line joining the points (0, 20) and (15, 0).

Let us test for origin (0, 0) in inequation (*i*), we have $0 \le 60$ which is true.

$\therefore$ The region for inequality (*i*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

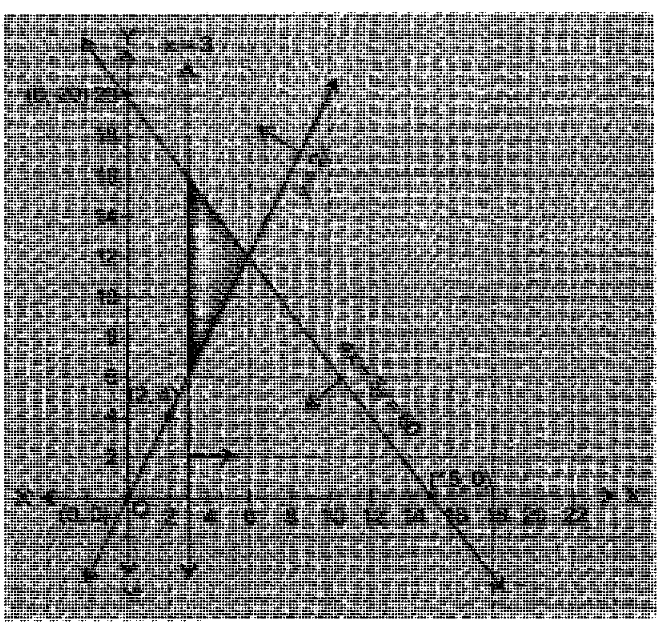
Now replacing $\ge$ by = in (*ii*), the corresponding equation is $y = 2x$

Table of values

x	0	2
y	0	4

$\therefore$ Graph of straight line $y = 2x$ is the line joining the points (0, 0) and (2, 4).

Now we select a point **not** on the line $y = 2x$. Since this line passes through the origin, we select a point other than origin say (0, 20). Putting $x = 0$ and $y = 20$ in (*ii*), we have $20 \ge 0$ which is true. Therefore region represented by (*ii*) is towards the side of the point (0, 20) (as indicated by an arrow) including the points on the line.



The inequality (*iii*) $x \geq 3$ represents the right half plane determined by the line $x = 3$, including points on the line. The inequalities (*iv*) $x \geq 0$, $y \geq 0$ represents the region in the first quadrant, including the points on the axes.

Hence, the common shaded region including the points on the lines, is the required solution region of the given system of inequalities.

14. $3x + 2y \leq 150$, $x + 4y \leq 80$, $x \leq 15$, $y, x \geq 0$

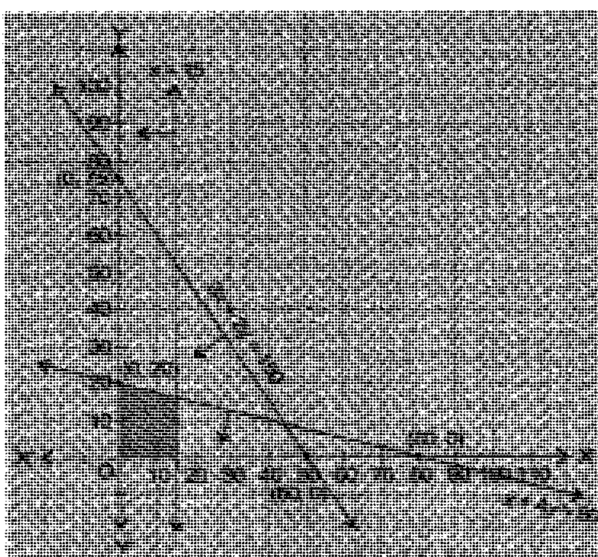
Sol. The given system of inequalities is

$$3x + 2y \leq 150 \quad \text{...}(i)$$

$$x + 4y \leq 80 \quad \text{...}(ii)$$

$$x \leq 15 \quad \text{...}(iii)$$

$$y, x \geq 0 \quad \text{...}(iv)$$



Replacing ≤ by = in (*i*), the corresponding equation is $3x + 2y = 150$

Table of values

x	0	50
y	75	0

∴ Graph of straight line $3x + 2y = 150$ is the line joining the points (0, 75) and (50, 0).

Let us test for origin (0, 0) in (*i*), we have $0 \leq 150$ which is true.

∴ The region for inequality (*i*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

Now replacing ≤ by = the corresponding equation is $x + 4y = 80$.

Table of values

x	0	80
y	20	0

∴ Graph of straight line $x + 4y = 80$ is the line joining the points (0, 20) and (80, 0).

Let us test for origin (0, 0) in (*i*), we have

$x \leq 80$ which is true.

∴ The region for inequality (*ii*) is in the origin side of the line (indicated by an arrow) including the points on the line.

The inequality (*iii*) $x \leq 15$ represents the left half plane determined by the vertical line $x = 15$, including points on the line. The inequalities (*iv*) $x \geq 0$, $y \geq 0$ represent the region in the first quadrant, including the points on the axes.

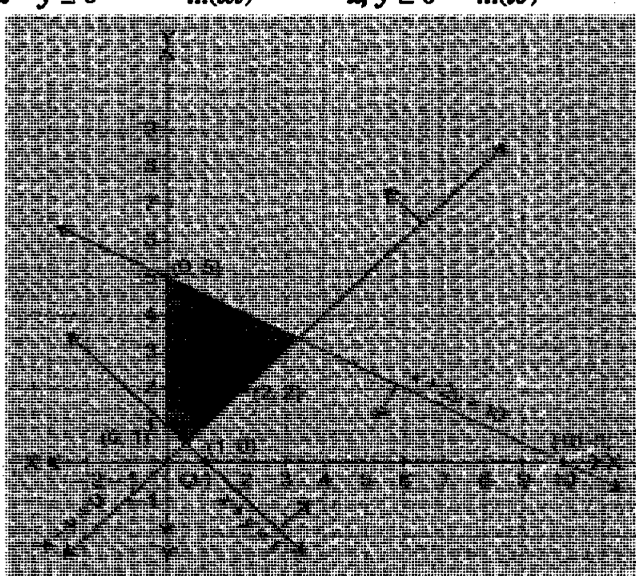
Hence the common shaded region including the points on the lines, is the required solution region of the given system of inequalities.

15. $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x, y \geq 0$

Sol. The given system of inequalities is

$x + 2y \leq 10$ ...(*i*) $x + y \geq 1$ ...(*ii*)

$x - y \leq 0$ ...(*iii*) $x, y \geq 0$ ...(*iv*)



Replacing ≤ by = in (*i*), the corresponding equation is
$x + 2y = 10$

Table of values

x	0	10
y	5	0

∴ Graph of straight line $x + 2y = 10$ is the line joining the points (0, 5) and (10, 0).

Let us test for origin (0, 0) in (*i*), we have $0 \le 10$ which is true.

∴ The region for inequality (*i*) is the region on the origin side of the line (indicated by an arrow) including the points on the line.

replacing ≥ by = in (*ii*) the corresponding equation is $x + y = 1$

Table of values

x	0	1
y	1	0

∴ Graph of the line $x + y = 1$ is the line joining the points (0, 1) and (1, 0).

Let us test for origin (0, 0) in (*ii*), we have

$0 \ge 1$ which is not true.

∴ The region for inequality (*ii*) is the rigion on the non-origin side of the line (indicated by an arrow) including the points on the line.

Now replacing ≤ by = in (*iii*), the corresponding equation is $x - y = 0$

Table of values

x	0	2
y	0	2

∴ Graph of the line $x - y = 0$ is the line joining the points (0, 0) and (2, 2).

Now we select a point **not** on the line $x - y = 0$ since this line passes through the origin. We select a point other than origin say (0, 5). Putting $x = 0$ and $y = 5$ in (*iii*), we have

$-5 \leq 0$ which is true. Therefore region represented by (*iii*) is towards the side of the point (0, 5) (as indicated by an arrow) including the points on the line.

The inequalities (*iv*) $x \geq 0$, $y \geq 0$ represent the region in the first quadrant, including the points on the axes.

Hence, the common shaded region, including points on the lines, is the required solution region of the given system of inequalities.

MISCELLANEOUS EXERCISE ON CHAPTER 6

(Page No.: 132)

Solve the inequalities in Exercises 1 to 6.

1. $2 \leq 3x - 4 \leq 5$

Sol. $2 \leq 3x - 4 \leq 5$

Adding 4 throughout, we get $2 + 4 \leq 3x - 4 + 4 \leq 5 + 4$

$$6 \leq 3x \leq 9$$

Dividing throughout by 3, we get

$$2 \leq x \leq 3$$

$\Rightarrow$ $x \in [2, 3]$.

2. $6 \leq -3(2x - 4) < 12$

Sol. $6 \leq -3(2x - 4) < 12$

$\Rightarrow$ $6 \leq -6x + 12 < 12$

Subtracting 12 throughout, we get $6 - 12 \leq -6x + 12 - 12 < 12 - 12$

$$-6 \leq -6x < 0$$

Dividing throughout by -6, which is negative, we get

$$1 \geq x > 0$$

$\Rightarrow$ $0 < x \leq 1$ $\Rightarrow$ $x \in (0, 1]$.

3. $-3 \leq 4 - \dfrac{7x}{2} \leq 18$

Sol. $-3 \leq 4 - \dfrac{7x}{2} \leq 18$

Subtracting 4 throughout, we get

$$-7 \le -\frac{7x}{2} \le 14$$

Multiplying throughout by $-\frac{2}{7}$, which is negative, we get

$$-\frac{2}{7}(-7) \ge -\frac{2}{7}\left(-\frac{7x}{2}\right) \ge -\frac{2}{7}(14)$$

$\Rightarrow \quad 2 \ge x \ge -4$

$\Rightarrow \quad -4 \le x \le 2$

$\Rightarrow \quad x \in [-4, 2].$

4. $-15 < \dfrac{3(x-2)}{5} \le 0$

Sol. $$-15 < \frac{3(x-2)}{5} \le 0$$

Multiplying throughout by 5, we get $-75 < 3(x-2) \le 0$

Dividing throughout by 3,

$$-25 < x - 2 \le 0$$

Adding 2 throughout, we get

$$-23 < x \le 2$$

$\Rightarrow \quad x \in (-23, 2].$

5. $-12 < 4 - \dfrac{3x}{-5} \le 2$

Sol. $$-12 < 4 - \frac{3x}{-5} \le 2$$

$\Rightarrow \quad -12 < 4 + \dfrac{3x}{5} \le 2$

Subtracting 4 throughout, we get

$$-16 < \frac{3x}{5} \le -2$$

Multiplying throughout by 5, we get $-80 < 3x \le -10$

Dividing throughout by 3,

$$-\frac{80}{3} < x \le -\frac{10}{3}$$

$$\Rightarrow \quad x \in \left(-\frac{80}{3}, -\frac{10}{3}\right].$$

6. $7 \le \dfrac{(3x+11)}{2} \le 11$

Sol. $7 \le \dfrac{(3x+11)}{2} \le 11$

Multiplying throughout by 2, we get

$$14 \le 3x + 11 \le 22$$

Subtracting 11 throughout, we get

$$3 \le 3x \le 11$$

Dividing throughout by 3, we get

$$1 \le x \le \frac{11}{3}$$

$$\Rightarrow \quad x \in \left[1, \frac{11}{3}\right].$$

Solve the inequalities in Exercises 7 to 10 and represent the solution graphically on number line.

7. $5x + 1 > -24,\ 5x - 1 < 24$

Sol. $5x + 1 > -24$ ...(*i*)

and $5x - 1 < 24$ ...(*ii*)

from (*i*), $5x > -24 - 1$

$\Rightarrow \quad 5x > -25 \quad \Rightarrow \quad x > -5$ ...(*iii*)

from (*ii*), $5x - 1 < 24$

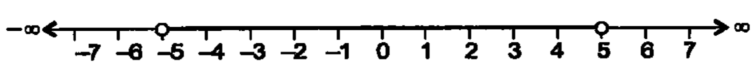
$\Rightarrow \quad 5x < 24 + 1 \quad \Rightarrow \quad 5x < 25$

$\Rightarrow \quad x < 5$ ...(*iv*)

From (*iii*) and (*iv*), we have $-5 < x < 5$

$\Rightarrow \ x \in (-5, 5)$

On the number line, the graph of solution set is shown by the bold line segment in the following figure.



8. $2(x - 1) < x + 5,\ 3(x + 2) > 2 - x$

Sol. $2(x - 1) < x + 5$ ...(*i*)

and $3(x + 2) > 2 - x$ ...(*ii*)

From (*i*), $2x - 2 < x + 5$

$\Rightarrow \quad 2x - x < 5 + 2 \quad \Rightarrow \quad x < 7$ ...(*iii*)

From (*ii*), $3x + 6 > 2 - x$

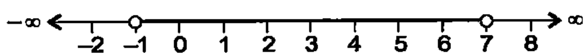
$\Rightarrow \quad 3x + x > 2 - 6 \quad \Rightarrow \quad 4x > -4$

$\Rightarrow \quad x > -1$ ...(*iv*)

From (*iii*) and (*iv*), we have $-1 < x < 7$

$\Rightarrow \quad x \in (-1, 7)$

On the number line, the graph of solution set is shown by the bold line segment is the following figure.



9. $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$

Sol. $3x - 7 > 2(x - 6)$ ...(*i*)

and $6 - x > 11 - 2x$ ...(*ii*)

From (*i*), $3x - 7 > 2x - 12$

$\Rightarrow \quad 3x - 2x > -12 + 7 \quad \Rightarrow \quad x > -5$ ...(*iii*)

From (*ii*), $-x + 2x > 11 - 6$

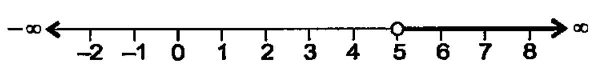
$\Rightarrow \quad x > 5$ ...(*iv*)

From (*iii*) and (*iv*), we have $x > 5$

($\because$ $x > 5 \quad \Rightarrow \quad x > -5$ also)

$\Rightarrow \quad 5 < x < \infty \quad \Rightarrow \quad x \in (5, \infty)$.

On the number line, the graph of solution set is shown by the bold ray in the following figure.



10. $5(2x - 7) - 3(2x + 3) \le 0$, $2x + 19 \le 6x + 47$.

Sol. $5(2x - 7) - 3(2x + 3) \le 0$ ...(*i*)

and $2x + 19 \le 6x + 47$ ...(*ii*)

From (*i*), $10x - 35 - 6x - 9 \le 0$

$\Rightarrow \quad 4x - 44 \le 0 \quad \Rightarrow \quad 4x \le 44$

$\Rightarrow \quad x \le 11$ ...(*iii*)

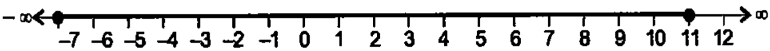
From (*ii*), $2x - 6x \Rightarrow 47 - 19$

$\Rightarrow \quad -4x \le 28 \quad \Rightarrow \quad x \ge -7$ ...(*iv*)

From (*iii*) and (*iv*), we have $-7 \le x \le 11$

$\Rightarrow \quad x \in [-7, 11]$

On the number line, the graph of solution set is shown by the bold line segment in the following figure.



11. A solution is to be kept between 68°F and 77°F. What is the range in temperature in degree Celsius (C) if the Celsius/Fahrenheit (F) conversion formula is given by

$$F = \frac{9}{5} C + 32?$$

Sol. Given, $68 < F < 77$

(Between 68°F and 77°F ⇒ ≠ 68 and ≠ 77)

Putting $F = \frac{9}{5} C + 32$ (given), we get

$$68 < \frac{9}{5} C + 32 < 77$$

Subtracting 32 throughout, we get

$$36 < \frac{9}{5} C < 45$$

Multiplying throughout by 5, we have $180 < 9C < 225$

Dividing by 9,

⇒ $20 < C < 25$

Thus, the required range of the temperature is **between** 20°C and 25°C.

12. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Sol. It is given that we have 640 litres of 8% boric acid solution.

Let x be number of litres of 2% boric acid solution.

∴ Total mixture = $(640 + x)$ litres

Now according to given

$$2\% \text{ of } x + 8\% \text{ of } (640) > 4\% \text{ of } (x + 640)$$

and $$2\% \text{ of } x + 8\% \text{ of } (640) < 6\% \text{ of } (x + 640)$$

i.e. $$\frac{2x}{100}+\frac{8(640)}{100}>\frac{4x}{100}+\frac{4(640)}{100}$$

and $$\frac{2x}{100}+\frac{8(640)}{100}<\frac{6x}{100}+\frac{6(640)}{100}$$

Multiplying every term by 100,

$$2x + 5120 > 4x + 2560$$

and $$2x + 5120 < 6x + 3840$$

or $$2560 > 2x \quad \text{and} \quad 1280 < 4x$$

or $$1280 > x \quad \text{and} \quad 320 < x$$

$\therefore$ $$320 < x < 1280$$

$\therefore$ Amount x of 2% boric acid solution to be added is more than 320 litres but less than 1280 litres.

13. How many litres of water will have to be added to 1125 litres of 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Sol. Let x litres of water be required to be added. Since pure water contains 0% acid, *i.e.*, no acid, we have

$$0\% \text{ of } x + 45\% \text{ of } 1125 > 25\% \text{ of } (1125 + x) \quad ...(i)$$

and $$0\% \text{ of } x + 45\% \text{ of } 1125 < 30\% \text{ of } (1125 + x) ...(ii)$$

From (*i*), $$\frac{45}{100}(1125) > \frac{25}{100}(1125 + x)$$

Multiplying throughout by 100, we get

$$45 \times 1125 > 25(1125 + x)$$

Dividing by 25, we get $\quad 45 \times 45 > 1125 + x$

$\Rightarrow \quad 2025 > 1125 + x \qquad \Rightarrow \quad 900 > x$

$\Rightarrow \quad x < 900 \qquad ...(iii)$

From (*ii*), $$\frac{45}{100}(1125) < \frac{30}{100}(1125 + x)$$

Multiplying throughout by 100, we get

$$45 \times 1125 < 30 \times (1125 + x)$$

Dividing by 15, $3 \times 1125 < 2\,(1125 + x)$

$\Rightarrow 3375 < 2250 + 2x$

$\Rightarrow 1125 < 2x$

Dividing by 2, we get $\frac{1125}{2} < x$

$\Rightarrow \quad 562.5 < x$ ...(*iv*)

From (*iii*) and (*iv*), $562.5 < x < 900$.

$\therefore$ Amount of water to be added is more than 562.5 litres and less than 900 litres.

14. IQ of a person is given by the formula

$$IQ = \frac{MA}{CA} \times 100,$$

where MA is mental age and CA is chronological age. If 80 ≤ IQ ≤ 140 for a group of 12 years old children, find the range of their mental age.

Sol. Given $80 \le IQ \le 140$ ..(*i*)

Let x be the mental age for chronological age 12.

$$\therefore \quad IQ = \frac{MA}{CA} \times 100 \quad \text{(given)}$$

$$= \frac{x}{12} \times 100$$

Putting this value of IQ in (*i*),

$$80 \le \frac{x}{12} \times 100 \le 140$$

Multiplying by 12,

$$960 \le 100x \le 1680$$

Dividing by 100, $9.6 \le x \le 16.8$

$\therefore$ Required Range of mental age for 12 years old children is from 9.6 to 16.8.

Remark: From $\Rightarrow$ Both 9.6 and 16.8 are included.