CLASS X (2020-21)
MATHEMATICS BASIC(241)
SAMPLE PAPER-2

Time : 3 Hours **Maximum Marks : 80**

General Instructions :
1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :
1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :
1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

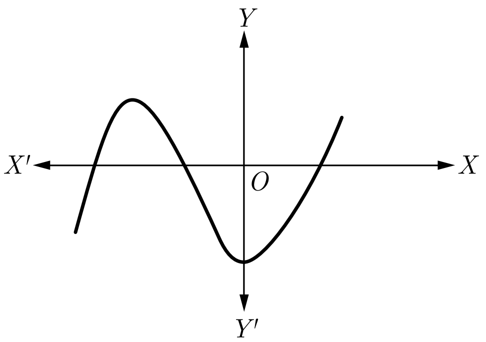
Section - I

1. Find the HCF and the LCM of 12, 21, 15.

Ans : [Board 2020 Delhi Standard]

We have
$$12 = 2 \times 2 \times 3$$
$$21 = 3 \times 7$$
$$15 = 3 \times 5$$
$$\text{HCF}(12, 21, 15) = 3$$
$$\text{LCM}(12, 21, 15) = 2 \times 2 \times 3 \times 5 \times 7 = 420$$

2. The graph of a polynomial is shown in Figure. What is the number of its zeroes?



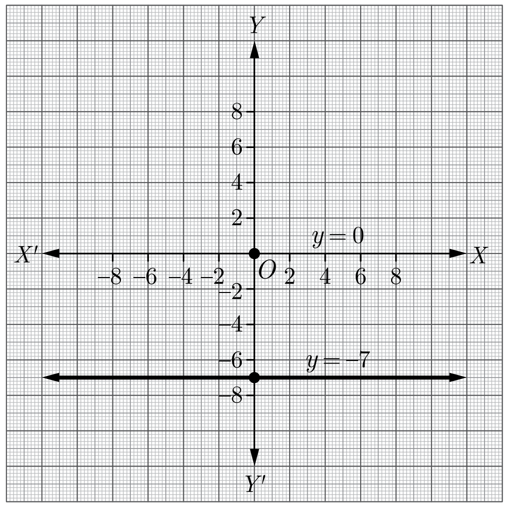
Ans : [Board 2020 Delhi Basic]

Since, the graph cuts the x-axis at 3 points, the number of zeroes of polynomial $p(x)$ is 3.

3. The pair of equations $y = 0$ and $y = -7$ has no solution. Justify.

Ans :

The given pair of equations are
$$y = 0 \quad y = -7$$



The pair of both equations are parallel to x-axis and we know that parallel lines never intersects. So, there is no solution of these lines.

4. What are the real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$?

Ans :

We have $x^{2/3} + x^{1/3} - 2 = 0$

Substituting $x^{1/3} = y$ we obtain,
$$y^2 + y - 2 = 0$$
$$(y-1)(y+2) = 0 \Rightarrow y = 1 \text{ or } y = -2$$

Thus $x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$

or $x^{1/3} = -2 \Rightarrow x = (-2)^3 = -8$

Hence, the real roots of the given equations are $1, -8$.

5. The n^{th} term of an AP is $(7 - 4n)$, then what is its common difference?

Ans : [Board 2020 Delhi Basic]

We have $a_n = 7 - 4n$

Putting $n = 1$, $a_1 = 7 - 4 = 3$

Putting $n = 2$, $a_2 = 7 - 8 = -1$

Common difference $d = a_2 - a_1 = -1 - 3 = -4$

or

In an AP, if the common difference $d = -4$, and the seventh term a_7 is 4, then find the first term.

Ans : [Board 2018]

We have $\quad d = -4$

and $\quad a_7 = 4$

Now $\quad a_n = a + (n-1)d$

$$a_7 = a + (7-1)d$$

$$4 = a + (7-1)(-4)$$

$$4 = a - 24 \Rightarrow a = 4 + 24 = 28$$

First term of the AP is 28.

6. ΔABC is isosceles with $AC = BC$. If $AB^2 = 2AC^2$, then find the measure of $\angle C$.

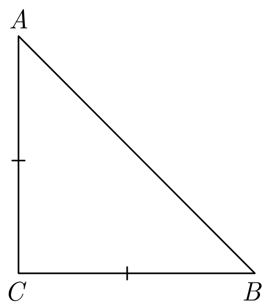
Ans : [Board 2020 Delhi Basic]

We have $AB^2 = 2AC^2$

$$AB^2 = AC^2 + AC^2$$

$$AB^2 = BC^2 + AC^2 \qquad (BC = AC)$$

It satisfies the Pythagoras theorem. Thus according to converse of Pythagoras theorem, ΔABC is a right angle triangle and $\angle C = 90°$.



7. Find the point on the x-axis which is equidistant from the points $A(-2, 3)$ and $B(5, 4)$?

Ans :

Let $P(x, 0)$ be a point on x-axis such that,

$$AP = BP$$

$$AP^2 = BP^2$$

$$(x+2)^2 + (0-3)^2 = (x-5)^2 + (0+4)^2$$

$$x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$$

$$14x = 28$$

$$x = 2$$

Hence required point is (2, 0).

or

If three points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then what is the value of λ?

Ans :

Let the given points are $A(0, 0)$, $B(3, \sqrt{3})$ and $C(3, \lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

$$\sqrt{(3-0)^2 + (\sqrt{3}-0)^2} = \sqrt{(3-0)^2 + (\lambda - 0)^2}$$

$$9 + 3 = 9 + \lambda^2$$

$$\lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}$$

8. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is ....

Ans : [Board 2020 Delhi Basic]

We have $\quad \tan(A+B) = \sqrt{3}$

$$= \tan 60°$$

Hence, $\quad A + B = 60° \quad$ ...(1)

Again, $\quad \tan(A-B) = \frac{1}{\sqrt{3}}$

$$= \tan 30°$$

$$A - B = 30° \quad ...(2)$$

Adding equation (1) and (2) we get

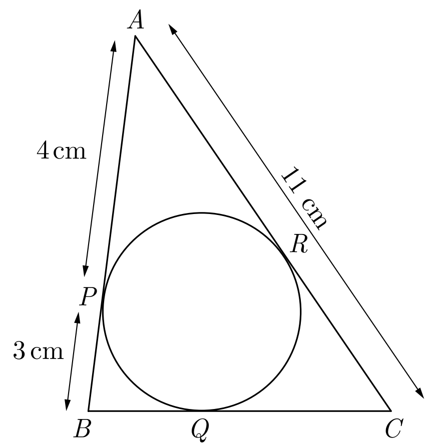
$$2A = 90° \Rightarrow A = 45°$$

9. The .... is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Ans :

line of sight

10. In figure, ΔABC is circumscribing a circle, the length of BC is .... cm.



Ans : [Board 2020 Delhi Standard]

Since AP and AR are tangents to the circle from external point A, we have

$$AP = AR = 4 \text{ cm}$$

Similarly, PB and BQ are tangents.

Therefore $\quad BP = BQ = 3 \text{ cm}$

Now, $\quad CR = AC - AR$

$$= 11 - 4 = 7 \text{ cm}$$

Similarly, CR and CQ are tangents.

Therefore $\quad CR = CQ = 7 \text{ cm}$

Now, $\quad BC = BQ + CQ = 3 + 7 = 10 \text{ cm}$

Hence, the length of BC is 10 cm.

or

If the angle between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

Ans : [Board Term-2 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°. Thus angle at the point of intersection of tangents

$$= 180° - 130° = 50°$$

11. In drawing a triangle, if $AB = 3$ cm, $BC = 2$ cm and $AC = 6$ cm. What is the possibility that a triangle cannot be drawn.

Ans : [Board Term-2 2014]

When $AB + BC < AC$ triangle cannot be drawn.
Here $3\,\text{cm} + 2\,\text{cm} < 6\,\text{cm}$. Hence ΔABC can not be drawn.

12. Find the ratio of volumes of two cones with same radii.

Ans :

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_1^2 h_2 \quad (r_1 = r_2)$$

$$h_1 : h_2$$

13. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then what is the median of the new set?

Ans :

Since, $n = 9$

then, median term $= \left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ item.

Now, last four observations are increased by 2, but median is 5^{th} observation, which is remaining unchanged. Hence there will be no change in median.

or

If the coordinates of the point of intersection of less than ogive and more than ogive is (13.5, 20), then find the value of median.

Ans :

The abscissa of point of intersection gives the median of the data. So, median is 13.5.

14. If a card is selected from a deck of 52 cards, then find the probability of its being a red face card?

Ans :

In a deck of 52 cards, there are 12 face cards i.e., 6 red and 6 black cards.

$$n(S) = 52$$

$$n(E) = 6$$

So, probability of getting a red face card,

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

or

A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. Find the number of outcomes favourable to E.

Ans :

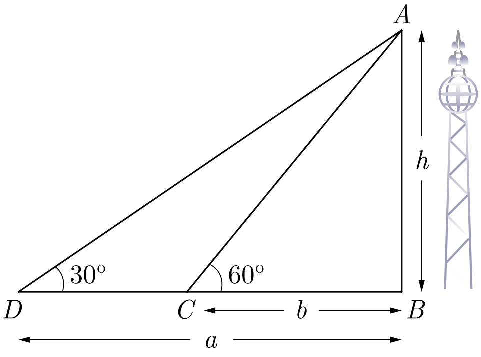
In a deck of 52 cards, there are 13 cards of heart and 1 is ace of heart.
Hence, the number of outcomes favourable

$$n(E) = 52 - 1 = 51$$

15. If the angles of elevation of the top of a tower from two points distant a and $b(a > b)$ from its foot and in the same straight line from it are respectively $30°$ and $60°$, then find the height of the tower.

Ans : [Board Term-2 2014]

Let the height of tower be h. As per given in question we have drawn figure below.



From ΔABD, $\frac{h}{a} = \tan 30°$

$$h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad ...(1)$$

From ΔABC, $\frac{h}{b} = \tan 60°$

$$h = b \times \sqrt{3} = b\sqrt{3} \quad ...(2)$$

From (1) we get $a = \sqrt{3}\,h$

From (2) get $b = \frac{h}{\sqrt{3}}$

Thus $a \times b = \sqrt{3}\,h \times \frac{h}{\sqrt{3}}$

$$ab = h^2$$

$$h = \sqrt{ab}$$

Hence, the height of the tower is $\sqrt{ab}$.

16. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm^3.

Ans : [Board Term-2 2012]

Let r be the radius of bowl, then circumference of bowl,

$$2\pi r = 132$$

$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Capacity i.e volume of the bowl,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 19404 \text{ cm}^3$$

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. John and Priya went for a small picnic. After having their lunch Priya insisted to travel in a motor boat. The speed of the motor boat was 20 km/hr. Priya being a Mathematics student wanted to know the speed of the current. So she noted the time for upstream and downstream.

She found that for covering the distance of 15 km the boat took 1 hour more for upstream than downstream.

(i) Let speed of the current be x km/hr. then speed of the motorboat in upstream will be
(a) 20 km/hr (b) $(20+x)$ km/hr
(c) $(20-x)$ km/hr (d) 2 km/hr

(ii) What is the relation between speed distance and time?
(a) speed = (distance)/time
(b) distance = (speed)/time
(c) time = speed × distance
(d) none of these

(iii) Which is the correct quadratic equation for the speed of the current ?
(a) $x^2+30x-200=0$
(b) $x^2+20x-400=0$
(c) $x^2+30x-400=0$
(d) $x^2-20x-400=0$

(iv) What is the speed of current ?
(a) 20 km/hour (b) 10 km/hour
(c) 15 km/hour (d) 25 km/hour

(v) How much time boat took in downstream ?
(a) 90 minute (b) 15 minute
(c) 30 minute (d) 45 minute

Ans :

(i) In this case speed of the motorboat in upstream will be $(20-x)$ km/hr.
Thus (c) is correct option.
(ii) distance = (speed)/time
Thus (b) is correct option.
(iii) As per question,

$$\frac{15}{20-x} = \frac{15}{20+x}+1$$
$$15(20+x) = 15(20-x)+(20-x)(20+x)$$
$$15x = -15x+(20^2-x^2)$$
$$30x = -x^2+400$$
$$x^2+30x-400 = 0$$

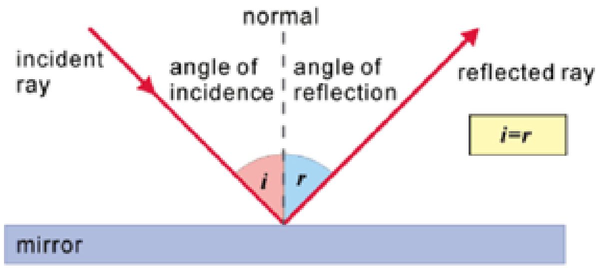
Thus (c) is correct option.
(iv) We have $x^2+30x-400 = 0$

$$x^2+40x-10x-400 = 0$$
$$x(x+40)-10x(x+40) = 0$$
$$(x+40)(x-10) = 0$$
$$x = 10, -40$$

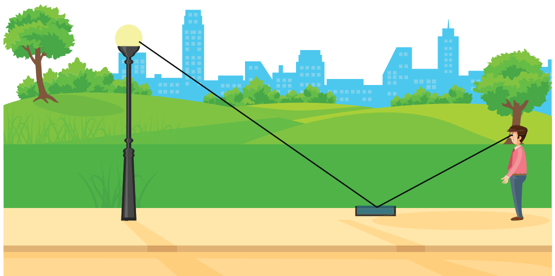
Here $x=10$ is only possible.
Thus (b) is correct option.
(v) In downstream speed of boat
$= 20+10 = 30$ km/hr

Time take to cover distance 15 km will be 30 minutes.
Thus (c) is correct option.

18. The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.



Ramesh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Ramesh's eye level is 1.5 m above the ground. The distance of Ramesh and the pole from the mirror are 1.8 m and 6 m respectively.



(i) Which criterion of similarity is applicable to similar triangles?
(a) SSA (b) ASA
(c) SSS (d) AA

(ii) What is the height of the pole?
(a) 6 metres (b) 8 metres
(c) 5 metres (d) 4 metres

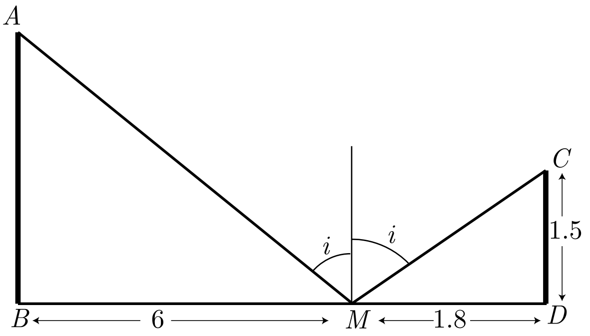
(iii) If angle of incidence is i, which of the following is correct relation?
(a) $\tan i = \frac{5}{6}$ (b) $\tan i = \frac{6}{5}$
(c) $\tan i = \frac{3}{5}$ (d) $\tan i = \frac{5}{3}$

(iv) Now Ramesh move behind such that distance between pole and Ramesh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Ramesh ?
(a) 7 metres (b) 3 metres
(c) 5 metres (d) 4 metres

(v) What is the distance between mirror and pole?
(a) 9 metres (b) 8 metres
(c) 12 metres (d) 10 metres

Ans :

(i) Since angle of incidence and angle of reflection are the same, we draw the figure as given below.



Now $\angle AMB = \angle CMD$

Also, $\angle ABM = \angle CDM = 90^\circ$

So, by AA similarity criterion,

$$\Delta AMB \sim \Delta CDM$$

Thus (d) is correct option.

(ii) As $\Delta ABM \sim \Delta CDM$ we obtain,

$$\frac{AB}{CD} = \frac{BM}{DM}$$

$$\frac{AB}{1.8} = \frac{5}{1.5}$$

$$AB = \frac{6}{1.8} \times 1.5 = 5 \text{ m}$$

Thus, the height of the pole is 5 metres.

Thus (c) is correct option.

(iii) From the geometry of diagram we have

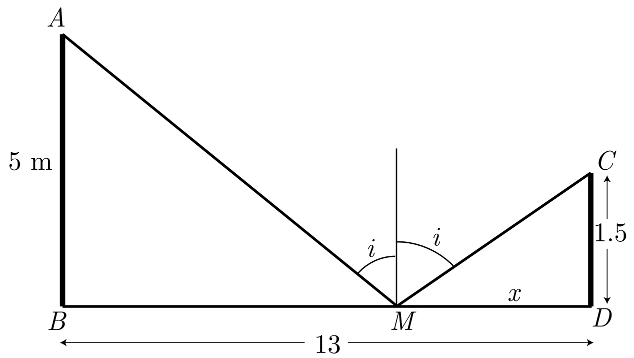
$$\angle MCD = i$$

$$\tan \angle MCD = \frac{MD}{CD}$$

$$\tan i = \frac{1.8}{1.5} = \frac{6}{5}$$

Thus (b) is correct option.

(iv) On the basis of given information we have drawn the figure as follows:



Once again due to AA similarity criterion,

$$\Delta AMB \sim \Delta CDM$$

$$\frac{5}{13-x} = \frac{1.5}{x}$$

$$\frac{1}{13-x} = \frac{0.3}{x}$$

$$x = 3.9 - 0.3x$$

$$1.3x = 3.9$$

$\Rightarrow$ $x = 3$

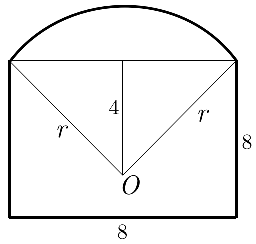
Thus (b) is correct option.

(v) Distance between mirror and pole,

$$= 13 - x = 13 - 3 = 10 \text{ m}$$

Thus (d) is correct option.

19. A barn is an agricultural building usually on farms and used for various purposes. A barn refers to structures that house livestock, including cattle and horses, as well as equipment and fodder, and often grain.



Ramkaran want to build a barn at his farm. He has made a design for it which is shown above. Here roof is arc of a circle of radius r at centre O.

(i) What is the value of radius of arc ?
 (a) $4\sqrt{3}$ m
 (b) $4\sqrt{2}$ m
 (c) $4\sqrt{3}$ m
 (d) $2\sqrt{2}$ m

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(ii) What is the curved width of roof ?
 (a) $2\pi\sqrt{3}$ m (b) $4\pi\sqrt{2}$ m
 (c) $2\pi\sqrt{2}$ m (d) $4\pi\sqrt{3}$ m

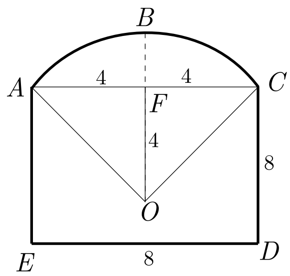
(iii) What is area of cross section of barn ?
 (a) $8(6+\pi)\,\text{m}^2$ (b) $4(6+\pi)\,\text{m}^2$
 (c) $8(3+\pi)\,\text{m}^2$ (d) $4(3+\pi)\,\text{m}^2$

(iv) If the length of the barn is 12 meters, what is the curved surface area of roof?
 (a) $32\sqrt{2}\pi\,\text{m}^2$ (b) $16\sqrt{2}\pi\,\text{m}^2$
 (c) $48\sqrt{2}\pi\,\text{m}^2$ (d) $24\sqrt{2}\pi\,\text{m}^2$

(v) What is the storage capacity of barn ?
 (a) $48(6+\pi)\,\text{m}^3$ (b) $48(6+\pi)\,\text{m}^3$
 (c) $96(6+\pi)\,\text{m}^3$ (d) $96(3+\pi)\,\text{m}^3$

Ans :

(i) We redraw the cross section of barn as shown below.



In right triangle ΔAFO,

$$AO = \sqrt{AF^2 + FO^2}$$

$$= \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ m}$$

Thus $AO = 4\sqrt{2}$ which is also radius of curved arc.

Thus (b) is correct option.

(ii) In right angle triangle ΔAFO is also isosceles triangle

Thus, $\angle FAO = \angle FOA = 45^\circ$

Similarly, $\angle FOC = 45^\circ$

Thus $\angle AOC = \angle AOF + \angle FOC$

$$= 45^\circ + 45^\circ = 90^\circ$$

Curved width $\dfrac{2\pi r\theta}{360^\circ} = \dfrac{2\pi \times 4\sqrt{2} \times 90^\circ}{360^\circ}$

$= 2\pi\sqrt{2}$ m

Thus (c) is correct option.

(iii) Area of cross section

$=$ Area of $AECD$ $+$

$+$ Area of section $ABCO$

$-$ Area of triangle ACO

$$= 8 \times 8 + \frac{\pi(4\sqrt{2})^2 \times 90^\circ}{360^\circ} - \frac{1}{2} \times 4 \times 8$$

$$= 64 + 8\pi - 16$$

$$= 48 + 8\pi = 8(6+\pi)$$

Thus (a) is correct option.

(iv) Curved surface area of roof

$$= 2\pi\sqrt{2} \times 12$$

$$= 24\sqrt{2}\,\pi \text{ m}^2$$

Thus (d) is correct option.

(v) Storage capacity of barn,

$=$ Cross section area $\times$ Length

$$= 8(6+\pi) \times 12$$

$$= 96(6+\pi)\text{ m}^3$$

Thus (c) is correct option.

20. Student-teacher ratio expresses the relationship between the number of students enrolled in a school and the number teachers employed by the school. Student-teacher ratio is important for a number of reasons. It can be used as a tool to measure teacher workload as well as the allocation of resources. A low student-teacher ratio indicates the burden on a single teacher of teaching multiple students as well as the lack of time that each student gets.

A survey was conducted in the 100 secondary school of Rajasthan and following frequency distribution table was prepared

Students per teacher	Number of School
20-25	5
25-30	15
30-35	25
35-40	30
40-45	15
45-50	10

(i) What is the upper limit of median class ?
(a) 25 (b) 40
(c) 35 (d) 35

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(ii) What is the median value of students per teacher?
(a) 25.67 (b) 37.67
(c) 35.83 (d) 39.67

(iii) What is the lower limit of model class ?
(a) 20 (b) 40
(c) 35 (d) 45

(iv) What is the model value of students per teacher ?
(a) 35.25 (b) 36.25
(c) 37.25 (d) 39.25

(v) What is the mean value of students per teacher ?
(a) 35.625 (b) 36.250
(c) 38.500 (d) 39.275

Ans :

(i) We prepare following cumulative frequency table to find median class.

Students per teacher	Number of School	c.f
20-25	5	5
25-30	15	20
30-35	25	45
35-40	30	75
40-45	15	90
45-50	10	100
	$N = 100$	

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 75 and the corresponding class is 35-40. Thus median class is 35-40 and upper limit is 40.

Thus (b) is correct option.

(ii) Median, $M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right)h$

$$= 35 + \frac{50-45}{30} \times 5$$

$$= 35 + \frac{5}{6} = \frac{215}{6} = 35.83$$

Thus (c) is correct option.

(iii) Class 35-40 has the maximum frequency 30, therefore this is model class. Lower limit of this class is 35.

Thus (c) is correct option.

(iv) Here, $l = 35$, $f_1 = 30$, $f_0 = 25$, $f_2 = 15$ and $h = 5$

Mode, $M_o = l + h\left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$$= 35 + \frac{30-25}{60-25-15} \times 5$$

$$= 35 + \frac{5}{20} \times 5$$

$$= 35 + 1.25 = 36.25$$

Thus (b) is correct option.

(v) Now $3M_d = M_o + 2M$

$$\frac{3 \times 215}{6} = 36.25 + 2M$$

$$2 \times 215 = 4 \times 36.25 + 8M$$

$$430 = 145 + 8M$$

$$8M = 430 - 145 = 285$$

$$M = \frac{285}{8} = 35.625$$

Thus (a) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Find HCF and LCM of 404 and 96 and verify that HCF $\times$ LCM = Product of the two given numbers.

Ans : [Board 2018]

We have
$$\begin{aligned} 404 &= 2\times 2\times 101 \\ &= 2^2\times 101 \\ 96 &= 2\times 2\times 2\times 2\times 2\times 3 \\ &= 2^5\times 3 \\ \text{HCF}(404,\,96) &= 2^2 = 4 \\ \text{LCM}(404,\,96) &= 101\times 2^5\times 3 = 9696 \\ \text{HCF}\times\text{LCM} &= 4\times 9696 = 38784 \end{aligned}$$

Also, $404\times 96 = 38784$

Hence, HCF $\times$ LCM = Product of 404 and 96

22. Find whether the lines represented by $2x+y=3$ and $4x+2y=6$ are parallel, coincident or intersecting.

Ans : [Board Term-1 2016, MV98HN3]

Here $a_1=2, b_1=1, c_1=-3$ and $a_1=4, b_2=2, c_2=-6$

If
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

then the lines are parallel.

Clearly
$$\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$$

Hence lines are coincident.

23. Find a relation between x and y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$.

Ans : [Board Term-2 SQP 2016]

Let $P(x,y)$ is equidistant from $A(-5,3)$ and $B(7,2)$, then we have
$$\begin{aligned} AP &= BP \\ \sqrt{(x+5)^2+(y-3)^2} &= \sqrt{(x-7)^2+(y-2)^2} \\ (x+5)^2+(y-3)^2 &= (x-7)^2+(y-2)^2 \\ 10x+25-6y+9 &= -14x+49-4y+4 \\ 24x+34 &= 2y+53 \\ 24x-2y &= 19 \end{aligned}$$

Thus $24x-2y-19=0$ is the required relation.

or

Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the co-ordinates of the point of division.

Ans : [Delhi Set I, II, III, 2016]

Let y -axis be divides the line-segment joining $A(5,-6)$ and $B(-1,-4)$ at the point $P(x,y)$ in the ratio $AP:PB = k:1$

Now, the coordinates of point of division P,
$$\begin{aligned} (x,y) &= \left(\frac{k(-1)+1(5)}{k+1},\ \frac{k(-4)+1(-6)}{k+1}\right) \\ &= \left(\frac{-k+5}{k+1},\ \frac{-4k-6}{k+1}\right) \end{aligned}$$

Since P lies on y axis, therefore $x=0$, which gives
$$\frac{5-k}{k+1} = 0 \Rightarrow k = 5$$

Hence required ratio is 5:1,

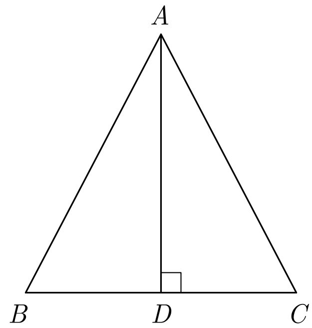
Now
$$y = \frac{-4(5)-6}{6} = \frac{-13}{3}$$

Hence point on y-axis is $\left(0,-\frac{13}{3}\right)$.

24. Find cosec30^o and $\cos 60^o$ geometrically.

Ans : [Board Term-1 2015]

Let a triangle ABC with each side equal to $2a$ as shown below.



In ΔABC, $\angle A = \angle B = \angle C = 60^o$

Now we draw AD perpendicular to BC, then
$$\begin{aligned} \Delta BDA &\cong \Delta CDA \\ BD &= CD \\ \angle BAD &= CAD = 30^o \qquad \text{by } CPCT \\ AD &= \sqrt{3}a \end{aligned}$$

In ΔBDA,
$$\text{cosec}\,30^o = \frac{AB}{BD} = \frac{2a}{a} = 2$$

and
$$\cos 60^o = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

or

If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta-7\sin\theta}{11\cos\theta+7\sin\theta}$.

Ans : [Board Term-1 2012]

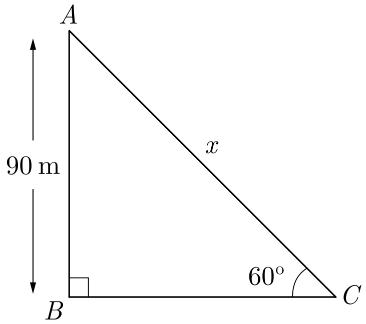
We have
$$\begin{aligned} 4\cos\theta &= 11\sin\theta \\ \text{or,}\quad \cos\theta &= \frac{11}{4}\sin\theta \end{aligned}$$

Now
$$\begin{aligned} \frac{11\cos\theta-7\sin\theta}{11\cos\theta+7\sin\theta} &= \frac{11\times\frac{11}{4}\sin\theta-7\sin\theta}{11\times\frac{11}{4}\sin\theta+7\sin\theta} \\ &= \frac{\sin\theta\left(\frac{121}{4}-7\right)}{\sin\theta\left(\frac{121}{4}+7\right)} \\ &= \frac{121-28}{121+28} = \frac{93}{149} \end{aligned}$$

25. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

Ans : [Board Term-2 2011, 2014]

As per given in question we have drawn figure below.



In right ΔABC, we have

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{90}{x}$$

$$x = \frac{90 \times 2}{\sqrt{3}} = \frac{180}{\sqrt{3}} = \frac{3 \times 60}{\sqrt{3}}$$

$$= 60\sqrt{3} = 60 \times 1.732$$

Hence length of string is 103.92 m.

26. Median of a data is 52.5 and its mean is 54, use empirical relationship between three measure of central tendency to find its mode.

Ans : [Board Term-1 2012]

Median $M_d = 52.5$

and mean $M = 54$

Now $3M_d = M_o + 2M$

$$3 \times 52.5 = M_o + 2 \times 54$$

Mode $M_o = 157.5 - 108 = 49.5$

27. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. [3]

Ans : [Board 2019 Delhi]

Assume that $\frac{2+\sqrt{3}}{5}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$\frac{2+\sqrt{3}}{5} = \frac{p}{q}$$

$$2+\sqrt{3} = \frac{5p}{q}$$

$$\sqrt{3} = \frac{5p}{q} - 2$$

$$\sqrt{3} = \frac{5p-2q}{q}$$

Since, p and q are co-prime integers, then $\frac{5p-2q}{q}$ is a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. So, our assumption is wrong. Therefore $\frac{2+\sqrt{3}}{5}$ is an irrational number.

28. Determine the values of m and n so that the following system of linear equation have infinite number of solutions :

$$(2m-1)x + 3y - 5 = 0$$

$$3x + (n-1)y - 2 = 0$$

Ans : [Board Term-1 2013, VKH6FFC; 2011, Set-66

We have $(2m-1)x + 3y - 5 = 0$ ...(1)

Here $a_1 = 2m-1, b_1 = 3, c_1 = -5$

$$3x + (n-1)y - 2 = 0 \quad ...(2)$$

Here $a_2 = 3, b_2 = (n-1), c_2 = -2$

For a pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$

or $2(2m-1) = 15$ and $5(n-1) = 6$

Hence, $m = \frac{17}{4}, n = \frac{11}{5}$

29. Find the 20^{th} term of an AP whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a + 2d = 7$ (1)

$$a_7 = 3a_3 + 2$$

$$a + 6d = 3 \times 7 + 2 = 23 \quad (2)$$

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$

$$a + 8 = 7 \Rightarrow a = -1$$

$$a_{20} = a + 19d = -1 + 19 \times 4 = 75$$

$$a_n = a + (n-1)d$$

$$= -1 + 4n - 4$$

$$= 4n - 5.$$

Hence n^{th} term is $4n-5$.

or

If 7^{th} term of an AP is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Ans : [Board Term-2 Delhi 2014]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$ (1)

$$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7} \quad (2)$$

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \Rightarrow d = \frac{1}{63}$$

Substituting the value of d in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$

$$a = \frac{1}{7} - \frac{8}{63}$$

$$= \frac{9-8}{63} = \frac{1}{63}$$

Thus $a_{63} = a + (63-1)d$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$

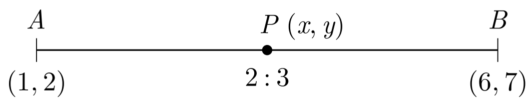
$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$.

30. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$.

Ans : [Board Term-2 OD 2015]

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP{:}PB = 2{:}3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{14+6}{5} = 4$$

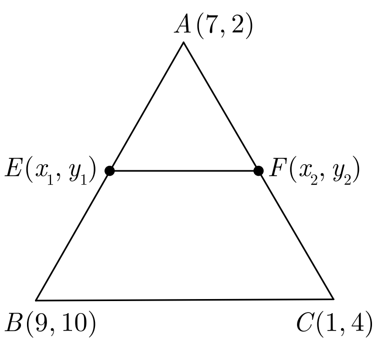
Thus $P(x, y) = (3, 4)$

or

The co-ordinates of the vertices of ΔABC are $A(7,2)$, $B(9,10)$ and $C(1,4)$. If E and F are the mid-points of AB and AC respectively, prove that $EF = \frac{1}{2}BC$.

Ans : [Board Term-2 2015]

Let the mid-points of AB and AC be $E(x_1, y_1)$ and $F(x_2, y_2)$. As per question, triangle is shown below.



Co-ordinates of point E,

$$(x_1, y_1) = \left(\frac{9+7}{2}, \frac{10+2}{2}\right) = (8, 6)$$

Co-ordinates of point F,

$$(x_2, y_2) = \left(\frac{7+1}{2}, \frac{2+4}{2}\right) = (4, 3)$$

Length, $$EF = \sqrt{(8-4)^2 + (6-3)^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= 5 \text{ units} \quad ...(1)$$

Length $$BC = \sqrt{(9-1)^2 + (10-4)^2}$$
$$= \sqrt{8^2 + 6^2}$$
$$= 10 \text{ units} \quad ...(2)$$

From equation (1) and (2) we get

$$EF = \frac{1}{2}BC$$ Hence proved.

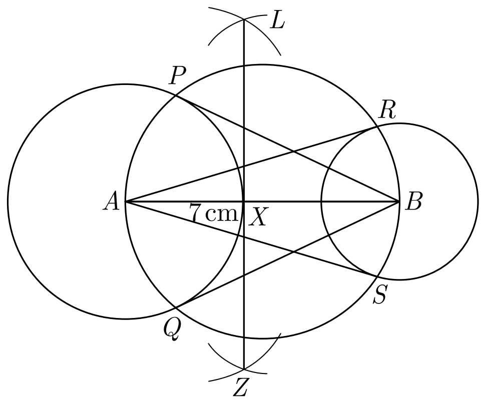
Section D

31. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Ans : [Board 2020 Delhi Standard]

Steps of construction :

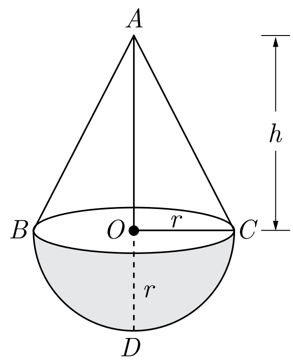
1. Draw a line segment AB of length 7 cm.
2. Draw a circle with A as centre and radius 3 cm.
3. Draw another circle with B as centre and radius 2 cm.
4. Draw another circle taking AB as diameter circle, which intersects first two circles at P and Q, R and S.
5. Join B to P, B to Q A to R and A to S. Hence, BP, BQ, AR and AS are the required tangents.



32. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

Ans : [Board 2020 OD Standard]

Let ABC be a cone, which is mounted on a hemisphere.



We have $OC = OD = r$

Curved surface area of the hemispherical part

$$= \frac{1}{2}(4\pi r^2) = 2\pi r^2$$

Slant height of a cone,

$$l = \sqrt{r^2 + h^2}$$

Curved surface area of a cone $= \pi rl$

$$= \pi r\sqrt{h^2 + r^2}$$

Since curved surface areas of the hemispherical part and the conical part are equal,

$$2\pi r^2 = \pi r\sqrt{h^2+r^2}$$

$$2r = \sqrt{h^2+r^2}$$

Squaring both of the sides, we have

$$4r^2 = h^2+r^2$$

$$4r^2-r^2 = h^2$$

$$3r^2 = h^2$$

$$\frac{r^2}{h^2} = \frac{1}{3}$$

$$\frac{r}{h} = \frac{1}{\sqrt{3}}$$

Hence, the ratio of the radius and the height is $1:\sqrt{3}$

33. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting :
(i) a red face card,
(ii) a spade,
(iii) either a king or a black cards.

Ans : [Board Term-2 2012, 2015]

Total cards, $n(S) = 52$

(i) Red face card

Total number of red-face card,

$$n(E_1) = 6$$

o232

P(red face cards)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

(ii) Spade card

Number of spade cards

$$n(E_2) = 13$$

P(Spade cards),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Black king or a red queen,

Number of kings = 4

Number of black cards = 26 − 2 = 24

Thus there are 4 favourable outcome.

$$n(E_3) = 24+4 = 28$$

P(a black Kind or a red queen)

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{28}{52} = \frac{7}{13}$$

34. If α and β are the zeroes of polynomial $p(x) = 3x^2+2x+1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Ans : [Board Term-1 2010, 2012]

We have $p(x) = 3x^2+2x+1$

Since α and β are the zeroes of polynomial $3x^2+2x+1$, we have

$$\alpha+\beta = -\frac{2}{3}$$

b137

and $$\alpha\beta = \frac{1}{3}$$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1+\beta_1 = \frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1+\beta}$$

$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$= \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$

$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

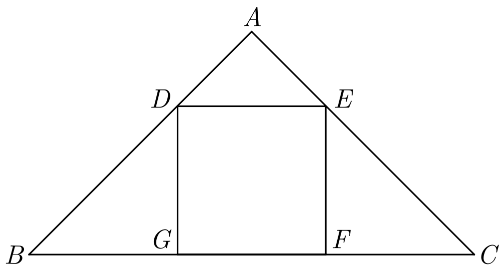
$$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$

Hence, Required polynomial

$$q(x) = x^2-(\alpha_1+\beta_1)2x+\alpha_1\beta_1$$

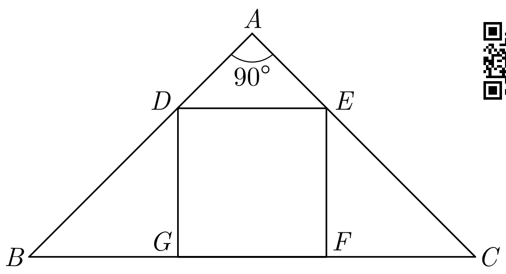
$$= x^2-2x+3$$

35. In the given figure, $DEFG$ is a square and $\angle BAC = 90^\circ$. Show that $FG^2 = BG\times FC$.



Ans : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



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In ΔADE and ΔGBD, we have

$$\angle DAE = \angle BGD \quad \text{[each } 90^\circ\text{]}$$

Due to corresponding angles we have

$$\angle ADE = \angle GDB$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta GBD$$

Now, in ΔADE and ΔFEC,

$$\angle EAD = \angle CFE \quad \text{[each } 90^\circ\text{]}$$

Due to corresponding angles we have

$$\angle AED = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta FEC$$

Since $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$ we have

$$\Delta GBD \sim \Delta FEC$$

Thus $$\frac{GB}{FE} = \frac{GD}{FC}$$

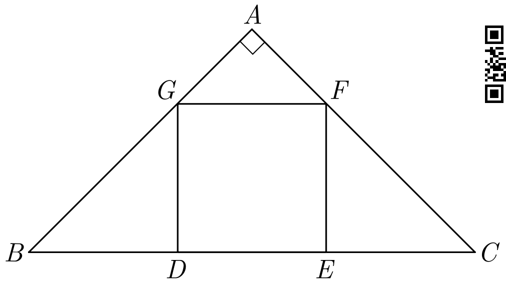
Since $DEFG$ is square, we obtain,

$$\frac{BG}{FG} = \frac{FG}{FC}$$

Therefore $FG^2 = BG \times FC$ Hence Proved

or

In Figure $DEFG$ is a square in a triangle ABC right angled at A. Prove that
(i) $\Delta AGF \sim \Delta DBG$
(ii) $\Delta AGF \sim \Delta EFC$



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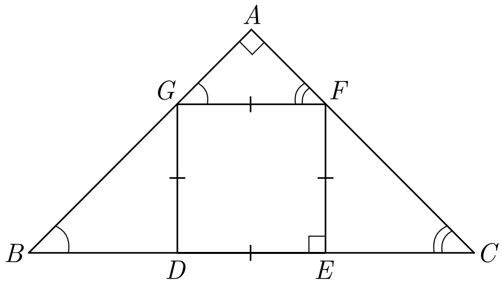
Ans : [Board 2020 Delhi, OD Basic]

We have redrawn the given figure as shown below.



Here ABC is a triangle in which $\angle BAC = 90°$ and $DEFG$ is a square.

(i) In ΔAGF and ΔDBG

$$\angle GAF = \angle BDG \quad \text{(each } 90°\text{)}$$

Due to corresponding angles,

$$\angle AGF = \angle GBD$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta DBG$$ Hence Proved

(ii) In ΔAGF and ΔEFC,

$$\angle GAF = \angle CEF \quad \text{(each } 90°\text{)}$$

Due to corresponding angles,

$$\angle AFG = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta EFC$$ Hence Proved

36. If $\tan\theta = \frac{1}{\sqrt{5}}$,

(1) Evaluate : $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$

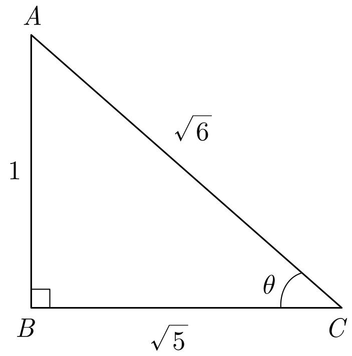
(2) Verify the identity : $\sin^2\theta + \cos^2\theta = 1$

Ans : [Board Term-1 2012]

We have $$\tan\theta = \frac{1}{\sqrt{5}}$$

We draw the triangle as shown below and write all dimensions.

h201



Now $$\cot\theta = \frac{1}{\tan\theta} = \sqrt{5}$$

$$\sin\theta = \frac{1}{\sqrt{6}}$$

$$\cos\theta = \frac{\sqrt{5}}{\sqrt{6}}$$

(1) $$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(1+\cot^2\theta) - (1+\tan^2\theta)}{(1+\cot^2\theta) + (1+\tan^2\theta)}$$

$$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta + \tan^2\theta}$$

$$= \frac{(\sqrt{5})^2 - (\frac{1}{5})^2}{2 + (\sqrt{5})^2 + (\frac{1}{\sqrt{5}})^2}$$

$$= \frac{5 - \frac{1}{5}}{2 + 5 + \frac{1}{5}} = \frac{25-1}{35+1} = \frac{24}{36} = \frac{2}{3}$$

(2) $$\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$

$$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6}$$

$$= 1$$ Hence proved.