CLASS X (2020-21)
MATHEMATICS BASIC(241)
SAMPLE PAPER-4

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

Section - I

1. Find the LCM of smallest two digit composite number and smallest composite number.

Ans : [Board 2020 SQP Standard]

Smallest two digit composite number is 10 and smallest composite number is 4.

$$\text{LCM }(10, 4) = 20$$

2. Find the quadratic polynomial, the sum of whose zeroes is -5 and their product is 6.

Ans : [Board 2020 Delhi Standard]

Let α and β be the zeroes of the quadratic polynomial, then we have

$$\alpha + \beta = -5$$

and $$\alpha\beta = 6$$

Now $$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6$$

3. Find whether the pair of linear equations $y = 0$ and $y = -5$ has no solution, unique solution or infinitely many solutions.

Ans :

The given variable y has different values. Therefore the pair of equations $y = 0$ and $y = -5$ has no solution.

4. Find the nature of roots of the quadratic equation $2x^2 - 3\sqrt{2}\,x + \frac{9}{4} = 0$

Ans :

We have $$2x^2 - 3\sqrt{2}\,x + \frac{9}{4} = 0$$

Here $$a = 2,\ b = -3\sqrt{2},\ c = \frac{9}{4}$$

Discriminant $$D = b^2 - 4ac$$
$$= (-3\sqrt{2})^2 - 4 \times 2 \times \frac{9}{4}$$
$$= 18 - 18 = 0$$

Thus, $2x^2 - 3\sqrt{2}\,x + \frac{9}{4} = 0$ has real and equal roots.

5. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.

Ans : [Board 2020 SQP Standard]

If any number is divisible by 2 and 5, it must be divisible by LCM of 2 and 5, i.e. 10.

Numbers between 102 .... 998 which are divisible by 2 and 5 are 110, 120, 130, ...990

Here $a = 110$, $d = 120 - 110 = 10$ and $a_n = 990$

$$a_n = a + (n-1)d$$
$$990 = 110 + (n-1)10$$
$$880 = 10(n-1)$$
$$88 = n - 1$$
$$n = 88 + 1 = 89$$

or

Is -150 a term of the AP 11, 8, 5, 2, ...?

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

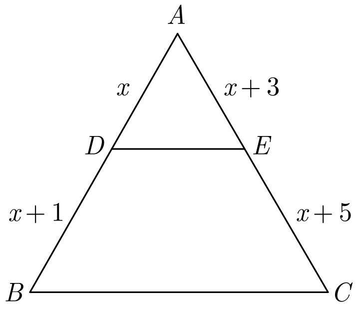
We have $$a = 11,\ d = -3,\ a_n = -150$$

Now $$a_n = a + (n-1)d$$
$$-150 = 11 + (n-1)(-3)$$
$$-150 = 11 - 3n + 3$$
$$3n = 164$$

or, $$n = \frac{164}{3} = 54.66$$

Since, 54.66 is not a whole number, -150 is not a term of the given AP

6. In $\Delta ABC, DE || BC$, find the value of x.



Ans : [Board Term-1 2016]

In the given figure $DE || BC$, thus

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

7. If the point $C(k, 4)$ divides the line segment joining two points $A(2, 6)$ and $B(5,1)$ in ratio 2 : 3, the value of k is ....

Ans : [Board 2020 Delhi Basic]

We have $m:n = 2:3$

By section formula,

$$\frac{mx_2 + nx_1}{m+n} = x$$

Now, $$\frac{2 \times 5 + 3 \times 2}{2+3} = k$$

$$\Rightarrow \quad k = \frac{16}{5}$$

or

If points $A(-3, 12)$, $B(7, 6)$ and $C(x, 9)$ are collinear, then the value of x is ....

Ans : [Board 2020 Delhi Basic]

If points are collinear, then area of triangle must be zero.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\frac{1}{2}[-3(6-9) + 7(9-12) + x(12-6)] = 0$$

$$\frac{1}{2}(9 - 21 + 6x) = 0$$

$$\frac{1}{2}(-12 + 6x) = 0$$

$$6x = 12 \Rightarrow x = 2$$

8. The value of $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) =$ ....

Ans : [Board 2020 Delhi Standard]

$$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$$

$$= \sec^2\theta(1-\sin^2\theta)$$

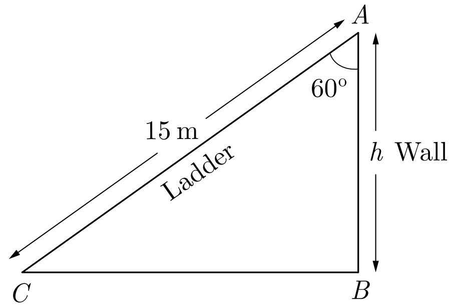
$$= \sec^2\theta \times \cos^2\theta$$

$$= \frac{1}{\cos^2\theta} \times \cos^2\theta = 1$$

9. A ladder 15 m long leans against a wall making an angle of 60° with the wall. Find the height of the point where the ladder touches the wall.

Ans : [Board Term-2 2014]

Let the height of wall be h. As per given in question we have drawn figure below.



$$\frac{h}{15} = \cos 60^\circ$$

$$h = 15 \times \cos 60^\circ$$

$$= 15 \times \frac{1}{2} = 7.5 \text{ m}$$

10. If a circle can be inscribed in a parallelogram how will the parallelogram change?

Ans : [Board Term-2, 2014]

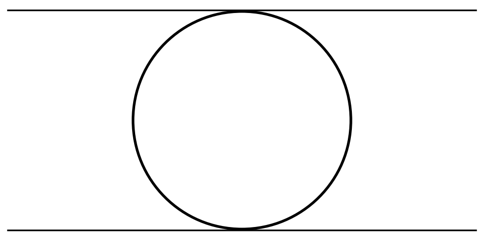
It changes into a rectangle or a square.

or

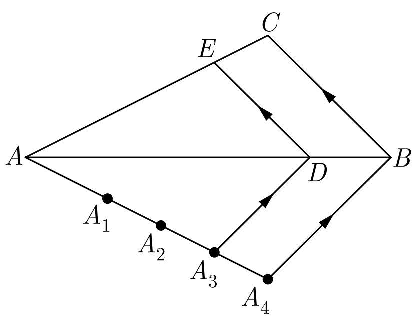
What is the maximum number of parallel tangents a circle can have on a diameter?

Ans : [Board Term-2 2012]

Tangent touches a circle on a distinct point. Only two parallel tangents can be drawn on the diameter of a circle. It has been shown in figure given below.



11. In figure, ΔADE is constructed similar to ΔABC, write down the scale factor.



Ans : [Board Term-2 2012]

Scale factor is $\frac{3}{4}$.

12. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. What is the length of the wire?

Ans :

Let the length of the wire be l. Since, metallic sphere is converted into a cylindrical shaped wire of length l, Volume of the metal used in wire is equal to the volume of the sphere.

$$\pi r^2 l = \frac{4}{3}\pi R^3$$

$$\pi \times \left(\frac{2}{2}\times\frac{1}{10}\right)^2 \times l = \frac{4}{3}\times\pi\times\left(\frac{6}{2}\right)^3$$

$$\pi \times \frac{1}{100}\times h = \frac{4}{3}\times\pi\times 3^3$$

$$\frac{l}{100} = 4\times 3^2 = 36$$

$$l = 3600\text{ cm} = 36\text{ m}$$

13. The mean weight of 9 students is 25 kg. If one more student is joined in the group the mean is unaltered, then find the weight of the 10th student.

(a) 25 kg (b) 24 kg

(c) 26 kg (d) 23 kg

Ans :

The sum of the weights of the 9 students $= 25\times 9 = 225$. If one more student is joined in the group, then total number of students is 10 and mean is 25.

Hence, the sum of the weights of the 10th students $= 25\times 10 = 250$.

Hence, the weight of the 10th student is $250 - 225 = 25$ kg.

However we can answer this question without any calculation. If mean is not altered on adding more data, then added data must be of mean value.

or

The mean and median of the data a, b and c are 50 and 35 respectively, where $a < b < c$. If $c - a = 55$, then find the value of $(b-a)$.

Ans :

Since, a, b and c and are in ascending order, therefore median is b i.e. $b = 35$.

Mean $\frac{a+b+c}{3} = 50$

$$a+b+c = 150$$

$$a+c = 150-35$$

$$= 115 \quad ...(1)$$

Also, it is given that $c - a = 55$ ...(2)

Subtracting equation (2) and (1), we get

$$a = 30$$

Hence, $b - a = 35 - 30 = 5$

14. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(a) 40 (b) 240

(c) 480 (d) 750

Ans :

Total number of sold tickets are 6000. Let she bought x tickets.

Now $n(S) = 6000$

$$n(E) = x$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$0.08 = \frac{x}{6000}$$

$$x = 0.08\times 6000 = 480$$

Hence, she bought 480 tickets.

or

One ticket is drawn at random from a bag containing tickets numbered 1 to 40. Find the probability that the selected ticket has a number which is a multiple of 5.

Ans :

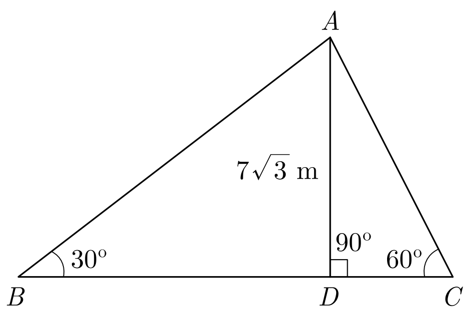
Multiples of 5 are 5, 10, 15, 20, 25, 30, 35 and 40 thus 8 outcome.

$$n(S) = 40$$

$$n(E) = 8$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{40} = \frac{1}{5}$$

15. In the given figure, if $AD = 7\sqrt{3}$ m, then find the value of BC.



Ans : [Board Term-2 2012]

Let $BD = x$ and $DC = y$

From ΔADB we get

$$\tan 30^\circ = \frac{7\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{7\sqrt{3}}{x}$$

$$x = 7\sqrt{3}\times\sqrt{3} = 21\text{ m}$$

From ΔADC,

$$\tan 60^\circ = \frac{7\sqrt{3}}{y}$$

$$\sqrt{3} = \frac{7\sqrt{3}}{y}$$

$$y = 7\text{ m.}$$

Now $BC = BD + DC$

$$= 21 + 7 = 28\text{ m.}$$

Hence, the value of BC is 28 m.

16. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius.

Ans : [Board Term-2, 2014]

Let the radius of spherical ball be r.

Volume of spherical ball = Volume of three balls

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi[3^3 + 4^3 + 5^3]$$

$$r^3 = 27 + 64 + 125 = 216$$

, $$r = 6 \text{ cm}$$

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Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. Auditorium, the part of a public building where an audience sits, as distinct from the stage, the area on which the performance or other object of the audience's attention is presented. In a large theatre an auditorium includes a number of floor levels frequently designed as stalls, private boxes, dress circle, balcony or upper circle, and gallery. A sloping floor allows the seats to be arranged to give a clear view of the stage. The walls and ceiling usually contain concealed light and sound equipment and air extracts or inlets and may be highly decorated.

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In an auditorium, seats are arranged in rows and columns. The number of rows are equal to the number of seats in each row. When the number of rows are doubled and the number of seats in each row is reduced by 10, the total number of seats increases by 300.

(i) If x is taken as number of row in original arrangement which of the following quadratic equation describes the situation ?
(a) $x^2 - 20x - 300 = 0$ (b) $x^2 + 20x - 300 = 0$
(c) $x^2 - 20x + 300 = 0$ (d) $x^2 + 20x + 300 = 0$

(ii) How many number of rows are there in the original arrangement?
(a) 20 (b) 40
(c) 10 (d) 30

(iii) How many number of seats are there in the auditorium in original arrangement ?
(a) 725 (b) 400
(c) 900 (d) 680

(iv) How many number of seats are there in the auditorium after re-arrangement.
(a) 860 (b) 990
(c) 1200 (d) 960

(v) How many number of columns are there in the auditorium after re-arrangement?
(a) 42 (b) 20
(c) 25 (d) 32

Ans :

(i) Since number of rows are equal to the number of seats in each row in original arrangement, total seats are x^2.

In new arrangement row are $2x$ and seats in each row are $x - 10$. Hence total $2x(x-10)$ seats are there.

Total seats are 300 more than previous seats so total number of seats are $x^2 + 300$.

Thus $$2x(x-10) = x^2 + 300$$
$$2x^2 - 20x = x^2 + 300$$
$$x^2 - 20x - 300 = 0$$

Thus (a) is correct option.

(ii) We have $$x^2 - 20x - 300 = 0$$
$$x^2 - 30x + 10x - 300 = 0$$
$$x(x-30) + 10(x-30) = 0$$
$$(x-30)(x+10) = 0 \Rightarrow x = 30, -10$$

Thus (d) is correct option.

(iii) Number of seats in original arrangement,
$$x^2 = 30^2 = 900$$

Thus (c) is correct option.

(iv) Total seats in rearrangement $= 30^2 + 300$
$$= 900 + 300 = 1200$$

Thus (c) is correct option.

(v) Number of row are 30 in original arrangement. In rearrangement number of rows are $2 \times 30 = 60$.

Number of Column after rearrangement,
$$= \frac{\text{Total seats}}{\text{Row}} = \frac{1200}{60} = 20 \text{ Column}$$

Thus (b) is correct option.

18. Rani wants to make the curtains for her window as shown in the figure. The window is in the shape of a rectangle, whose width and height are in the ratio $2:3$. The area of the window is 9600 square cm.

(i) What is the shape of the window that is uncovered?
(a) Right triangle (b) Equilateral triangle
(c) Isosceles triangle (d) Rectangle

(ii) What will be the ratio of two sides of each curtain (other than hypotenuse) ?
(a) 1 : 3 (b) 2 : 3
(c) 1 : 1 (d) 3 : 2

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(iii) What are the dimensions of the window ?
(a) 40 cm × 80 cm (b) 20 cm × 60 cm
(c) 80 cm × 120 cm (d) 40 cm × 120 cm

(iv) What will be the perimeter of the window ?
(a) 200 cm (b) 100 cm
(c) 400 cm (d) 450 cm

(v) How much window area is covered by the curtains?
(a) 50 % (b) 75 %
(c) 25 % (d) 80 %

Ans :

(i) It is isosceles triangle.
Thus (c) is correct option.
(ii) Let $2x$ be the width of window, then $3x$ will be height of window because ratio is 2 : 3. Now width of single curtain will be x because it is half of window. Length of single curtain is equal the height of window.
Thus ratio is $\frac{x}{3x} = \frac{1}{3}$
Thus (a) is correct option.

(iii) Area, $9600 = 2x \times 3x = 6x^2$

$$x^2 = \frac{9600}{6} = 1600$$

$$x = \sqrt{1600} = 40 \text{ cm}$$

Width, $2x = 80\text{ cm}$

Length, $3x = 120\text{ cm}$
Thus (c) is correct option.

(iv) Perimeter,

$$P = 2(80 + 120) = 400 \text{ cm}$$
Thus (c) is correct option.

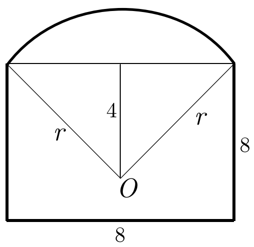
(v)Area of both curtains$= 2 \times \left(\frac{1}{2} \times 40 \times 120\right)$

$$= 40 \times 120 = 4800$$

$$\text{Window area} = \frac{4800}{9600} \times 100 = 50\%$$

Thus (a) is correct option.

19. A barn is an agricultural building usually on farms and used for various purposes. A barn refers to structures that house livestock, including cattle and horses, as well as equipment and fodder, and often grain.



Ramkaran want to build a barn at his farm. He has made a design for it which is shown above. Here roof is arc of a circle of radius r at centre O.

(i) What is the value of radius of arc ?
(a) $4\sqrt{3}$ m (b) $4\sqrt{2}$ m
(c) $4\sqrt{3}$ m (d) $2\sqrt{2}$ m

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(ii) What is the curved width of roof ?
(a) $2\pi\sqrt{3}$ m (b) $4\pi\sqrt{2}$ m
(c) $2\pi\sqrt{2}$ m (d) $4\pi\sqrt{3}$ m

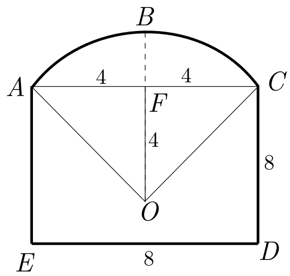
(iii) What is area of cross section of barn ?
(a) $8(6+\pi)\,\text{m}^2$ (b) $4(6+\pi)\,\text{m}^2$
(c) $8(3+\pi)\,\text{m}^2$ (d) $4(3+\pi)\,\text{m}^2$

(iv) If the length of the barn is 12 meters, what is the curved surface area of roof?
(a) $32\sqrt{2}\,\pi\,\text{m}^2$ (b) $16\sqrt{2}\,\pi\,\text{m}^2$
(c) $48\sqrt{2}\,\pi\,\text{m}^2$ (d) $24\sqrt{2}\,\pi\,\text{m}^2$

(v) What is the storage capacity of barn ?
(a) $48(6+\pi)\,\text{m}^3$ (b) $48(6+\pi)\,\text{m}^3$
(c) $96(6+\pi)\,\text{m}^3$ (d) $96(3+\pi)\,\text{m}^3$

Ans :

(i) We redraw the cross section of barn as shown below.



In right triangle ΔAFO,

$$AO = \sqrt{AF^2 + FO^2}$$
$$= \sqrt{4^2 + 4^2}$$
$$= 4\sqrt{2} \text{ m}$$

Thus $AO = 4\sqrt{2}$ which is also radius of curved arc.
Thus (b) is correct option.
(ii) In right angle triangle ΔAFO is also isosceles triangle

Thus, $\angle FAO = \angle FOA = 45^\circ$

Similarly, $\angle FOC = 45^\circ$

Thus $\angle AOC = \angle AOF + \angle FOC$

$$= 45^\circ + 45^\circ = 90^\circ$$

Curved width $\frac{2\pi r\theta}{360^\circ} = \frac{2\pi \times 4\sqrt{2} \times 90^\circ}{360^\circ}$

$$= 2\pi\sqrt{2} \text{ m}$$

Thus (c) is correct option.
(iii) Area of cross section

$=$ Area of $AECD$ $+$

$+$ Area of section $ABCO$ $-$ Area of triangle ACO

$$= 8 \times 8 + \frac{\pi(4\sqrt{2})^2 \times 90^\circ}{360^\circ} - \frac{1}{2} \times 4 \times 8$$

$$= 64 + 8\pi - 16$$

$$= 48 + 8\pi = 8(6 + \pi)$$

Thus (a) is correct option.
(iv) Curved surface area of roof

$$= 2\pi\sqrt{2} \times 12$$
$$= 24\sqrt{2}\,\pi\,\text{m}^2$$

Thus (d) is correct option.

(v) Storage capacity of barn,

$$= \text{Cross section area} \times \text{Length}$$
$$= 8(6+\pi) \times 12$$
$$= 96(6+\pi)\,\text{m}^3$$

Thus (c) is correct option.

20. Amul, is an Indian dairy cooperative society, based at Anand in the Gujarat. Formed in 1946, it is a cooperative brand managed by a cooperative body, the Gujarat Co-operative Milk Marketing Federation Ltd. (GCMMF), which today is jointly owned by 36 lakh (3.6 million) milk producers in Gujarat. Amul spurred India's White Revolution, which made the country the world's largest producer of milk and milk products.

Survey manager of Amul dairy has recorded monthly expenditures on milk in 100 families of a housing society. This is given in the following frequency distribution :

Monthly expenditure (in Rs.)	Number of families
0-175	10
175-350	14
350-525	15
525-700	x
700-875	28
875-1050	7
1050-1225	5

(i) How many families spend between Rs 350- 700 on milk ?
(a) 21 (b) 38
(c) 17 (d) 36

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(ii) What is the upper limit of median class ?
(a) 1225 (b) 875
(c) 1050 (d) 700

(iii) What is the median expenditure on milk?
(a) 601.4 (b) 636.5
(c) 616.6 (d) 624.5

(iv) What is the lower limit of model class ?
(a) 1225 (b) 875
(c) 1050 (d) 700

(v) What is the model expenditure on milk?
(a) 734.25 (b) 743.75
(c) 801.25 (d) 820.25

Ans :

(i) Since number of families is 100,

$$10+14+15+x+28+7+5 = 100$$
$$79+x = 100$$
$$x = 100-79 = 21$$

Thus $15+21 = 36$ families spend between Rs 350-700 on milk.

Thus (d) is correct option.

(ii) We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-175	10	10
157-350	14	24
350-525	15	39
525-700	21	60
700-875	28	88
875-1050	7	95
1050-1225	5	100
	$N = 100$	

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 525-700. Thus median class is 525-700 and upper limit is 700.

Thus (d) is correct option.

(iii) Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$= 525 + \frac{50-39}{21} \times 175$$
$$= 525 + \frac{11}{21} \times 175$$
$$= 525 + 91.6 = 616.6$$

Thus (d) is correct option.

(iv) Class 700-875 has the maximum frequency 28, therefore this is model class and lower limit is 700.

Thus (d) is correct option.

(v) Here $l = 700$, $f_0 = 21$, $f_1 = 28$ $f_2 = 7$, $h = 175$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$$= 700 + \left(\frac{28-21}{2\times 28 - 21 - 7}\right) \times 175$$
$$= 700 + \frac{7}{28} \times 175$$
$$= 700 + 43.75 = 743.75$$

Thus (b) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Given that HCF $(306, 1314) = 18$. Find LCM $(306, 1314)$. [2]

Ans : [Board Term-1 2013]

We have $\text{HCF}(306, 1314) = 18$

$\text{LCM}(306, 1314) = ?$

Let $a = 306$ and $b = 1314$, then we have

$$\text{LCM}(a,b) \times \text{HCF}(a,b) = a \times b$$

Substituting values we have

$$\text{LCM}(a,b) \times 18 = 306 \times 1314$$

$$\text{LCM}(a,b) = \frac{306 \times 1314}{18}$$

$$\text{LCM}(306, 1314) = 22,338$$

or

Check whether $(15)^n$ can end with digit 0 for any $n \in N$. [2]

Ans : [Board Term-1 2012]

If the number $(15)^n$, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.
That is, the prime factorization of $(15)^n$ would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(15)^n = (3 \times 5)^n$ are 3 and 5. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(15)^n$. Since there is no prime factor 2, $(15)^n$ cannot end with the digit zero.

22. For what value of k, the pair of linear equations $kx - 4y = 3$, $6x - 12y = 9$ has an infinite number of solutions ?

Ans : [Board Term-1 2012]

We have $kx - 4y - 3 = 0$

and $6x - 12y - 9 = 0$

where, $a_1 = k, b_1 = 4, c_1 = -3$

$a_2 = 6, b_2 = -12, c_2 - 9$

Condition for infinite solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$$

Hence, $k = 2$

or

For what value of k, $2x + 3y = 4$ and $(k+2)x + 6y = 3k + 2$ will have infinitely many solutions ?

Ans : [Board Term-1 2012]

We have $2x + 3y - 4 = 0$

and $(k+2)x + 6y - (3k+2) = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -4$

and $a_2 = k+2, b_2 = 6, c_3 = -(3k+2)$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or, $$\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

From $\frac{2}{k+2} = \frac{3}{6}$ we have

$3(k+2) = 2 \times 6 \Rightarrow (k+2) = 4 \Rightarrow k = 2$

From $\frac{3}{6} = \frac{4}{3k+2}$ we have

$3(3k+2) = 4 \times 6 \Rightarrow (3k+2) = 8 \Rightarrow k = 2$

Thus $k = 2$

23. Prove that the point $(3,0)$, $(6,4)$ and $(-1,3)$ are the vertices of a right angled isosceles triangle.

Ans : [Board Term-2 OD 2016]

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now $AB^2 = (3-6)^2 + (0-4)^2$

$= 9 + 16 = 25$

$BC^2 = (6+1)^2 + (4-3)^2$

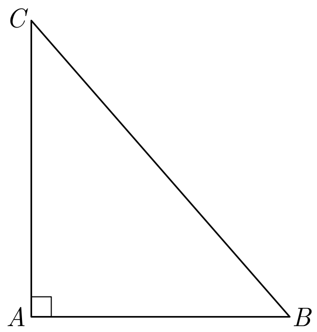
$= 49 + 1 = 50$

$CA^2 = (-1-3)^2 + (3-0)^2$

$= 16 + 9 = 25$

$AB^2 = CA^2$ or, $AB = CA$

Hence triangle is isosceles.



Also, $25 + 25 = 50$

or, $AB^2 + CA^2 = BC^2$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

24. If $\cos(A - B) = \frac{\sqrt{3}}{2}$ and $\sin(A + B) = \frac{\sqrt{3}}{2}$, find $\sin A$ and B, where $(A + B)$ and $(A - B)$ are acute angles.

Ans : [Board Term-1 2012]

We have $\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$A - B = 30^\circ$ ...(1)

Also $\sin(A + B) = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$A + B = 60^\circ$ ...(2)

Adding equations (1) and (2), we obtain,

$2A = 90^\circ$

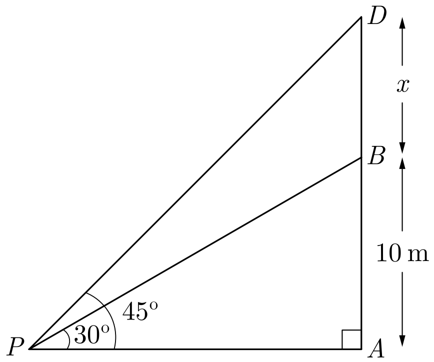
$A = 45^\circ$

Substituting this value of A in equation (1), we get $B = 15^\circ$

25. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top the of the building and the angle of elevation of the length of the flagstaff from P is 45°. Find the length of the flagstaff and distance of building from point P. [Take $\sqrt{3} = 1.732$]

Ans : [Board Term-2 2011, Delhi 2012, 2013]

Let height of flagstaff be $BD = x$. As per given in question we have drawn figure below.



$$\tan 30^\circ = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{AP}$$

$$AP = 10\sqrt{3}$$

Distance of the building from P,

$$= 10 \times 1.732 = 17.32 \text{ m}$$

Now $$\tan 45^\circ = \frac{AD}{AP}$$

$$1 = \frac{10 + x}{17.32}$$

$$x = 17.32 - 10.00 = 7.32 \text{ m}$$

Hence, length of flagstaff is 7.32 m.

26. The mode of the following frequency distribution is 36. Find the missing frequency f.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	8	10	f	16	12	6	7

Ans : [Board 2020 OD Basic]

Mode is 36 which lies in class 30-40, therefore this is model class.

Here, $f_0 = f$, $f_1 = 16$, $f_2 = 12$, $l = 30$ and $h = 10$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$36 = 30 + \frac{16 - f}{2 \times 16 - f - 12} \times 10$$

$$6 = \frac{16 - f}{20 - f} \times 10$$

$$120 - 6f = 160 - 10f$$

$$4f = 40 \Rightarrow f = 10$$

Thus (d) is correct option.

27. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. [3]

Ans : [Board 2019 OD]

Assume that $2 + 5\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$2 + 5\sqrt{3} = \frac{p}{q}, \quad q \neq 0$$

$$5\sqrt{3} = \frac{p}{q} - 2$$

$$5\sqrt{3} = \frac{p - 2q}{q}$$

$$\sqrt{3} = \frac{p - 2q}{5q}$$

Here $\sqrt{3}$ is irrational and $\frac{p-2q}{5q}$ is rational because p and q are co-prime integers. But rational number cannot be equal to an irrational number. Hence $2 + 5\sqrt{3}$ is an irrational number.

or

Write the smallest number which is divisible by both 306 and 657.

Ans : [Board 2019 OD]

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers Here, the given numbers are 306 and 657.

$$306 = 6 \times 51 = 3 \times 2 \times 3 \times 17$$

$$657 = 9 \times 73 = 3 \times 3 \times 73$$

$$\text{LCM}(306, 657) = 2 \times 3 \times 3 \times 17 \times 73$$

$$= 22338$$

Hence, the smallest number which is divisible by 306 and 657 is 22338.

28. The Present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Ans : [Board Term-1 2012, Set-39]

Let the sum of the ages of the 2 children be x and the age of the father be y years.

Now $$y = 2x$$

$$2x - y = 0 \quad ...(1)$$

and $$20 + y = x + 40$$

$$x - y = -20 \quad ...(2)$$

Subtracting (2) from (1), we get

$$x = 20$$

From(1), $$y = 2x = 2 \times 20 = 40$$

Hence, the age of the father is 40 years.

29. Determine an AP whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6.

Ans : [Board Term-2 2015]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a_3 = 9$$

$$a + 2d = 9 \quad ...(1)$$

and $$a_8 - a_5 = 6$$

$$(a + 7d) - (a + 4d) = 6$$

$$3d = 6$$

$$d = 2$$

Substituting this value of d in (1), we get

$$a + 2(2) = 9$$

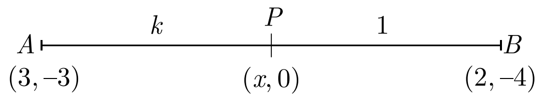
$$a = 5$$

So, AP is 5, 7, 9, 11, ...

30. Find the ratio in which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Ans : [Board Term-2 Delhi 2014]

We know that y co-ordinate of any point on the $x-$ axis will be zero. Let $(x,\ 0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$. Using section formula for y co-ordinate we have

$$0 = \frac{1(-3)+k(7)}{1+k}$$

g150

$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have

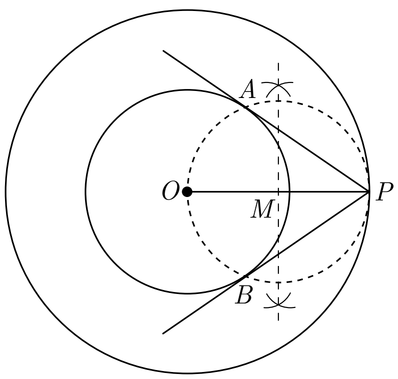
$$x = \frac{1(3)+k(-2)}{1+k} = \frac{3-2\times\frac{3}{7}}{1+\frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $\left(\frac{3}{2},0\right)$.

31. Draw two concentric circles of radii 2 cm and 5 cm. Take a point P on the outer circle and construct a pair of tangents PA and PB to the smaller circle. Measure PA.

Ans : [Board 2019 OD Standard]

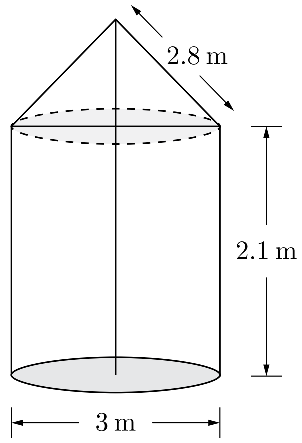
1. Draw a circle with centre O and radius 2 cm.
2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on outer circle and join OP.
4. Draw perpendicular bisector of OP which intersect OP at M.
5. Draw a circle with centre M which intersects inner circle at points A and B.
6. Join AP and BP. Thus AP and BP are required tangents.



$$PA = \sqrt{5^2-2^2}$$
$$= \sqrt{21} = 4.6 \text{ cm}$$

32. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use $\pi = \frac{22}{7}$.

m142



Ans : [Board Term-2 OD 2016]

Area of canvas required will be surface area of tent.

Height of cylinder $= 2.1$ m

Radius of cylinder $=$ radius of cone $= \frac{3}{2}$ m

Slant height of cone $= 2.8$ m

Surface area of tent,

$$= C.S.A \text{ of cone} + C.S.A \text{ of cylinder.}$$
$$= \pi rl + 2\pi rh = \pi r(l+2h)$$

Thus $$\pi r(l+2h) = \frac{22}{7}\times\frac{3}{2}(2.8+2\times 2.1)$$
$$= \frac{33}{7}\times 7 = 33 \text{ m}^2$$

$$\text{Total Cost} = 33\times 500 = 16{,}500 Rs$$

33. Five cards, ten, Jack, Queen, King and Ace of diamonds are well shuffled. One card is picked up from them.

(i) Find the probability that the drawn card is Queen.
(ii) If Queen is put aside, then find the probability that the second card drawn is an ace.

Ans : [Board Term-2 2014]

We have 5 cards and thus there are 5 possible outcomes.

$$n(S) = 5$$

(i) drawn card is queen
No. of favourable outcomes,

$$n(E_1) = 1$$

o239

P(queen), $$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{5}$$

(ii) second card drawn is an ace
Since, queen was kept, number of all possible outcomes

$$n(S) = 5-1 = 4$$

Number of favourable outcomes

$$n(E_2) = 1$$

P(second card drawn is an ace),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{4}$$

or

A box contains cards, number 1 to 90. A card is drawn at random from the box. Find the probability that the selected card bears a :

(i) Two digit number.

(ii) Perfect square number

Ans : [Board Term-2 Delhi Compt. 2017]

We have 90 cards and thus there are 90 possible outcomes.

$$n(S) = 90$$

(i) No. of cards having 2 digit number $90 - 9 = 81$.
Number of favourable outcomes,

$$n(E_1) = 81$$

P(selected card bears two digit number)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) Perfect square number between 1 to 90 are 1, 4, 9, 16, 25, 36, 49, 64 and 81 i.e. 9 numbers.
No. of favourable outcomes,

$$n(E_2) = 9$$

o241

P(perfect square numbers)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{9}{90} = \frac{1}{10}$$

34. If α and β are zeroes of the polynomial $p(x) = 6x^2 - 5x + k$ such that $\alpha - \beta = \frac{1}{6}$, Find the value of k.

Ans : [Board 2007]

We have $p(x) = 6x^2 - 5x + k$

Since α and β are zeroes of

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$$p(x) = 6x^2 - 5x + k,$$

Sum of zeroes, $\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha - \beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$$

Hence, $k = 1$.

or

If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2 + a)x^2 + 61x + 6a$. Find the value of β and α.

Ans :

We have $p(x) = (a^2 + a)x^2 + 61x + 6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta + \frac{1}{\beta} = -\frac{61}{a^2 + a}$

or, $\frac{\beta^2 + 1}{\beta} = \frac{-61}{a^2 + a}$ ...(1)

Product of zeroes $\beta\frac{1}{\beta} = \frac{6a}{a^2 + a}$

b140

or, $1 = \frac{6}{a + 1}$

$$a + 1 = 6$$

$$a = 5$$

Substituting this value of a in (1) we get

$$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$$

$$30\beta^2 + 30 = -61\beta$$

$$30\beta^2 + 61\beta + 30 = 0$$

Now $\beta \frac{-61 \pm \sqrt{(-61)^2 - 4 \times 30 \times 30}}{2 \times 30}$

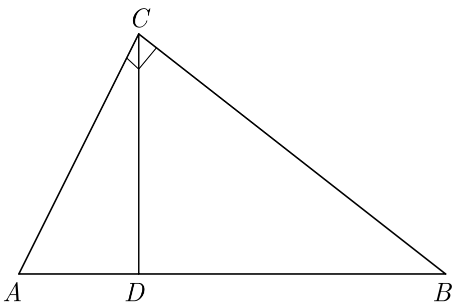
$$= \frac{-61 \pm \sqrt{3721 - 3600}}{60}$$

$$\frac{-61 \mp 11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

35. In Figure , $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



Ans : [Board 2019 Delhi]

In ΔACB we have

$$\angle ACB = 90°$$

and $CD \perp AB$

Thus $AB^2 = CA^2 + CB^2$ ...(1)

In ΔCAD, $\angle ADC = 90°$, thus we have

$$CA^2 = CD^2 + AD^2 \quad ...(2)$$

and in ΔCDB, $\angle CDB = 90°$, thus we have

$$CB^2 = CD^2 + BD^2 \quad ...(3)$$

Adding equation (2) and (3), we get

$$CA^2 + CB^2 = 2CD^2 + AD^2 + BD^2$$

Substituting AB^2 from equation (1) we have

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$$AB^2 = 2CD^2 + AD^2 + BD^2$$

$$AB^2 - AD^2 = BD^2 + 2CD^2$$

$$(AB + AD)(AB - AD) = BD^2 + 2CD^2$$

$$(AB + AD)BD - BD^2 = 2CD^2$$

$$BD[(AB + AD) - BD] = 2CD^2$$

$$BD[AD + (AB - BD)] = 2CD^2$$

$$BD[AD + AD] = 2CD^2$$

$$BD \times 2AD = 2CD^2$$

$$CD^2 = BD \times AD$$ Hence Proved

36. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Ans : [Board-Term 1 2009

We have $\tan A + \sin A = m$

and $\tan A - \sin A = n$

h211

$$m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$

$$= (\tan^2 A + \sin^2 A + 2\sin A \tan A) - (\tan^2 A + \sin^2 A - 2\sin A \tan A)$$

$$= \tan^2 A + \sin^2 A + 2\sin A \tan A - \tan^2 A - \sin^2 A + 2\sin A \tan A$$

$$= 4\sin A \tan A$$

$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A\cos^2 A}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}}$$

$$= 4\frac{\sin A \times \sin A}{\cos A}$$

$$= 4\sin A \times \frac{\sin A}{\cos A}$$

$$= 4\sin A \tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved