CLASS X (2020-21)
MATHEMATICS BASIC(241)
SAMPLE PAPER-7

Time : 3 Hours **Maximum Marks : 80**

General Instructions :
1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :
1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :
1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

Section - I

1. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after how many places of decimal?

Ans : [Board 2020 Delhi Standard]

Rational number,

$$\frac{14587}{1250} = \frac{14587}{2^1 \times 5^4} = \frac{14587}{2^1 \times 5^4} \times \frac{2^3}{2^3}$$
$$= \frac{14587 \times 8}{2^4 \times 5^4} = \frac{116696}{(10)^4}$$
$$= 11.6696$$

a248

Hence, given rational number will terminate after four decimal places.

or

If two positive integers a and b are written as $a = x^3 y^2$ and $b = xy^3$, where x, y are prime numbers, then Find HCF (a, b).

Ans :

We have
$$a = x^3 y^2 = x \times x \times x \times y \times y$$
$$b = xy^3 = x \times y \times y \times y$$
$$\text{HCF}(a, b) = \text{HCF}(x^3 y^2, xy^3)$$
$$= x \times y \times y = xy^2$$

a249

HCF is the product of the smallest power of each common prime factor involved in the numbers.

2. What is the lowest value of $x^2 + 4x + 2$?

Ans :

We have
$$x^2 + 4x + 2 = (x^2 + 4x + 4) - 2$$
$$= (x+2)^2 - 2$$

Here $(x+2)^2$ is always positive and its lowest value is zero. Thus lowest value of $(x+2)^2 - 2$ is -2 when $x + 2 = 0$.

b230

3. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

Ans :

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel, then

c104

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or
$$\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

4. Find the nature of roots of the quadratic equation $x^2 - 4x + 3\sqrt{2} = 0$.

Ans :

We have $x^2 - 4x + 3\sqrt{2} = 0$

Here $a = 1$, $b = -4$ and $c = 3\sqrt{2}$

Now
$$D = b^2 - 4ac = (-4)^2 - 4(1)(3\sqrt{2})$$
$$= 16 - 12\sqrt{2}$$
$$= 16 - 12 \times (1.41)$$
$$= 16 - 16.92 = -0.92$$
$$b^2 - 4ac < 0$$

d336

Hence, the given equation has no real roots.

or

Find the nature of roots of the quadratic equation $x^2 + 4x - 3\sqrt{2} = 0$.

Ans :

We have $x^2 + 4x - 3\sqrt{2} = 0$

Here $a = 1$, $b = 4$ and $c = -3\sqrt{2}$

Now
$$D = b^2 - 4ac$$
$$= (4)^2 - 4(1)(-3\sqrt{2})$$
$$= 16 + 12\sqrt{2} > 0$$

d337

Hence, the given equation has two distinct real roots,

5. Find the ratio in which x-axis divides the line segment joining $A(2, -3)$ and $B(5, 6)$.

Ans : [Board 2020 OD Basic]

Let point $P(x,\ 0)$ on x-axis divide the segment joining points $A(2,\ -3)$ and $B(5,\ 6)$ in ratio $k\ :\ 1$, then

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$0 = \frac{6k-3}{k+1}$$

$$6k = 3 \Rightarrow k = \frac{1}{2}$$

Therefore ratio is 1 : 2.

6. If $\cos(\alpha+\beta) = 0$, then find $\sin(\alpha-\beta)$ in terms of β.

Ans :

We have, $\cos(\alpha+\beta) = 0 = \cos 90^\circ$ $[\cos 90^\circ = 0]$

$$\alpha + \beta = 90^\circ$$

$$\alpha = 90^\circ - \beta$$

Now, $\sin(\alpha-\beta) = \sin(90^\circ - \beta - \beta)$

$$= \sin(90^\circ - 2\beta)$$

$$= \cos 2\beta$$

Thus (b) is correct option.

7. If the height and length of the shadow of a man are equal, then find the angle of elevation of the sun.

Ans :

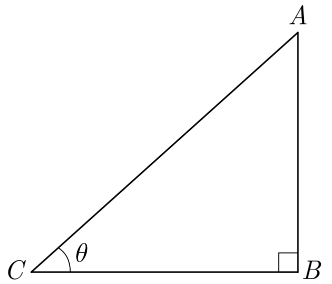
Let AB be the height of a man and BC be the shadow of a man.

$$AB = BC$$

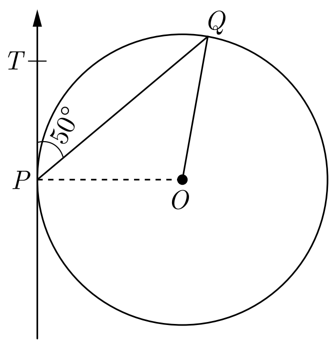
In ΔABC, $\tan\theta = \frac{AB}{BC}$

$$\frac{AB}{AB} = \tan\theta$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$



8. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with PQ. Find the angle $\angle POQ$.



Ans : [Board 2020 OD Basic]

Due to angle between radius and tangent,

$$\angle OPT = 90^\circ$$

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Also, $OP = OQ$ [Radii of a circle]

Since equal opposite sides have equal opposite angles,

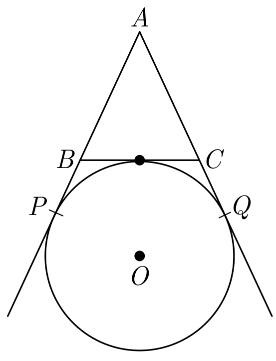
$$\angle OPQ = \angle OQP = 40^\circ$$

$$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$$

$$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

or

In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then what is the length of AP?



Ans : [Board 2020 Delhi Basic]

Due to tangents from external points,

$BP = BR, CR = CQ$, and $AP = AQ$

Perimeter of ΔABC,

$$AB + BC + AC = AB + BR + RC + AC$$

$$5 + 4 + 6 = AB + BP + CQ + AC$$

$$15 = AP + AQ$$

$$15 = 2AP$$

Thus $AP = \frac{15}{2} = 7.5$ cm

Thus (d) is correct option.

9. To divide a line segment AB in the ratio 2 : 5, first a ray AX is drawn, so that $\angle BAX$ is an acute angle and then at equal distance points are marked on the ray AX such that the minimum number of these point is____

Ans :

We know that, to divide a line segment AB in the ratio $m:n$, first draw a ray AX which makes an acute $\angle BAX$ then, marked $m+n$ points at equal distance.

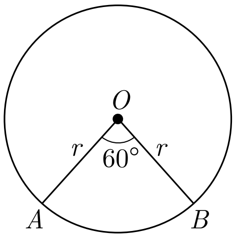
Here, $m = 2,\ n = 5$

Minimum number of these points $= 2 + 5 = 7$

10. What is the perimeter of the sector with radius 10.5 cm and sector angle 60^o.

Ans : [Board Term-2 2012]

As per question the digram is shown below.



Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$$

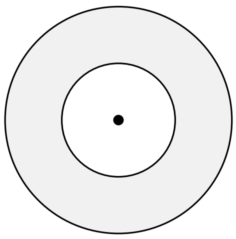
$$= 21 + 11 = 32 \text{ cm}$$

or

If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring.

Ans : [Board Term-2 Delhi 2013]

As per question statement figure is shown below.



Circumference of the outer circle, $2\pi r_1 = 88$ cm

$$r_1 = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

Circumference of the outer circle, $2\pi r_2 = 66$ cm

$$r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm} = 10.5 \text{ cm}$$

Width of the ring,

$$r_1 - r_2 = 14 - 10.5 \text{ cm} = 3.5 \text{ cm}$$

11. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then what is the ratio of the slant height of the cone to the height of the cylinder?

Ans :

$$\pi rl = 2\pi rh$$

$$\frac{l}{h} = \frac{2}{1}$$

12. A solid piece of iron in the form of a cuboid of dimensions $49\,\text{cm} \times 33\,\text{cm} \times 24\,\text{cm}$, is moulded to form a solid sphere. What is the radius of the sphere ?

Ans :

Volume of the sphere = Volume of the cuboid

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24 = 38808\,\text{cm}^3$$

$$4 \times \frac{22}{7} r^3 = 38808 \times 3$$

$$r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 441 \times 21$$

$$r^3 = 21 \times 21 \times 21$$

$$r = 21\,\text{cm}$$

13. Find the mode of the following grouped frequency distribution.

Class	**Frequency**
3-6	2
6-9	5
9-12	10
12-15	23
15-18	21
18-21	12
21-24	03

Ans :

We observe that the class 12-15 has maximum frequency 23. Therefore, this is the modal class.

We have, $l = 12$, $h = 3$, $f_1 = 23$, $f_0 = 10$ and $f_2 = 21$

$$M_o = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$$

$$= 12 + \frac{13}{15} \times 3$$

$$= 12 + \frac{13}{5}$$

$$= 14.6$$

14. Consider the data:

Class	6 5 - 85	8 5 - 105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	13	20	14	7	4

What is the difference of the upper limit of the median class and the lower limit of the modal class?

Ans :

Class	**Frequency**	**Cumulative frequency**
65-85	4	7
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	7	63
185-205	4	67

Here, $\frac{N}{2} = \frac{67}{2} = 33.5$, which lies in the interval $125 - 145$. Hence, upper limit of median class is 145.

Here, we see that the highest frequency is 20 which lies in 125-145. Hence, the lower limit of modal class is 125.

Required difference

= Upper limit of median class

− Lower limit of modal class

$$= 145 - 125 = 20$$

15. Find the class marks of the classes 15-35 and 45-60.

Ans : [Board 2020 OD Standard]

Class mark of $15-35 = \frac{15+35}{2} = \frac{50}{2} = 25$

and class mark of $45-60 = \frac{45+60}{2} = \frac{105}{2}$
$= 52.5$

16. If $P(E) = 0.20$, then what is the probability of 'not E'?

Ans : [Board Term-2, 2012]

We have
$$P(E) = 0.20$$
$$P(\text{not}E) = 1 - P(E)$$
$$= 1 - 0.20 = 0.80$$

or

If the probability of winning a game is $\frac{5}{11}$, find the probability of losing the game.

Ans : [Board Term-2, 2014]

Probability of winning the game,
$$P(E) = \frac{5}{11}$$
Probability of losing the game
$$P(\overline{E}) = 1 - P(E)$$
$$= 1 - \frac{5}{11} = \frac{6}{11}$$

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. An barrels manufacturer can produce up to 300 barrels per day. The profit made from the sale of these barrels can be modelled by the function $P(x) = -10x^2 + 3500x - 66000$ where $P(x)$ is the profit in rupees and x is the number of barrels made and sold.

Based on this model answer the following questions:

(i) When no barrels are produce what is a profit loss?
(a) Rs 22000 (b) Rs 66000
(c) Rs 11000 (d) Rs 33000

(ii) What is the break even point ? (Zero profit point is called break even)
(a) 10 barrels (b) 30 barrels
(c) 20 barrels (d) 100 barrels

(iii) What is the profit/loss if 175 barrels are produced
(a) Profit 266200
(b) Loss 266200
(c) Profit 240250
(d) Loss 240250

(iv) What is the profit/loss if 400 barrels are produced
(a) Profit Rs 466200
(b) Loss Rs 266000
(c) Profit Rs 342000
(d) Loss Rs 342000

(v) What is the maximum profit which can manufacturer earn?
(a) Rs 240250 (b) Rs 480500
(c) Rs 680250 (d) Rs 240250

Ans :

(i) When no barrels are produced, $x = 0$
$$P(x) = 0 + 0 - 66000$$
$$P(x) = -66000 \text{ Rs}$$
Thus (b) is correct option.

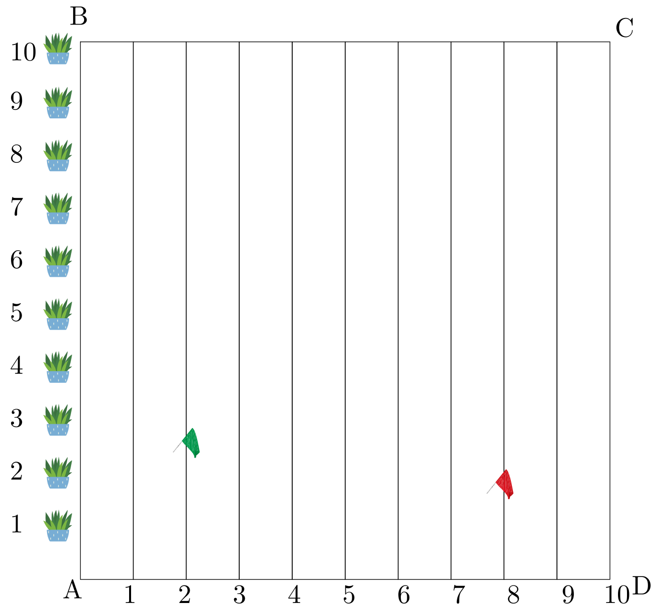
(ii) At break-even point $P(x) = 0$, thus
$$0 = -10x^2 + 3500x - 66000$$
$$x^2 + 350x + 6600 = 0$$
$$x^2 - 330x - 20x + 6600 = 0$$
$$x(x-330) - 20(x+330) = 0$$
$$(x-330)(x-20) = 0$$
$$x = 20, 330$$
Thus (c) is correct option.

(iii) $P(175) = -10(175)^2 + 3500(175) - 66000$
$= 240250$
Thus (c) is correct option.

(iv) $P(400) = -10(400)^2 + 3500(400) - 66000$
$= -266000$ Rs
Thus (b) is correct option.

(v) Rearranging the given equation we have
$$P(x) = -10x^2 + 3500x - 66000$$
$$= -10(x^2 - 350x + 6600)$$
$$= -10[(x-175)^2 - 30625 + 6600]$$
$$= -10[(x-175)^2 - 24025]$$
$$= -10(x-175)^2 + 240250$$
From above equation it is clear that maximum value of $P(x)$ is Rs 240250.
Thus (a) is correct option.

18. To conduct sports day activities, in a rectangular shaped school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AB, as shown in figure. Nishtha runs $\frac{1}{4}$th the distance AB on the 2nd line and posts a green flag. Suman runs $\frac{1}{5}$th the distance AB on the 8th line and posts a red flag.



(i) What is the position of green flag ?
(a) (2, 25) (b) (25, 4)
(c) (25,2) (d) (4, 25)

(ii) What is the position of red flag ?
(a) (20, 4) (b) (8, 20)
(c) (20, 8) (d) (4, 20)

(iii) What is the distance between both the flags?
(a) $\sqrt{51}$ (b) $3\sqrt{3}$
(c) $\sqrt{61}$ (d) $2\sqrt{3}$

(iv) What is the distance of red flag from point A ?
(a) $4\sqrt{29}$ (b) $2\sqrt{29}$
(c) $8\sqrt{15}$ (d) $16\sqrt{3}$

(v) If Rakhi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?
(a) (20, 4) (b) (22.5, 5)
(c) (4, 20) (d) (5, 22.5)

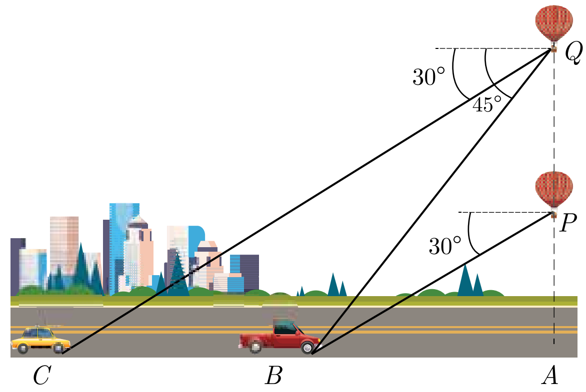
Ans :

(i) $\frac{1}{4}$th of the AD corresponds to $y = \frac{100}{4} = 25$ and 2nd line corresponds to $x = 2$. Thus coordinates of green flag point are (2, 25).
Thus (a) is correct option.
(ii) $\frac{1}{5}$th of the AD corresponds to $y = \frac{100}{5} = 20$ and 8th line corresponds to $x = 8$. Thus coordinates of red point are (8, 20).
Thus (b) is correct option.

(iii)
$$d = \sqrt{(8-2)^2 + (20-25)^2}$$
$$= \sqrt{6^2 + 5^2}$$
$$= \sqrt{61}$$
Thus (c) is correct option.

(iv)
$$d = \sqrt{8^2 + 20^2} = 4\sqrt{29}$$
Thus (a) is correct option.
(v) Mid point of green flag and red flag
$$= \left(\frac{2+8}{2}, \frac{25+20}{2}\right)$$
$$= (5, 22.5)$$
Thus (d) is correct option.

19. A hot air balloon is a type of aircraft. It is lifted by heating the air inside the balloon, usually with fire. Hot air weighs less than the same volume of cold air (it is less dense), which means that hot air will rise up or float when there is cold air around it, just like a bubble of air in a pot of water. The greater the difference between the hot and the cold, the greater the difference in density, and the stronger the balloon will pull up.



Lakshman is riding on a hot air balloon. After reaching at height x at point P, he spots a lorry parked at B on the ground at an angle of depression of 30°. The balloon rises further by 50 metres at point Q and now he spots the same lorry at an angle of depression of 45° and a car parked at C at an angle of depression of 30°.

(i) What is the relation between the height x of the balloon at point P and distance d between point A and B?
(a) $x = 3d$ (b) $d = 3x$
(c) $d^2 = 3x^2$ (d) $3d^2 = x^2$

(ii) When balloon rises further 50 metres, then what is the relation between new height y and d?
(a) $y = d + 50$ (b) $d = y$
(c) $y = \sqrt{3}\,d$ (d) $\sqrt{3}\,y = d$

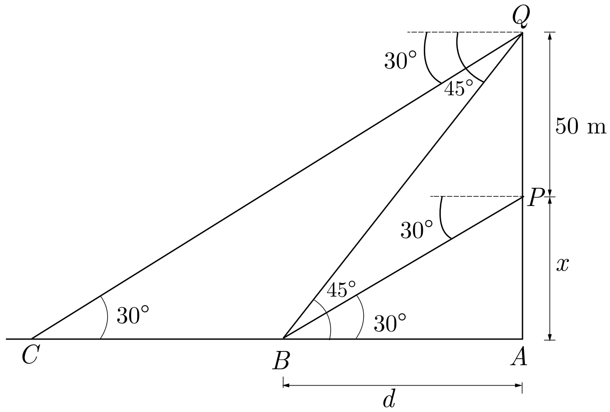
(iii) What is the new height of the balloon at point Q ?
(a) $50(\sqrt{3}+3)$ m (b) $25(\sqrt{3}+1)$ m
(c) $50(\sqrt{3}+1)$ m (d) $25(\sqrt{3}+3)$ m

(iv) What is the distance AB on the ground ?
(a) $50(\sqrt{3}+3)$ m
(b) $25(3+3\sqrt{3})$ m
(c) $50(\sqrt{3}+1)$ m
(d) $25(\sqrt{3}+3)$ m

(v) What is the distance AC on the ground ?
(a) $75(1+\sqrt{3})$ m
(b) $25(1+\sqrt{3})$ m
(c) $50(1+\sqrt{3})$ m
(d) $25(\sqrt{3}+3)$ m

Ans :

(i) We make the diagram as per given information.



In ΔAPB, $\tan 30^\circ = \frac{AP}{AB}$

$$\frac{1}{\sqrt{3}} = \frac{x}{d}$$

$$d = \sqrt{3}x \Rightarrow d^2 = 3x^2$$

Thus (c) is correct option.

(ii) In ΔBAQ,

$$\tan 45^\circ = \frac{AQ}{AB}$$

$$AB = AQ$$

$$d = y$$

Thus (b) is correct option.

(iii) From (i) and (ii) we have

$$d = \sqrt{3}x \text{ and } d = y$$

Since point Q is 50 m above point P, Thus

$$y = x + 50$$

Thus $\quad d = x + 50$

Solving above equations we get

$$\sqrt{3}x = x + 50$$

$$x(\sqrt{3} - 1) = 50$$

$$x = \frac{50}{(\sqrt{3} - 1)} = 25(\sqrt{3} + 1)$$

$$y = x + 50$$

$$= 25(\sqrt{3} + 1) + 50$$

$$= 25\sqrt{3} + 25 + 50$$

$$= 25(\sqrt{3} + 3)$$

Thus (d) is correct option.

(iv) The distance AB on the ground is d and which is equal to

$$d = \sqrt{3}x$$

or $\quad d = y = 25(\sqrt{3} + 3)$

Thus (d) is correct option.

(v) In ΔCAQ,

$$\tan 30^\circ = \frac{AQ}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{AC}$$

$$= \frac{25(\sqrt{3} + 3)}{AC}$$

$$AC = 25\sqrt{3}(\sqrt{3} + 3)$$

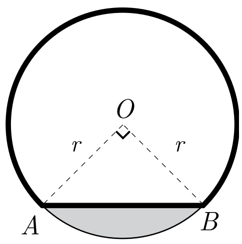
$$= 25(3 + 3\sqrt{3})$$

$$= 75(1 + \sqrt{3})$$

Thus (a) is correct option.

20. Atal Tunnel (also known as Rohtang Tunnel) is a highway tunnel built under the Rohtang Pass in the eastern Pir Panjal range of the Himalayas on the Leh-Manali Highway in Himachal Pradesh. At a length of 9.02 km, it is the longest tunnel above 10,000 feet (3,048 m) in the world and is named after former Prime Minister of India, Atal Bihari Vajpayee. The tunnel reduces the travel time and overall distance between Manali and Keylong on the way to Leh. Moreover, the tunnel bypasses most of the sites that were prone to road blockades, avalanches, and traffic snarls.

Earth is excavated to make a railway tunnel. The tunnel is a cylinder of radius 7 m and length 450 m. A level surface is laid inside the tunnel to carry the railway lines. Figure given below shows the circular cross - section of the tunnel. The level surface is represented by AB, the centre of the circle is O and $\angle AOB = 90^\circ$. The space below AB is filled with rubble (debris from the demolition buildings).



(i) How much volume of earth is removed to make the tunnel ?
 (a) 58700 m^3 (b) 61400 m^3
 (c) 62700 m^3 (d) 69300 m^3

(ii) If the cost of excavation of 1 cubic meter is Rs 250, what is the total cost of excavation?
 (a) Rs 17325000 (b) Rs 34650000
 (c) Rs 8662500 (d) Rs 12677500

(iii) A coating is to be done on the surface of inner curved part of tunnel. What is the area of tunnel to be being coated ?
 (a) 12300 m^2 (b) 14850 m^2
 (c) 15250 m^2 (d) 21200 m^2

(iv) Costing of coating is Rs 30 per m^2. What is the total cost of coating ?
(a) Rs 5588000 (b) Rs 445500
(c) Rs 339900 (d) Rs 228800

(v) How much volume of debris is required to fill the ground surface of tunnel ?
(a) 3500 m^3 (b) 14000 m^3
(c) 7000 m^3 (d) 10500 m^3

Ans :

(i) Cross-section area of tunnel to be excavated $= \pi r^2$
Volume of earth to be removed,

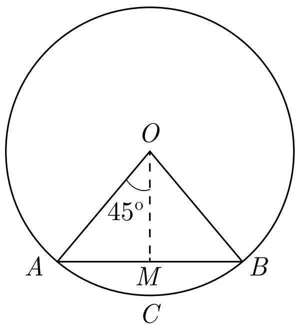
$$\pi r^2 l = \frac{22}{7} \times 7 \times 7 \times 450$$

$$= 69300 \text{ m}^3$$

Thus (d) is correct option.
(ii) Total cost of excavation

$$= 69300 \times 250 = ₹\,1732500$$

Thus (a) is correct option.
(iii) The geometry of cross-section is shown below.



Triangle OAB is isosceles triangle having right angle at O.
Length of curved part of cross-section,

$$= \frac{2\pi r(360^\circ - 90^\circ)}{360^\circ}$$

$$= \frac{2 \times \frac{22}{7} \times 7(360^\circ - 90^\circ)}{360^\circ}$$

$$= \frac{2 \times 22 \times 270^\circ}{360^\circ} = 33 \text{ m}$$

Total curved surface area of tunnel

= Length of curved part of cross-section × Length of tunnel

$$= 33 \times 450 = 14850 \text{ m}^2$$

Thus (b) is correct option.
(iv) Cost of coating on curved part,

$$= 14850 \times 30$$

$$= ₹\,445500$$

Thus (b) is correct option.
(v) Cross-section area of debris part of tunnel

= Area of $OACB$ − Area of $\Delta\, OAB$

$$= \frac{\pi r^2}{4} - \frac{r^2}{2}$$

$$= \frac{\frac{22}{7} \times 7 \times 7}{4} - \frac{7 \times 7}{2}$$

$$= \frac{11 \times 7}{2} - \frac{7 \times 7}{2}$$

$$= \frac{4 \times 7}{2} = 14 \text{ m}^2$$

Volume of debris required

$$= 14 \times 500 = 7000 \text{ m}^3$$

Thus (c) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Find the value of k such that the polynomial $x^2 - (k+6)x + 2(2k+1)$ has sum of its zeros equal to half of their product.

Ans : [Board 2019 Delhi]

Let α and β be the roots of given quadratic equation

$$x^2 - (k+6)x + 2(2k+1) = 0$$

Now sum of roots,. $\alpha + \beta = -\frac{-(k+6)}{1} = k+6$

Product of roots, $\alpha\beta = \frac{2(2k+1)}{1} = 2(2k+1)$

According to given condition,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$k + 6 = \frac{1}{2}[2(2k+1)]$$

$$k + 6 = 2k + 1 \Rightarrow k = 5$$

d310

Hence, the value of k is 5.

or

Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

Ans : [Board 2019 OD]

We have $2x^2 - 4x + 3 = 0$
Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 2$ $b = -4$, $c = 3$

Now $D = b^2 - 4ac$

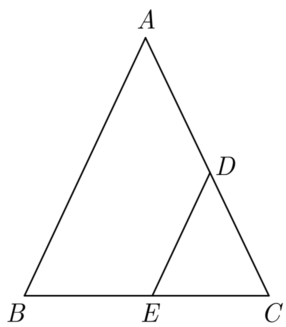
$$= (-4)^2 - 4(2) \times (3)$$

$$= -8 < 0 \text{ or } (-\text{ve})$$

d311

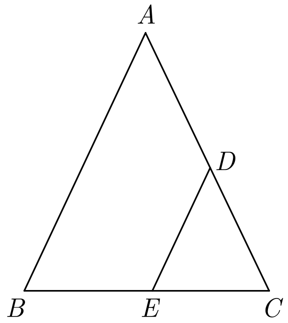
Hence, the given equation has no real roots.

22. In the figure of $\Delta\, ABC$, the points D and E are on the sides CA, CB respectively such that $DE || AB$, $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$. Then, find x.



OR

In the figure of $\Delta\, ABC$, $DE || AB$. If $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$, then find the value of x.



Ans : [Board Term-1 2015, 2016]

We have $\frac{CD}{AD} = \frac{CE}{BE}$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Alternative Method :

In ABC, $DE \parallel AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$

$$\frac{CD}{CA-CD} = \frac{CE}{CB-CE}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$

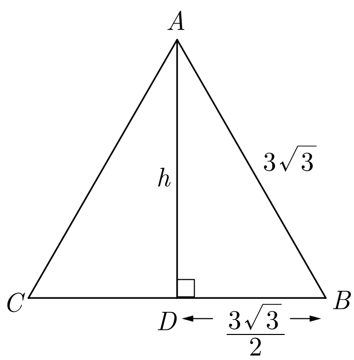
$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

$$27 = h^2 + \frac{27}{4}$$

$$h^2 = 27 - \frac{27}{4} = \frac{81}{4}$$

$$h = \frac{9}{2} = 4.5 \text{ cm}$$

23. Prove that : $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Ans : [Board 2020 OD Basic]

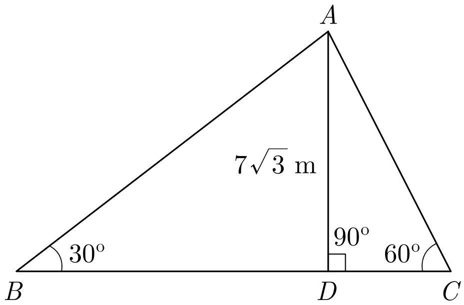
$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta}$$

$$= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$$

$$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$$

$$= \cos^2\theta - \sin^2\theta \quad \text{Hence Proved}$$

24. In the given figure, if $AD = 7\sqrt{3}$ m, then find the value of BC.



Ans : [Board Term-2 2012]

Let $BD = x$ and $DC = y$

From ΔADB we get

$$\tan 30^\circ = \frac{7\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{7\sqrt{3}}{x}$$

$$x = 7\sqrt{3} \times \sqrt{3} = 21 \text{ m}$$

From ΔADC,

$$\tan 60^\circ = \frac{7\sqrt{3}}{y}$$

$$\sqrt{3} = \frac{7\sqrt{3}}{y}$$

$$y = 7 \text{ m.}$$

Now $$BC = BD + DC$$

$$= 21 + 7 = 28 \text{ m.}$$

Hence, the value of BC is 28 m.

25. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5.

Ans : [Board 2020 OD Basic]

Number of possible outcomes,

$$n(S) = 6^2 = 36$$

The favourable outcomes are (sum less than 5) {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2) and (3, 1)} i.e. 6 outcomes.

Number of favourable outcome,

$$n(E) = 6$$

P (have sum less than 5)

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

26. If a number x is chosen at random from the numbers -3, -2, -1. 0, 1, 2, 3, then find the probability of $x^2 < 4$.

Ans : [Board 2020 OD Standard]

Possible outcome are -3, -2, -1. 0, 1, 2, 3 i.e 7 outcomes.

Thus $n(S) = 7$

Favourable outcomes are $x^2 < 4$ i.e. $= -1$, 0, 1.

$$n(E) = 3$$

$$P(x^2 < 4), \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{7}$$

o180

27. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

Ans : [Board Term-1 2011, Set-25]

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha + \beta = -\frac{-5}{a} = \frac{5}{a}$

b126

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

28. Solve for x :

$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}; x \neq 1,2,3$$

Ans : [Board Term-2 OD 2016]

We have $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$

$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2x-4}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2}{(x-1)(x-3)} = \frac{2}{3}$$

$$3 = (x-1)(x-3)$$

$$x^2 - 4x + 3 = 3$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

d135

Thus $x = 0$ or $x = 4$

29. Find the point on y-axis which is equidistant from the points $(5,-2)$ and $(-3,2)$.

Ans : [Board 2019 Delhi]

We have point $A = (5,-2)$ and $B = (-3,2)$

Let $C(0,a)$ be point on y-axis.

According to question, point C is equidistant from A and B.

Thus $AC = BC$

Using distance formula we have

$$\sqrt{(0-5)^2 + (a+2)^2} = \sqrt{(0+3)^2 + (a-2)^2}$$

$$\sqrt{25 + a^2 + 4 + 4a} = \sqrt{9 + a^2 + 4 - 4a}$$

$$25 + a^2 + 4 + 4a = 9 + a^2 + 4 - 4a$$

$$8a = -16 \Rightarrow a = -2$$

g280

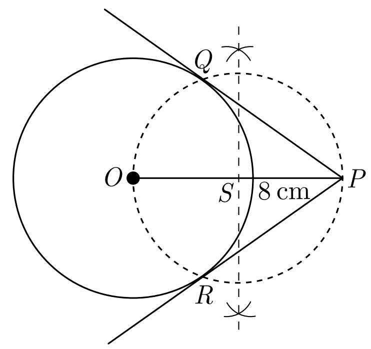
Hence, point on y-axis is $(0-2)$.

30. Construct a pair tangents PQ and PR to a circle of radius 4 cm from a point P outside the circle 8 cm away from the centre. Measure PQ and PR.

Ans : [Board Term-2 2014]

Steps of Construction :

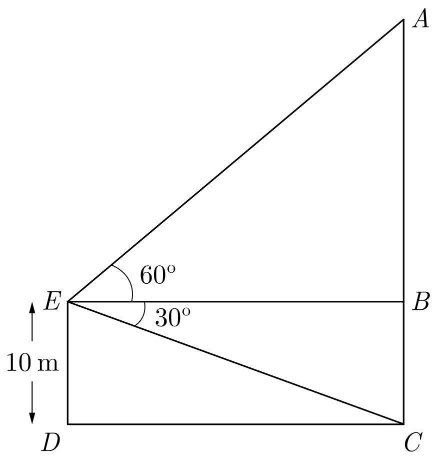
1. Draw a line segment OP of length 8 cm.
2. Draw a circle with centre O and radius 4 cm.
3. Taking OP as diameter draw another circle which intersects the first circle at Q and R.
4. Join P to Q and P to R. On measuring, we get $PQ = PR = 5$ cm



31. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Ans : [Board Term-2 OD 2016]

As per given in question we have drawn figure below. Here AC is height of hill and man is at E. $ED = 10$ is height of ship from water level.



In ΔBCE, $BC = EC = 10$ m and

$$\angle BEC = 30^o$$

Now $$\tan 30^o = \frac{BC}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{BE}$$

$$BE = 10\sqrt{3}$$

Since $BE = CD$, distance of hill from ship

$$CD = 10\sqrt{3}\,\text{m} = 10 \times 1.732 \text{ m}$$

$$= 17.32 \text{ m}$$

Now in ΔABE, $\angle AEB = 60^o$

where $AB = h$, $BE = 10\sqrt{3}\,\text{m}$

and $$\angle AEB = 60^o$$

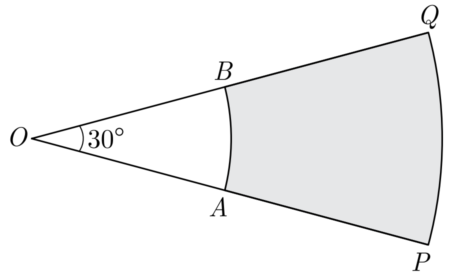
Thus $$\tan 60^o = \frac{AB}{BE}$$

$$\sqrt{3} = \frac{AB}{10\sqrt{3}}$$

$$AB = 10\sqrt{3} \times \sqrt{3} = 30\,m$$

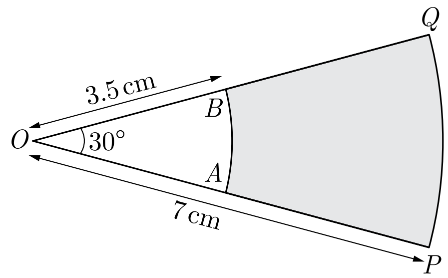
Thus height of hill $AB + 10 = 40$ m

32. In Figure, PQ and AB are two arcs of concentric circles of radii 7 cm and 3.5 cm respectively, with centre O. If $\angle POQ = 30^\circ$, then find the area of shaded region.



Ans : [Board 2020 OD Basic]

We redraw the given figure as below.



Area of shaded region

$$\pi[R^2 - r^2]\frac{\theta}{360^\circ} = \frac{22}{7}[7^2 - (3.5)^2]\frac{30^\circ}{360^\circ}$$

$$= \frac{22}{7}(7 + 3.5)(7 - 3.5) \times \frac{1}{12}$$

$$= \frac{22}{7} \times 10.5 \times 3.5 \times \frac{1}{12}$$

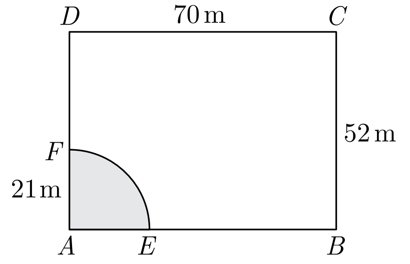
$$= 9.625\,\text{cm}^2$$

or

A horse is tethered to one corner of a rectangular field of dimensions $70\,\text{m} \times 52\,\text{m}$, by a rope of length 21 m. How much area of the field can it graze?

Ans : [Board 2020 OD Basic]

As per information given in question we have drawn the figure below.



Length of the rope is 21 cm.

Shaded portion $AEFA$ indicates the area in which the horse can graze. Clearly it is the area of a quadrant of a circle of radius, $r = 21$ m.

Area of quadrant,

$$\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times (21)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$$

$$= 346.5\,\text{m}^2$$

Hence, the graze area is 346.5 m^2

33. If the mean of the following data is 14.7, find the values of p and q.

Class	0-6	6-12	12-18	18-24	24-30	30-36	36-42	Total
Frequency	10	p	4	7	q	4	1	40

Ans : [Board Term-1 2013]

Class	x_i	f_i	$f_i x_i$
0-6	3	10	30
6-12	9	p	$9p$
12-18	15	4	60
18-24	21	7	147
24-30	27	q	$27q$
30-36	33	4	132
36-42	39	1	39
	Total	$\sum f_i = 26 + p + q = 40$	$\sum f_i x_i = 408 + 9p + 27q$

We have $$\sum f_i = 40,$$

$$26 + p + q = 40$$

$$p + q = 14 \qquad ...(1)$$

Mean $$M = \frac{\sum x_i f_i}{\sum f_i}$$

$$14.7 = \frac{408 + 9p + 27q}{40}$$

$$588 = 408 + 9p + 27q$$

$$180 = 9p + 27q$$

$$p + 3q = 20 \qquad ...(2)$$

Subtracting equation (1) from (2) we have,

$$2q = 6 \Rightarrow q = 3$$

Substituting this value of q in equation (2) we get

$$p = 14 - q = 14 - 3 = 11$$

Hence, $p = 11$, $q = 3$

or

Find the mean and mode of the following frequency distribution :

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	8	10	15	7	4	3

Ans : [Board Term-1 2013]

We prepare following table to find mean.

Classes	x_i	f_i	$f_i x_i$
0-10	5	3	15
10-20	15	8	120
20-30	25	10	250
30-40	35	15	525
40-50	45	7	315
50-60	55	4	220
60-70	65	3	195
		$\sum f_i = 50$	$\sum f_i x_i = 1640$

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{1640}{50} = 32.8$

Class 30-40 has the maximum frequency 35, therefore this is model class.

Here $l = 30,\ f_1 = 15,\ f_2 = 7,\ f_0 = 10,\ h = 10$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 30 + \frac{15 - 10}{30 - 10 - 7} \times 10$$

$$= 30 + \frac{5}{13} \times 10$$

$$= 30 + \frac{50}{13}$$

$$= 30 + 3.85 = 33.85$$

n222

34. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

Ans : [Board Term-1 2012 Set-25]

We have $n^2 - n = n(n-1)$

Thus $n^2 - n$ is product of two consecutive positive integers.

Any positive integer is of the form $2q$ or $2q+1$, for some integer q.

a144

Case 1 : $n = 2q$

If $n = 2q$ we have

$$n(n-1) = 2q(2q-1) = 2m,$$

where $m = q(2q-1)$ which is divisible by 2.

Case 1 : $n = 2q+1$

If $n = 2q+1$, we have

$$n(n-1) = (2q+1)(2q+1-1) = 2q(2q+1) = 2m$$

where $m = q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

or

Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7 + 2\sqrt{3}$ is also an irrational number.

Ans : [Board Term-1 2012]

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b}, \quad (a, b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 9b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

a164

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Let us assume that $7 + 2\sqrt{3}$ be rational equal to a, then we have

$$7 + 2\sqrt{3} = \frac{p}{q} \quad q \neq 0 \text{ and } p \text{ and } q \text{ are co-primes}$$

$$2\sqrt{3} = \frac{p}{q} - 7 = \frac{p - 7q}{q}$$

or $$\sqrt{3} = \frac{p - 7q}{2q}$$

Here $p - 7q$ and $2q$ both are integers, hence $\sqrt{3}$ should be a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence our assumption is not correct and $7 + 2\sqrt{3}$ is irrational.

35. Solve $x + y = 5$ and $2x - 3y = 4$ by elimination method and the substitution method.

Ans : [Board Term-1 2015]

By Elimination Method :

We have, $x + y = 5$ ...(1)

and $2x - 3y = 4$ ...(2)

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x + y) + (2x - 3y) = 3 \times 5 + 4$$

or, $$3x + 3y + 2x - 3y = 15 + 4$$

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

c155

Substituting $x = \frac{19}{5}$ in equation (1),

$$\frac{19}{5} + y = 5$$

$$y = 5 - \frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

By Substituting Method :

We have, $x + y = 5$ ...(1)

and $2x - 3y = 4$ ...(2)

From equation (1), $y = 5 - x$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$2x - 3(5-x) = 4$$

$$2x - 15 + 3x = 4$$

$$5x = 19$$

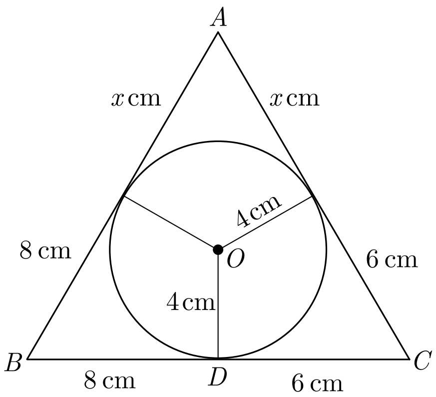
$$x = \frac{19}{5}$$

Substituting this value of x in equation (3), we get

$$y = 5 - \frac{19}{5} = \frac{6}{5}$$

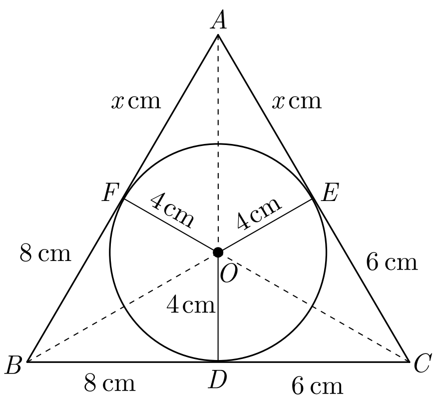
Hence $x = \frac{19}{5}$ and $y = \frac{6}{5}$

36. In the figure, the ΔABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find AB and AC.



Ans : [Board Term-2 Delhi 2014, 2012]

We redraw the given circle by joining AO, BO and CO shown in figure below. Let length of AF be x.



Since length of tangents from an external point to a circle are equal,

At A, $AF = AE = x$ (2)

At B $BF = BD = 8$ cm (3)

At C $CD = CE = 6$ cm (4)

Now $AB = x + 8$

$AC = x + 6$

$BC = 8 + 6 = 14$ cm

Perimeter of circle

$$p = AB + BC + CA$$

$$= x + 8 + 14 + x + 6$$

$$= 2(x + 14)$$

Semi-perimeter of circle

$$s = \tfrac{1}{2}p = x + 14$$

Area or triangle ΔABC

$$\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{48x^2 + 672x} \quad (1)$$

Area or triangle ΔABC,

$$\Delta ABC = \tfrac{1}{2}rp$$

$$= \tfrac{1}{2} \times 4 \times 2(x+14)$$

$$= 4(x+14) \quad (2)$$

From equation (1) and (2) we have

$$48x^2 + 672x = 16(x+14)^2$$

$$48x(x+14) = 16(x+14)^2$$

$$3x = x + 14$$

or, $x = 7$

Thus $AC = 6 + 7 = 13$ cm

and AB $8 + 7 = 15$ cm.