CLASS X (2020-21)
MATHEMATICS BASIC(241)
SAMPLE PAPER-8

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

Section - I

1. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then what is the LCM of (p, q) ?

Ans :

a250

We have $p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

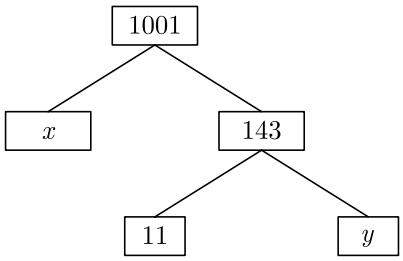
$\text{LCM}(p, q) = \text{LCM}(ab^2, a^3b)$

$= a \times b \times b \times a \times a = a^3b^2$

LCM is the product of the greatest power of each prime factor involved in the numbers.

or

What are the values of x and y in the given figure ?



Ans :

a251

We have $1001 = x \times 143 \Rightarrow x = 7$

$143 = y \times 11 \Rightarrow y = 13$

Hence $x = 7,\ y = 13$

2. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then what is the value of k

Ans :

b231

Sum of the zeroes, $6 = \frac{3k}{2}$

$k = \frac{12}{3} = 4$

3. Find whether the lines represented by $2x + y = 3$ and $4x + 2y = 6$ are parallel, coincident or intersecting.

Ans : [Board Term-1 2016]

Ans :

Here $a_1 = 2, b_1 = 1, c_1 = -3$ and $a_1 = 4, b_2 = 2, c_2 = -6$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

c105

then the lines are parallel.

Clearly $\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$

Hence lines are coincident.

4. Find the nature of roots of the quadratic equation $x^2 - 4x - 3\sqrt{2} = 0$

Ans :

d338

We have $x^2 - 4x - 3\sqrt{2} = 0$

Here $a = 1,\ b = -4$ and $c = -3\sqrt{2}$

Now $D = b^2 - 4ac$

$= (-4)^2 - 4(1)(-3\sqrt{2})$

$= 16 + 12\sqrt{2} > 0$

Hence, the given equation has two distinct real roots.

or

Find the nature of roots of the quadratic equation $3x^2 + 4\sqrt{3}\,x + 4$.

Ans :

d339

We have $3x^2 + 4\sqrt{3}\,x + 4 = 0$

Here, $a = 3,\ b = 4\sqrt{3}$ and $c = 4$

Now $D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48 = 0$

Hence, the equation has real and equal roots.

5. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then what is the value of y ?

Ans : [Board 2020 Delhi Basic]

Since, centre is the mid-point of end points of the diameter.

$$(3,\ 5) = \left(\frac{4+2}{2},\ \frac{7+y}{2}\right)$$

g301

Comparing both the sides, we get

$$5 = \frac{7+y}{2}$$

$$7+y = 10 \Rightarrow y = 3$$

6. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90^\circ$, then what is the value of $\tan 5\alpha$?

Ans :

We have $\cos 9\alpha = \sin\alpha$ where $9\alpha < 90^\circ$

$$\sin(90^\circ - 9\alpha) = \sin\alpha$$

$$90^\circ - 9\alpha = \alpha$$

$$10\alpha = 90^\circ \Rightarrow \alpha = 9^\circ$$

h312

$$\tan 5\alpha = \tan(5 \times 9^\circ)$$

$$= \tan 45^\circ = 1 \qquad [\tan 45^\circ = 1]$$

7. If the height and length of the shadow of a man are equal, then find the angle of elevation of the sun.

Ans :

Let AB be the height of a man and BC be the shadow of a man.

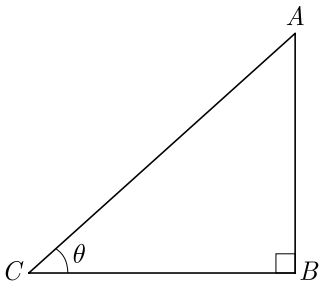
$$AB = BC$$

i216

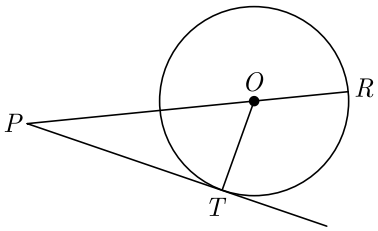
In ΔABC, $\tan\theta = \frac{AB}{BC}$

$$\frac{AB}{AB} = \tan\theta$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$



8. In figure, on a circle of radius 7 cm, tangent PT is drawn from a point P such that $PT = 24$ cm. If O is the centre of the circle, then what is the length of PR ?



Ans : [Board 2020 Delhi Basic]

Tangent at any point of a circle is perpendicular to the radius at the point of contact.

Thus $OT \perp PT$

j234

Now in right-angled triangle PTO

$$OP^2 = OT^2 + PT^2$$

$$= (7)^2 + (24)^2$$

$$= 49 + 576$$

$$= 625$$

Thus $OP = 25$ cm

Since $OR = OT$ because of radii of circle,

$$PR = OP + OR$$

$$= 25 + 7$$

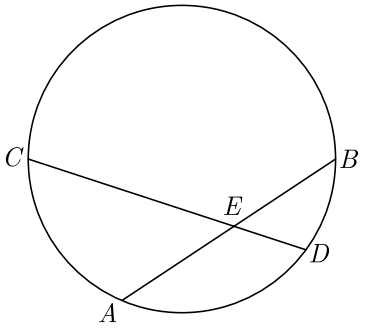
$$= 32 \text{ cm}$$

or

Two chords AB and CD of a circle intersect at E such that $AE = 2.4$ cm, $BE = 3.2$ cm and $CE = 1.6$ cm. What is the length of DE?

Ans :

j235



Applying the rule, $AE \times EB = CE \times ED$

$$2.4 \times 3.2 = 1.6 \times ED$$

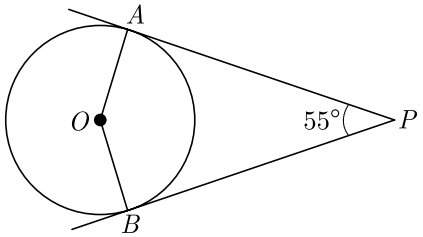
$$ED = 4.8 \text{ cm}$$

9. To draw a pair of tangents to a circle which are inclined to each other at an angle of 55°, it is required to draw tangents at the end points of these two radii of the circle, what is the angle between two radii?

Ans :

According to the question we can draw the following diagram.

k175



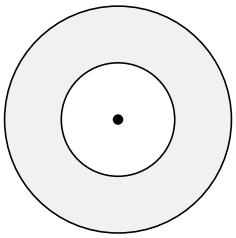
From figure,

$$\angle AOB + \angle APB = 180^\circ$$

$$\angle AOB = 180^\circ - \angle APB$$

$$= 180^\circ - 55^\circ = 125^\circ$$

10. Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, find the area of the shaded ring shaped region in square cm.



I103

Ans : [Board Term-2 2012]

$$\begin{aligned}\text{Area of circle} &= \pi r^2\\ \text{Area of the shaded region} &= \pi(2)^2 - \pi(1)^2\\ &= 4\pi - \pi\\ &= 3\pi \text{ sq cm}\end{aligned}$$

or

The diameter of two circle with centre A and B are 16 cm and 30 cm respectively. If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C.

Ans : [Board Term-2 2012]

Let the radius of circle with centre C be r.
According to question we have,

I104

$$\begin{aligned}\pi(8)^2 + \pi(15)^2 &= \pi r^2\\ 64\pi + 225\pi &= \pi r^2\\ 289\pi &= \pi r^2\\ r^2 &= 289 \text{ or } R = 17 \text{ cm}\end{aligned}$$

Circumference of circle

$$\begin{aligned}2\pi r &= 2\pi \times 17\\ &= 34\pi \text{ cm}\end{aligned}$$

11. Find the ratio of lateral surface areas of two cylinders with equal height.

Ans :

m290

$$2\pi Rh : 2\pi rh = R : r$$

12. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. What is the diameter of each sphere?

Ans :

Volume of the twelve solid sphere is equal to the volume of cylinder.

m298

$$\begin{aligned}V_{12\,\text{sphere}} &= V_{\text{cylinder}}\\ 12 \times \frac{4}{3}\pi r^3 &= \pi\left(\frac{2}{1}\right)^2 \times 16\\ 16\pi r^3 &= 16\pi\\ r^3 &= 1\\ \Rightarrow \quad r &= 1\,\text{cm}\end{aligned}$$

Diameter of each sphere, $d = 2r$

$$= 2 \times 1 = 2\,\text{cm}$$

13. If median is 137 and mean is 137.05, then what is the value of mode ?

Ans :

n116

$$\begin{aligned}M_o &= 3M_d - 2M\\ &= 3(137) - 2(137.05)\\ &= 411 - 274.10 = 136.90\end{aligned}$$

14. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below

Class	Frequency
13.8-14	2
14-14.2	4
14.2-14.4	5
14.4-14.6	71
14.6-14.8	48
14.8-15	20

What is the number of athletes who completed the race in less than 14.6 second?

Ans :

The number of athletes who completed the race in less than 14.6 second,

n124

$$= 2 + 4 + 5 + 71 = 82$$

15. If the mean of the first n natural number is 15, then find n.

Ans : [Board 2020 Delhi Standard]

Given : 1, 2, 3, 4, ... to n terms.
The sum of first n natural numbers

n151

$$S_n = \frac{n(n+1)}{2}$$

Mean, $$M = \frac{n(n+1)}{2 \times n}$$

$$15 = \frac{n(n+1)}{2 \times n}$$

$$15 = \frac{n+1}{2}$$

$$n + 1 = 30 \Rightarrow n = 29$$

16. If E be an event such that $P(E) = \frac{3}{7}$, what is $P(\text{not } E)$ equal to?

Ans : [Board Term-2, 2014]

We have $P(E) = \frac{3}{7}$

o145

$$\begin{aligned}P(\text{not } E) &= 1 - P(E)\\ &= 1 - \frac{3}{7} = \frac{4}{7}\end{aligned}$$

or

A bag contains lemon flavoured candies only. Shalini takes out one candy without looking into the bag. What is the probability that she takes out an orange flavoured candy?

Ans : [Board Term-2, 2012]

Bag contains only lemon flavoured candies. So, getting an orange flavoured candy is an impossible.

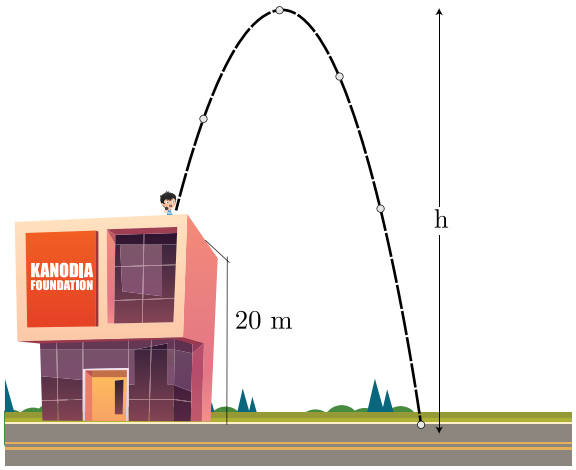
o146

$$P(E) = 0$$

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. Lavanya throws a ball upwards, from a rooftop, which is 20 m above from ground. It will reach a maximum height and then fall back to the ground. The height of the ball from the ground at time t is h, which is given by $h = -4t^2 + 16t + 20$.



(i) What is the height reached by the ball after 1 second?
(a) 64 m (b) 128 m
(c) 32 m (d) 20 m

b404

(ii) What is the maximum height reached by the ball?
(a) 54 m (b) 44 m
(c) 36 m (d) 18 m

(iii) How long will the ball take to hit the ground?
(a) 4 seconds (b) 3 seconds
(c) 5 seconds (d) 6 seconds

(iv) What are the two possible times to reach the ball at the same height of 32 m?
(a) 1 and 3 seconds (b) 1 and 4 seconds
(c) 1 and 2 seconds (d) 1 and 5 seconds

(v) Where is the ball after 5 seconds ?
(a) at the ground (b) rebounds
(c) at highest point (d) fall back

Ans :

(i) Height is given by,

$$h = -4t^2 + 16t + 20$$

At $t = 1$ second,

$$h = -4(1)^2 + 16(1) + 20 = 32 \text{ m}$$

Thus (c) is correct option.

(ii) Rearranging the given equation, by completing the square,

$$\begin{aligned} h &= -4(t^2 - 4t - 5) \\ &= -4(t^2 - 4t + 4 - 4 - 5) \\ &= -4[(t-2)^2 - 9] \\ &= -4(t-2)^2 + 36 \end{aligned}$$

Height is maximum, at $t = 2$, thus

$$h_{max} = 0 + 36 = 36 \text{ m}$$

Thus (c) is correct option.

(iii) When ball hits the ground, $h = 0$, thus

$$\begin{aligned} -4t^2 + 16t + 20 &= 0 \\ t^2 - 4t - 5 &= 0 \\ (t-5)(t+1) &= 0 \end{aligned}$$

Thus $t = 5$ or $t = -1$. Since, time cannot be negative, the $t = 5$ seconds is correct answer.

Thus (c) is correct option.

(iv) Since,

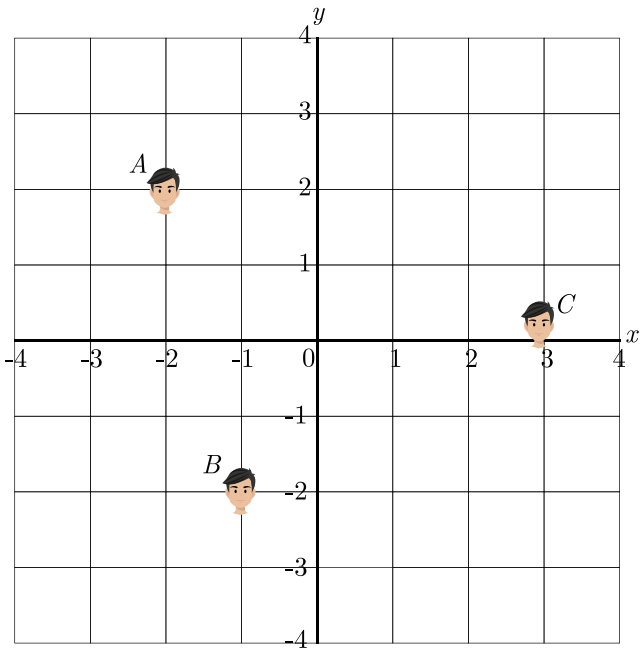
$$\begin{aligned} h &= -4t^2 + 16t^2 + 20 \\ 32 &= -4t^2 + 16t^2 + 20 \\ 8 &= -t^2 + 4t^2 + 5 \\ t^2 - 4t + 3 &= 0 \\ t^2 + 3t - t + 3 &= 0 \\ (t-1)(t-3) &= 0 \Rightarrow t = 3, 1 \end{aligned}$$

Thus (a) is correct option.

(v) From (iii) at $t = 5$ we have $h = 0$. Thus it will hit ground, then after that ball will rebound.

Thus (b) is correct option.

18. Ajay, Bhigu and Colin are fast friend since childhood. They always want to sit in a row in the classroom . But teacher doesn't allow them and rotate the seats row-wise everyday. Bhigu is very good in maths and he does distance calculation everyday. He consider the centre of class as origin and marks their position on a paper in a co-ordinate system. One day Bhigu make the following diagram of their seating position.



(i) What are the coordinates of point A?
(a) $(2, 2)$ (b) $(2, -2)$
(c) $(-2, 2)$ (d) $(-2, -2)$

g402

(ii) What is the distance of point A from origin ?
(a) 8 (b) $2\sqrt{2}$
(c) 4 (d) $4\sqrt{2}$

(iii) What is the distance between A and B ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $\sqrt{17}$ (d) $2\sqrt{5}$

(iv) What is the distance between B and C ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $2\sqrt{17}$ (d) $2\sqrt{5}$

(v) A point D lies on the line segment between points A and B such that $AD:DB=4:3$. What are the the coordinates of point D ?
(a) $\left(\frac{10}{7},\frac{2}{7}\right)$ (b) $\left(\frac{2}{7},\frac{7}{7}\right)$
(c) $\left(-\frac{10}{7},-\frac{2}{7}\right)$ (d) $\left(-\frac{2}{7},-\frac{7}{7}\right)$

Ans :

(i) It may be seen easily from figure that coordinates of point A are $(-2,2)$.
Thus (c) is correct option.

(ii) $OA = \sqrt{(0+2)^2+(0-2)^2} = 2\sqrt{2}$
Thus (b) is correct option.

(iii) It may be seen easily from figure that coordinates of point A are $(-1,-2)$.

$$AB = \sqrt{(-2+1)^2+(2+2)^2}$$
$$= \sqrt{1+4^2} = \sqrt{17}$$

Thus (c) is correct option.

(iv) It may be seen easily from figure that coordinates of point A are $(0,\ 3)$.

$$BC = \sqrt{(-1-3)^2+(-2-0)^2}$$
$$= \sqrt{4^2+4} = 2\sqrt{5}$$

Thus (d) is correct option.

(v) We have $A(-2,2)$ and $B(-1,-2)$

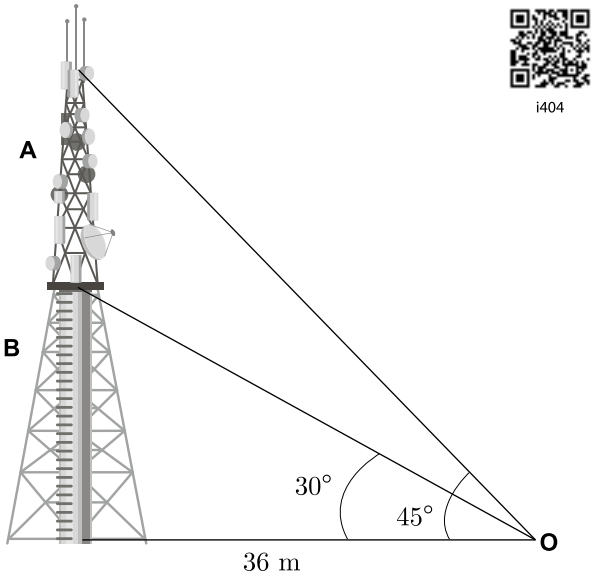
$$\frac{m}{n} = \frac{4}{3}$$

$$x = \frac{mx_2+nx_1}{m+n}$$
$$= \frac{-1(4)+3(-2)}{4+3} = \frac{-10}{7}$$

$$y = \frac{my_2+hy_1}{m+n}$$
$$= \frac{-2(4)+3(2)}{4+3} = \frac{-2}{7}$$

Thus (c) is correct option.

19. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure, including microwave dishes. They are among the tallest human-made structures. There are 2 main types: guyed and self-supporting structures.
On a similar concept, a radio station tower was built in two sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 m. From point O, the angle of elevation of the top of section B is 30° and the angle of elevation of the top of section A is 45°.



(i) What is the height of the section B ?
(a) $12\sqrt{3}$ m (b) $12\sqrt{2}$ m
(c) $8\sqrt{3}$ m (d) $4\sqrt{2}$ m

(ii) What is the height of the section A ?
(a) $12(2-\sqrt{2})$ (b) $24(2-\sqrt{2})$
(c) $12(3-\sqrt{3})$ (d) $24(3-\sqrt{3})$

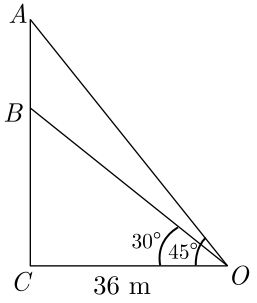
(iii) What is the length of the wire structure from the point O to the top of section A ?
(a) $32\sqrt{2}$ m (b) $24\sqrt{3}$ m
(c) $28\sqrt{3}$ m (d) $36\sqrt{2}$ m

(iv) What is the length of the wire structure from the point O to the top of section B ?
(a) $12\sqrt{3}$ m (b) $24\sqrt{3}$ m
(c) $28\sqrt{3}$ m (d) $16\sqrt{3}$ m

(v) What is the angle of depression from top of tower to point O ?
(a) 30° (b) 45°
(c) 15° (d) 75°

Ans :

(i) We make the following diagram as per given information.



In ΔBCO $\tan 30^\circ = \frac{BC}{OC}$

$$BC = OC\tan 30^\circ$$
$$BC = 36\times\frac{1}{\sqrt{3}} = 12\sqrt{3} \text{ m}$$

Thus (a) is correct option.

(ii) In ΔACO,

$$\tan 45^\circ = \frac{AC}{OC} = 1$$

Thus $AC = OC = 36$ m

Now, $AB = AC - BC$

$= 36 - 12\sqrt{3} = 12(3 - \sqrt{3})$ m

Thus (c) is correct option.

(iii) In ΔACO,

$$\cos 45^\circ = \frac{OC}{OA}$$

$$\frac{1}{\sqrt{2}} = \frac{36}{OA}$$

$$OA = 36\sqrt{2} \text{ m}$$

Thus (d) is correct option.

(iv) In ΔBCO,

$$\cos 30^\circ = \frac{OC}{OB}$$

$$\frac{\sqrt{3}}{2} = \frac{36}{OB}$$

$$OB = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 24\sqrt{3} \text{ m}$$

Thus (b) is correct option.

(v) It is clear from figure that angle of elevation from point O to top of tower is 45°. This is equal to the angle of depression from top of tower to point O.

Thus (b) is correct option.

20. A bakery is an establishment that produces and sells flour-based food baked in an oven such as bread, cookies, cakes, pastries, and pies. Some retail bakeries are also categorized as cafés, serving coffee and tea to customers who wish to consume the baked goods on the premises.

Tania runs a bakery shop and her bakery is very famous for tasty biscuits. The amount of mixture required to make one biscuit is 18 cu cm. Before it is cooked, the mixture is rolled into a sphere. After the biscuit is cooked, the biscuit becomes a cylinder of radius 3 cm and height 0.7 cm. The increase in volume is due to air being trapped in the biscuit. Biscuits are packed in a cylindrical card box of height 14 cm. The arrangement of biscuits is shown below.

(i) What is the volume of the biscuits after it is cooked ?

(a) 17.8 cu cm (b) 18.7 cu cm

(c) 19.8 cu cm (d) 21.2 cu cm

m404

(ii) What is the volume of air trapped, while cooking the biscuit ?

(a) 1.8 cu cm (b) 0.7 cu cm

(c) 1.5 cu cm (d) 3.2 cu cm

(iii) How many biscuits will be there in a box ?

(a) 120 (b) 70

(c) 140 (d) 60

(iv) How much space is vacant in box after biscuits are packed ?

(a) 940 cm^3 (b) 792 cm^3

(c) 846 cm^3 (d) 912 cm^3

(v) If weight of 7 biscuits is 50 grams, what will be the weight of box of biscuits?

(a) 750 gram (b) 1.4 kg

(c) 900 gram (d) 1 kg

Ans :

(i) Volume of the biscuit,

$$= \pi r^2 h = \frac{22}{7} \times 3^2 \times 0.7 = 19.8 \text{ cu cm}$$

Thus (c) is correct option.

(ii) Volume of air trap

= Volume of biscuit − Volume of sphere

$= 19.8 - 18 = 1.8$ cu cm

Thus (a) is correct option.

(iii) In a layer, 7 biscuits are arranged whose height is 0.7 cm.

Thus total layer in box,

$$= \frac{14}{0.7} = 20 \text{ layer}$$

Total biscuits in box $= 20 \times 7 = 140$ biscuits

Thus (c) is correct option.

(iv) From figure it is clear that radius of box is 3 times of biscuit radius i.e. $3 \times 3 = 9$.

Volume of box $= \pi R^2 H$

$= \frac{22}{7} \times 9 \times 9 \times 14$

$= 22 \times 9 \times 9 \times 2$

$= 3564 \text{ cm}^3$

Volume of biscuits $= \pi r^2 h \times 140$

$= 19.8 \times 140$

$= 2772 \text{ cm}^3$

Vacant volume $= 3564 - 2772$

$= 792 \text{ cm}^3$

Thus (b) is correct option.

(v) Weight of 7 biscuits = 50 grams

Weight of 140 biscuits $= \frac{50}{7} \times 140$

$= 1000$ grams

$= 1$ kg

Thus (d) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Ans : [Board Term-2, 2012]

We have $6x^2 - x - 2 = 0$

$6x^2 + 3x - 4x - 2 = 0 \quad (3 \times 4 = 2 \times 6)$

$3x(2x+1) - 2(2x+1) = 0$

$(2x+1)(3x-2) = 0$

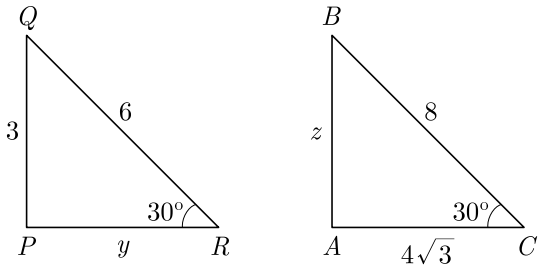
$3x - 2 = 0$ or $2x + 1 = 0$

$x = \frac{2}{3}$ or $x = -\frac{1}{2}$

Hence roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

d107

22. In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y + z$.



Ans : [Board Term-1 2010]

In the given figure $\Delta ABC \sim \Delta PQR$,

f112

Thus $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$

$\frac{z}{3} = \frac{8}{6}$ and $\frac{8}{6} = \frac{4\sqrt{3}}{y}$

$z = \frac{8 \times 3}{6}$

and $y = \frac{4\sqrt{3} \times 6}{8}$

$z = 4$ and $y = 3\sqrt{3}$

Thus $y + z = 3\sqrt{3} + 4$

23. Prove that $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = 1$.

Ans : [Board 2020 Delhi Basic]

h280

$$\text{LHS} = \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\text{cosec}^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1 = \text{RHS}$$

or

Prove that $\frac{\text{cosec}\,\theta}{\text{cosec}\,\theta - 1} + \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta + 1} = 2\sec^2\theta$.

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta - 1} + \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta + 1}$$

$$= \text{cosec}\,\theta\left[\frac{1}{\text{cosec}\,\theta - 1} + \frac{1}{\text{cosec}\,\theta + 1}\right]$$

$$= \text{cosec}\,\theta\left[\frac{\text{cosec}\,\theta + 1 + \text{cosec}\,\theta - 1}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}\right]$$

$$= \text{cosec}\,\theta\left(\frac{2\,\text{cosec}\,\theta}{\text{cosec}^2\theta - 1}\right)$$

h283

$$= \frac{2\,\text{cosec}^2\theta}{\text{cosec}^2\theta - 1} = \frac{2\,\text{cosec}^2\theta}{\cot^2\theta}$$

$$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2}{\cos^2\theta}$$

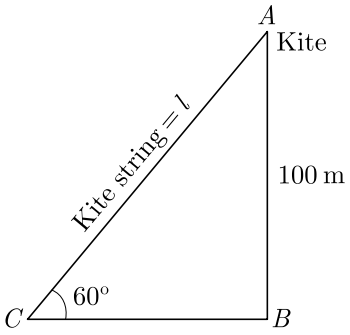
$$= 2\sec^2\theta = \text{RHS}$$ Hence Proved

24. Find the length of kite string flying at 100 m above the ground with the elevation of 60°.

Ans : [Board Term-2, 2012]

Let the length of kite string $AC = l$. As per given in question we have drawn figure below.

i114



Here $\angle ACB = 60°$, height of kite $AB = 100$ m.

From ΔABC, we have

$\sin 60° = \frac{AB}{BC}$

$\frac{\sqrt{3}}{2} = \frac{100}{l}$

$l = \frac{2 \times 100}{\sqrt{3}} = \frac{200}{\sqrt{3}}$ m

$= \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$ m

Hence length the kite string is $\frac{200\sqrt{3}}{3}$

25. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Ans : [Board 2020 OD Basic]

Total number of possible outcomes,

$n(S) = 10 + 25 = 35$

o176

Total number of prizes,

$n(E) = 10$

Probability of getting a prize,

$P(E) = \frac{n(E)}{n(S)} = \frac{10}{35} = \frac{2}{7}$

26. A bag contains cards with numbers written on it from 1–80. A card is pulled out at random. Find the probability that the card shows a perfect square.

Ans : [Board Term-2 2016]

We have $S = \{1,\ 2.....80\}$
Number of possible outcomes,

$$n(S) = 80$$

Favourable outcome are $\{1, 4, 9, 16, 25, 36, 49, 64\}$
Number of favourable outcomes,

$$n(E) = 8$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{80} = \frac{1}{10}$$

o181

27. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Ans : [Board Term-1 2011, Set-40]

We have $f(x) = 3x^2 - 8x + 2k + 1$
Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha + \beta = -\left(-\frac{8}{3}\right)$

b127

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $$\alpha = \frac{1}{3}$$

Product of zeroes, $\alpha \times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7 \times \frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3} - 1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

or

Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Ans : [Board Term-2 2015]

We have $f(x) = 2x^2 - 3x + 1$
If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

b128

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$p(x) = x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$
$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$
$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$
$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$
$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

28. Solve for x : $\sqrt{3}\,x^2 - 2\sqrt{2}\,x - 2\sqrt{3} = 0$

Ans : [Board Term-2, OD 2015, Foreign 2014]

We have
$$\sqrt{3}\,x^2 - 2\sqrt{2}\,x - 2\sqrt{3} = 0$$
$$\sqrt{3}\,x^2 - [3\sqrt{2} - \sqrt{2}]x - 2\sqrt{3} = 0$$
$$\sqrt{3}\,x^2 - 3\sqrt{2}\,x + \sqrt{2}\,x - 2\sqrt{3} = 0$$
$$\sqrt{3}\,x^2 - \sqrt{3}\sqrt{3}\sqrt{2}\,x + \sqrt{2}\,x - \sqrt{2}\sqrt{2}\sqrt{3} = 0$$
$$\sqrt{3}\,x(x - \sqrt{3}\sqrt{2}) + \sqrt{2}\,(x - \sqrt{2}\sqrt{3}) = 0$$
$$\sqrt{3}\,x[x - \sqrt{6}] + \sqrt{2}[x - \sqrt{6}] = 0$$
$$(x - \sqrt{6})(\sqrt{3}\,x + \sqrt{2}) = 0$$

d136

Thus $x = \sqrt{6} = -\sqrt{\frac{2}{3}}$

or

Solve for x : $x^2 + 5x - (a^2 + a - 6) = 0$

Ans : [Board Term-2 Foreign Set I 2015]

We have $x^3 + 5x - (a^2 + a - 6) = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

d138

Thus $$x = \frac{-5 \pm \sqrt{25 + 4(a^2 + a - 6)}}{2}$$

$$= \frac{-5 \pm \sqrt{25 + 4a^2 + 4a - 24}}{2}$$

$$= \frac{-5 \pm \sqrt{4a^2 + 4a + 1}}{2}$$

$$= \frac{-5 \pm (2a + 1)}{2}$$

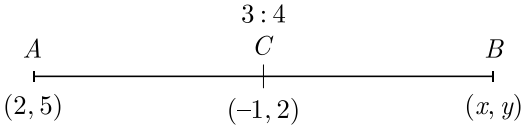
$$= \frac{2a - 4}{2}, \frac{-2a - 6}{2}$$

Thus $x = a - 2,\ x = -(a + 3)$

29. If the point $C(-1, 2)$ divides internally the line segment joining the points $A(2, 5)$ and $B(x, y)$ in the ratio 3:4, find the value of $x^2 + y^2$.

Ans : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



We have $\frac{AC}{BC} = \frac{3}{4}$

g144

Applying section formula for x co-ordinate,

$$-1 = \frac{3x + 4(2)}{3 + 4}$$

$$-7 = 3x + 8 \Rightarrow x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y + 4(5)}{3 + 4}$$

$$14 = 3y + 20 \Rightarrow y = -2$$

Thus (x, y) is $(-5, -2)$.

Now $x^2 + y^2 = (-5)^2 + (-2)^2$

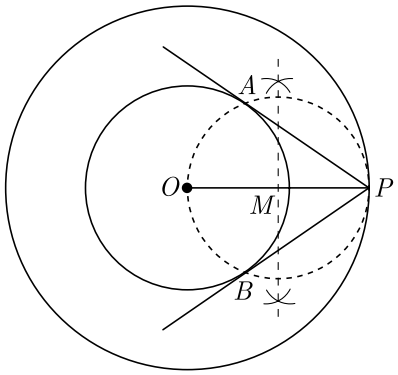
$= 25 + 4 = 29$

30. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

Ans : [Board Term-2, 2013]

Steps of Construction :

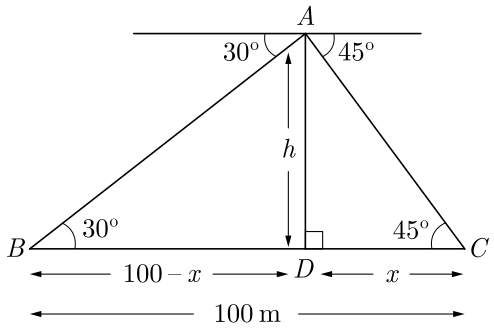
1. Draw a circle with centre O and radius 4 cm.
2. Draw another circle with centre O and radius 6 cm.
3. Take a point P on outer circle and join OP.
4. Draw perpendicular bisector of OP which intersect OP at M.
5. Draw a circle with centre M which intersects inner circle at points A and B.
6. Join AP and BP. Thus AP and BP are required tangents.



31. Two ships are approaching a light house from opposite directions. The angle of depression of two ships from top of the light house are 30° and 45°. If the distance between two ships is 100 m, Find the height of light-house.

Ans : [Board Term-2 Foreign 2014]

As per given in question we have drawn figure below. Here AD is light house of height h and BC is the distance between two ships.



We have $BC = 100$ m

In ΔADC, $\tan 45^\circ = \frac{h}{x} \Rightarrow h = x$

In ΔABD, $\tan 30^\circ = \frac{h}{100 - x}$

i134

$$\frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$$100 - x = h\sqrt{3}$$

$$100 - h = h\sqrt{3} \qquad h = x$$

$$100 = h + h\sqrt{3}$$

$$= h(1 + \sqrt{3})$$

$$h = \frac{100}{1 + \sqrt{3}}$$

$$= \frac{100}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$

$$= \frac{100(\sqrt{3} - 1)}{3 - 1}$$

$$= 50(\sqrt{3} - 1)$$

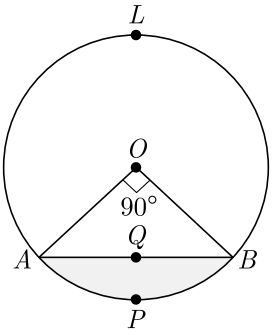
$$= 50(1.732 - 1)$$

$$= 50 \times 0.732$$

Thus height of light house is 36.60 m.

32. In the given figure, a chord AB of the circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $ALBQA$. (Use $\pi = 3.14$)

I133



Ans : [Board Term-2 Foreign 2016]

Area of sector $OAPB$,

$$= \frac{90}{360}\pi(10)^2 = 25\pi \text{ cm}^2$$

Area of ΔAOB,

$$= \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

Area of minor segment $AQBP$,

$$= (25\pi - 50) \text{ cm}^2$$

$$= 25 \times 3.14 - 50$$

$$= 78.5 - 50$$

$$= 28.5 \text{ cm}^2$$

Also area of circle $= \pi(10)^2$

$$= 3.14 \times 100 = 314 \text{ cm}^2$$

Area of major segment $ALBQA = 314 - 28.5$

$$= 285.5 \text{ cm}^2$$

33. Find the mean and median for the following data :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	16	36	34	6

Ans : [Board Term-1 2011]

We prepare following cumulative frequency table to find median class.

Class	x_i **(class marks)**	f_i	$f_i x_i$	*c.f.*
0-10	5	8	40	8
10-20	15	16	240	24
20-30	25	36	900	60
30-40	35	34	1190	94
40-50	45	6	270	100
		$\sum f_i = 100$	$\sum f_i x_i = 2640$	

Mean $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{2640}{100} = 26.4$

We have $N = 100 \; ; \; \dfrac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 20-30. Thus median class is 20-30.

Median, $M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right)h$

$= 20 + \dfrac{50 - 24}{36} \times 10$

$= 20 + 7.22 = 27.22$

34. Show that there is no positive integer n, for which $\sqrt{n-1} + \sqrt{n-1}$ is rational.

Ans : [Board Term-1 2012]

Let us assume that there is a positive integer n for which $\sqrt{n-1} + \sqrt{n-1}$ is rational and equal to $\frac{p}{q}$, where p and q are positive integers and $(q \neq 0)$.

$$\sqrt{n-1} + \sqrt{n-1} = \frac{p}{q} \quad ...(1)$$

or, $$\frac{q}{p} = \frac{1}{\sqrt{n-1} + \sqrt{n+1}}$$

$$= \frac{\sqrt{n-1} - \sqrt{n+1}}{(\sqrt{n-1} + \sqrt{n+1})(\sqrt{n-1} - \sqrt{n+1})}$$

$$= \frac{\sqrt{n-1} - \sqrt{n+1}}{(n-1) - (n+1)}$$

or $$\frac{q}{p} = \frac{\sqrt{n-1} - \sqrt{n+1}}{-2}$$

$$\sqrt{n+1} - \sqrt{n-1} = \frac{2q}{p} \quad ...(2)$$

Adding (1) and (2), we get

$$2\sqrt{n+1} = \frac{p}{q} + \frac{2q}{p} = \frac{p^2 + 2q^2}{pq} \quad ...(3)$$

Subtracting (2) from (1) we have

$$2\sqrt{n-1} = \frac{p^2 - 2q^2}{pq} \quad ...(4)$$

From (3) and (4), we observe that $\sqrt{n+1}$ and $\sqrt{n-1}$ both are rational because p and q both are rational. But it possible only when $(n+1)$ and $(n-1)$ both are perfect squares. But they differ by 2 and two perfect squares never differ by 2. So both $(n+1)$ and $(n-1)$ cannot be perfect squares, hence there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

35. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Ans : [Board Term-1 2012, NCERT]

Let the actual speed of the train be s and actual time taken t.

Distance = Speed × Time

$= st$ km

According to the given condition, we have

$$st = (s+10)(t-2)$$

$$st = st - 2s + 10t - 20$$

$$2s - 10t + 20 = 0$$

$$s - 5t = -10 \quad (1)$$

and $$st = (s-10)(t+3)$$

$$st = st + 3s - 10t - 30$$

$$3s - 10t = 30 \quad ...(2)$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$3 \times (s - 5t) - (3s - 10t) = -3 \times 10 - 30$$

$$-5t = -60 \Rightarrow t = 12$$

Substituting value of t equation (1),

$$s - 5 \times 12 = -10$$

$$s = -10 + 60 = 50$$

Hence, the distance covered by the train

$= 50 \times 12 = 600$ km.

or

If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Ans : [Board Term-1 2012]

Let the number of red balls be x and white balls be y. According to the question,

$$\frac{y}{2} = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad ...(1)$$

and $$3(x+y) - 7y = 6$$

or $$3x - 4y = 6 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by we have

$$6x - 9y = 0 \quad ...(3)$$

$$6x - 8y = 12 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

Substituting $y = 12$ in equation (1),

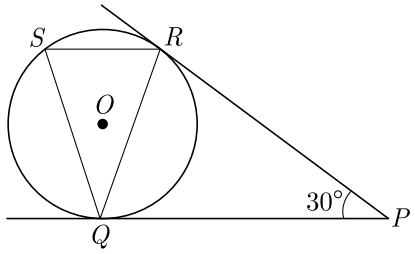
$$2x - 36 = 0$$

$$x = 18$$

Hence, number of red balls = 18

and number of white balls = 12

36. In the figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



Ans : [Board Term-2 Delhi 2015]

Since length of tangents from an external point to a circle are equal,

$$PR = PQ$$

Now $$\angle PRQ = \angle PQR = \frac{180^\circ - 30^\circ}{2}$$

$$= \frac{150^\circ}{2} = 75^\circ$$

Since $SR \parallel QP$, $\angle SRQ$ and $\angle RQP$ are alternate angle,

$$\angle SRQ = \angle RQP = 75^\circ$$

j223

Thus $$SQ = RQ$$

and $$\angle RSQ = \angle SRQ = 75^\circ$$

In triangle ΔAQR,

$$\angle SQR + \angle QSR + \angle QRS = 180^\circ$$

$$\angle SQR + 75^\circ + 75^\circ = 180^\circ$$

$$\angle SQR = 180^\circ - 150^\circ = 30^\circ$$

Thus $\angle SQR = 30^\circ$.