CLASS X (2020-21)
MATHEMATICS STANDARD (041)
SAMPLE PAPER-10

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

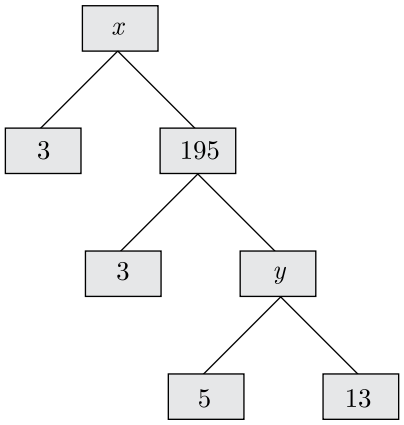
Part–B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

Section - I

1. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1 2015]

We have $y = 5\times13 = 65$

and $x = 3\times195 = 585$

a110

or

Write the rational number $\frac{7}{75}$ will have a terminating decimal expansion. or a non-terminating repeating decimal.

Ans : [Board 2018 SQP]

We have $\frac{7}{75} = \frac{7}{3\times5^2}$

a157

The denominator of rational number $\frac{7}{75}$ can not be written in form 2^m5^n So it is non-terminating repeating decimal expansion.

2. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then what is the value of $x+y$?

Ans :

The numbers that can be formed are xy and yx. Hence, $(10x+y)+(10y+x) = 11(x+y)$. If this is a perfect square than $x+y = 11$.

c242

3. Two triangles are similar if their corresponding sides are ..... .

Ans : [Board 2020 OD Standard]

in the same ratio.

f225

4. What is the distance of the point $(-12,\ 5)$ from the origin ?

Ans :

The distance between the origin and the point (x, y) is $\sqrt{x^2+y^2}$.

Therefore, the distance between the origin and point $(-12, 5)$

$$d = \sqrt{(-12-0)^2+(5-0)^2}$$
$$= \sqrt{144+25} = \sqrt{169}$$
$$= 13 \text{ units}$$

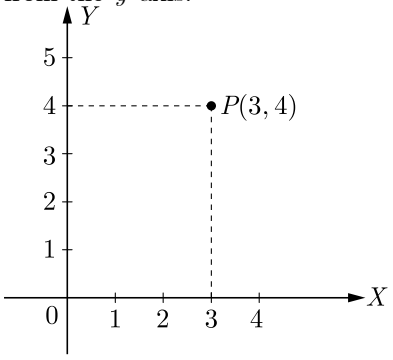
g295

or

What is the distance of point $P(3, 4)$ from x-axis?

Ans : [Board 2020 Delhi Basic]

Point $P(3, 4)$ is 4 units from the x-axis and 3 units from the y-axis.



g296

5. If $\cos A = \frac{4}{5}$, then find the value of $\tan A$.

Ans :

We have $\cos A = \frac{4}{5}$

h308

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

$\text{Perpendicular} = \sqrt{5^2 - 4^2} = \sqrt{25-16} = 3$

Now, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

6. If $\sin\theta - \cos\theta = 0$, then find the value of $(\sin^4\theta + \cos^4\theta)$

Ans :

We have, $\sin\theta - \cos\theta = 0$

h316

$\sin\theta = \cos\theta$

$\sin\theta = \sin(90^\circ - \theta)$

$\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

7. The ..... of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

Ans :

i198

angle of elevation

or

The ..... of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

Ans :

i199

angle of depression

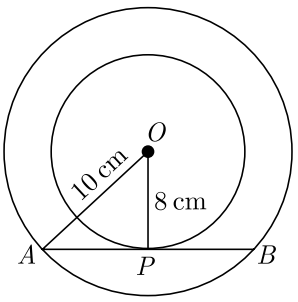
8. Two concentric circles are of radii 10 cm and 8 cm, then find the length of the chord of the larger circle which touches the smaller circle.

Ans :

Let O be the centre of the concentric circles of radii 10 cm and 8 cm, respectively. Let AB be a chord of the larger circle touching the smaller circles at P.

Then, $AP = PB$ and $OP \perp AB$

j242



Applying Pythagoras theorem in ΔOPA, we have

$OA^2 = OP^2 + AP^2$

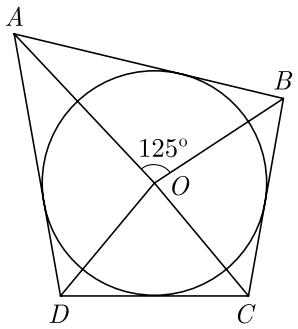
$100 = 64 + AP^2$

$AP^2 = 100 - 64 = 36 \Rightarrow AP = 6$ cm

$AB = 2AP = 2 \times 6 = 12$ cm

or

In given figure, if $\angle AOB = 125^\circ$, then find the angle $\angle COD$?



j243

Ans :

We know that, a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

i.e. $\angle AOB + \angle COD = 180^\circ$

$125^\circ + \angle COD = 180^\circ$

$\angle COD = 180^\circ - 125^\circ = 55^\circ$

9. To find a point P on the line segment $AB = 6$ cm, such that $\frac{AP}{AB} = \frac{2}{5}$, in which ratio the line segment AB is divided.

Ans : [Board Term-2 2012]

The line segment AB is divided in the ratio $AP : PB = 2 : (5-2) = 2 : 3$

k180

or

A line Segment AB is divided at point P such that $\frac{PB}{AB} = \frac{3}{7}$, then find the ratio $AP : PB$.

Ans : [Board Term-2, 2012]

Here, $AB = 7, PB = 3$

k181

Thus $AP = AB - PB = 7 - 3 = 4$

$AP : PB = 4 : 3$

10. Find the area of circle that can be inscribed in a square of side 10 cm.

Ans : [Board Term-2 2012]

Radius of the circle $= \frac{10}{2} = 5$ cm

l110

Area of the circle,

$\pi r^2 = \pi \times (5)^2 = 25\pi\ cm^2$

11. If the radius of a circle is doubled, what about its area?

Ans : [Board Term-2 2012]

Let the radius of the circle be r, then area will be πr^2

l114

Now if radius is doubled,

Area $= \pi(2r)^2 = 4\pi r^2 = 4 \times \pi r^2$

The area will be 4 times the area of the first circle.

12. For finding the popular size of readymade garments, which central tendency is used?

Ans :

For finding the popular size of ready made garments, mode is the best measure of central tendency.

n104

13. For the following distribution find the sum of lower limits of the median class and modal class.

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

Ans :

n120

Here,

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Now, $\frac{N}{2} = \frac{33}{2} = 33$, which lies in the interval 10-15. Therefore, lower limit of the median class is 10.
The highest frequency is 20, which lies in the interval 15-20. Therefore, lower limit of modal class is 15.
Hence, required sum is $10 + 15 = 25$.
Thus (b) is correct option.

14. A bag contains 3 red and 2 blue marbles. If a marble is drawn at random, then find the probability of drawing a blue marble.

Ans :

There are 5 marbles in the bag. Out of these 5 marbles one can be choose in 5 ways. Since, the bag contains 2 blue marbles. Therefore, one blue marble can be drawn in 2 ways.

$$n(S) = 5$$
$$n(E) = 2$$

o277

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}$$

15. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting a red king.

Ans : [Board 2020 OD Basic]

Total no. of cards, $n(S) = 52$
Number of red kings, $n(E) = 2$

o138

P(a red king), $P(E) = \frac{n(E)}{n(S)} = \frac{2}{52}$
$= \frac{1}{26}$

16. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from box, find the probability that it bears a prime number less than 23.

Ans : [Board Term-2, 2012]

Number of possible outcomes,

$$n(S) = 90$$

Prime numbers less than 23 are 2, 3, 5, 7, 11, 13, 17, 19.
Number of favourable outcomes

$$n(E) = 8$$

P(prime no. less than 23)

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{90} = \frac{4}{45}$$

o158

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. Pyramid, in architecture, a monumental structure constructed of or faced with stone or brick and having a rectangular base and four sloping triangular sides meeting at an apex. Pyramids have been built at various times in Egypt, Sudan, Ethiopia, western Asia, Greece, Cyprus, Italy, India, Thailand, Mexico, South America, and on some islands of the Pacific Ocean. Those of Egypt and of Central and South America are the best known.

The volume and surface area of a pyramid with a square base of area a^2 and height h is given by

$$V = \frac{ha^2}{3} \text{ and } S = a^2 + 2a\sqrt{\left(\frac{a}{2}\right)^2 + h^2}$$

A pyramid has a square base and a volume of $3y^3 + 18y^2 + 27y$ cubic units.

(i) If its height is y, then what polynomial represents the length of a side of the square base ?
(a) $9(y+3)$
(b) $9(y+3)^2$
(c) $3(y+3)$
(d) $3(y+3)^2$

b406

(ii) If area of base is 576 metre, what is the side of base?
(a) 24 metre (b) 16 metre
(c) 13 metre (d) 12 metre

(iii) What is the height of pyramid at above area of base ?
(a) 4 metre (b) 6 metre
(c) 5 metre (d) 12 metre

(iv) What is the ratio of length of side to the height ?
(a) $\frac{1}{5}$ (b) $\frac{2}{5}$
(c) $\frac{5}{24}$ (d) $\frac{24}{5}$

(v) What is surface area of pyramid ?
(a) 800 m^2 (b) 2400 m^2
(c) 1200 m^2 (d) 1600 m^2

Ans :

(i) $V(y) = 3y^3 + 18y^2 + 27y$
$= 3y(y^2 + 6y + 9)$
$= 3y(y+3)^2$

If y represent height, then comparing its volume with standard volume, we have

$$h\frac{a^2}{3} = 3y(y+3)^2$$
$$y\frac{a^2}{3} = 3y(y+3)^2$$
$$a^2 = 9(y+3)^2$$
$$a = 3(y+3)$$

Thus (c) is correct option.

(ii) $a^2 = 576 \Rightarrow a = 24$ unit

Thus (a) is correct option.

a. (iii) At $a = 24$ meter,

$$24 = 3(y+3)$$
$$8 = y+3$$
$$y = 5 \text{ metre}$$

Thus (c) is correct option.

b. (iv) We have $a = 24$ and $y = 5$.

$$\frac{a}{y} = \frac{24}{5}$$

Thus (c) is correct option.

(v) We have $S = a^2 + 2a\sqrt{(\frac{a}{2})^2 + h^2}$

c. We have $a = 24$ and $y = 5$.

Thus $S = 24^2 + 2 \times 24\sqrt{(\frac{24}{2})^2 + 5^2}$
$= 2 \times 24(12 + \sqrt{12^2 + 5^2})$
$= 48(12 + 13)$
$= 1200 \text{ m}^2$

Thus (c) is correct option.

18. Optimal pricing strategy : The director of the Blue Rose Theatre must decide what to charge for a ticket to the comedy drama. If the price is set too low, the theatre will lose money; and if the price is too high, people won't come. From past experience she estimates that the profit P from sales (in hundreds) can be approximated by $P(x) = -x^2 + 22x - 40$ where x is the cost of a ticket and $0 \le x \le 25$ hundred rupees.

(i) What is the lowest cost of a ticket that would allow the theatre to break even?
(a) Rs 3 hundred (b) Rs 4 hundred
(c) Rs 2 hundred (d) Rs 1 hundred

(ii) What is the highest cost that the theatre can charge to break even?
(a) Rs 16 hundred (b) Rs 14 hundred
(c) Rs 4 hundred (d) Rs 20 hundred

(iii) If theatre charge Rs 4 hundred for each ticket, what is the profit/loss ?
(a) Loss Rs 1600 (b) Profit Rs 1600
(c) Loss Rs 3200 (d) Profit Rs 3200

(iv) If theatre charge Rs 25 hundred for each ticket, what is the profit/loss ?
(a) Loss Rs 11500 (b) Profit Rs 8500
(c) Loss Rs 8500 (d) Profit Rs 11500

(v) What is the maximum profit which can be earned by theatre ?
(a) Rs 4000 (b) Rs 8100
(c) Rs 6100 (d) Rs 4200

Ans :

(i) At break even $P(x) = 0$, thus

d406

$$-x^2 + 22x - 40 = 0$$
$$x^2 - 22x + 40 = 0$$
$$(x-2)(x-20) = 0 \Rightarrow x = 2, 20$$

Thus (c) is correct option.

(ii) Theatre can charge Rs 20 hundred also. This is also break even point.

Thus (d) is correct option.

(iii) At, $x = 4$, we have

$$P(2) = -(4)^2 + 22 \times 4 - 40 = 32$$

Thus (d) is correct option.

(iv) At, $x = 25$, we have

$$P(5) = -(25)^2 + 22 \times 25 - 40 = -115$$

Thus (a) is correct option.

(v) We have $P(x) = -x^2 + 22x - 40$

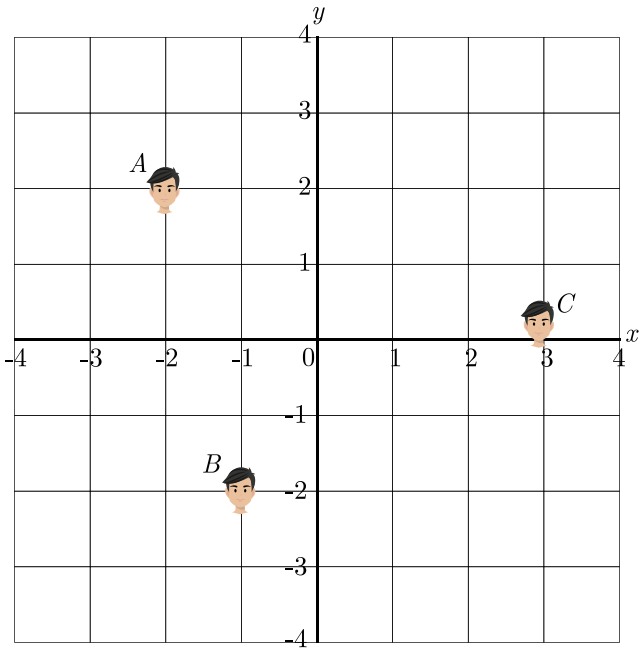
Rearranging the profit equation we have

$$P(x) = -(x^2 - 22x + 121 - 81)$$
$$= -(x-11)^2 + 81$$

From above equation it is clear that maximum value of above equation is 81.

Thus (b) is correct option.

19. Ajay, Bhigu and Colin are fast friend since childhood. They always want to sit in a row in the classroom . But teacher doesn't allow them and rotate the seats row-wise everyday. Bhigu is very good in maths and he does distance calculation everyday. He consider the centre of class as origin and marks their position on a paper in a co-ordinate system. One day Bhigu make the following diagram of their seating position.



(i) What are the coordinates of point A?
(a) $(2,2)$ (b) $(2,-2)$
(c) $(-2,2)$ (d) $(-2,-2)$

g402

(ii) What is the distance of point A from origin ?
(a) 8 (b) $2\sqrt{2}$
(c) 4 (d) $4\sqrt{2}$

(iii) What is the distance between A and B ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $\sqrt{17}$ (d) $2\sqrt{5}$

(iv) What is the distance between B and C ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $2\sqrt{17}$ (d) $2\sqrt{5}$

(v) A point D lies on the line segment between points A and B such that $AD:DB=4:3$. What are the the coordinates of point D ?
(a) $\left(\frac{10}{7},\frac{2}{7}\right)$ (b) $\left(\frac{2}{7},\frac{7}{7}\right)$
(c) $\left(-\frac{10}{7},-\frac{2}{7}\right)$ (d) $\left(-\frac{2}{7},-\frac{7}{7}\right)$

Ans :

(i) It may be seen easily from figure that coordinates of point A are $(-2,2)$.
Thus (c) is correct option.

(ii) $OA = \sqrt{(0+2)^2+(0-2)^2} = 2\sqrt{2}$
Thus (b) is correct option.

(iii) It may be seen easily from figure that coordinates of point A are $(-1,-2)$.

$$AB = \sqrt{(-2+1)^2+(2+2)^2}$$
$$= \sqrt{1+4^2} = \sqrt{17}$$

Thus (c) is correct option.

(iv) It may be seen easily from figure that coordinates of point A are $(0,\ 3)$.

$$BC = \sqrt{(-1-3)^2+(-2-0)^2}$$
$$= \sqrt{4^2+4} = 2\sqrt{5}$$

Thus (d) is correct option.

(v) We have $A(-2,2)$ and $B(-1,-2)$

$$\frac{m}{n} = \frac{4}{3}$$

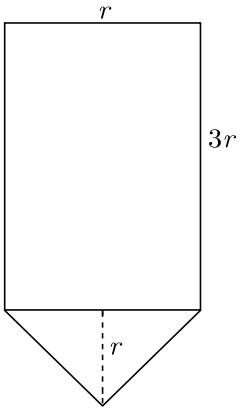
$$x = \frac{mx_2+nx_1}{m+n} = \frac{-1(4)+3(-2)}{4+3} = \frac{-10}{7}$$

$$y = \frac{my_2+hy_1}{m+n} = \frac{-2(4)+3(2)}{4+3} = \frac{-2}{7}$$

Thus (c) is correct option.

20. The advantages of cone bottom tanks are found in nearly every industry, especially where getting every last drop from the tank is important. This type of tank has excellent geometry for draining, especially with high solids content slurries as these cone tanks provide a better full-drain solution. The conical tank eliminates many of the problems that flat base tanks have as the base of the tank is sloped towards the centre giving the greatest possible full-drain system in vertical tank design.

Rajesh has been given the task of designing a conical bottom tank for his client. Height of conical part is equal to its radius. Length of cylindrical part is the 3 times of its radius. Tank is closed from top. The cross section of conical tank is given below.



m406

(i) If radius of cylindrical part is taken as 3 meter, what is the volume of above conical tank ?
(a) 120π m^3 (b) 90π m^3
(c) 60π m^3 (d) 30π m^3

(ii) What is the area of metal sheet used to make this conical tank ? Assume that tank is covered from top.
(a) $27(7+\sqrt{2})\pi$ m^2 (b) $9(7+\sqrt{2})\pi$ m^2
(c) $27(5+\sqrt{2})\pi$ m^2 (d) $9(5+\sqrt{2})\pi$ m^2

(iii) What is the ratio of volume of cylindrical part to the volume of conical part?
(a) 6 (b) 9
(c) $\frac{1}{6}$ (d) $\frac{1}{9}$

(iv) The cost of metal sheet is Rs 2000 per square meter and fabrication cost is 1000 per square meter. What is the total cost of tank ?
(a) Rs $27000(7+\sqrt{2})\pi$ (b) Rs $54000(7+\sqrt{2})\pi$
(c) Rs $27000(5+\sqrt{2})\pi$ (d) Rs $54000(5+\sqrt{2})\pi$

(v) A oil is to be filled in the tank. The density of oil is 1050 kg per cubic meter. What is the weight of oil filled in tank ?
(a) 195 Tonne (b) 200 Tonne
(c) 297 Tonne (d) 174 Tonne

Ans :

(i) Length of cylindrical part is three times of radius of conical part and height of conical part is equal to its radius.
If we assume r be the common radius of cylindrical part and conical part, height of conical part will be r and length of cylindrical part will be $3r$.

Volume of conical tank = Volume of cylindrical part + Volume of conical part

$$= \pi r^2 l + \frac{1}{3}\pi r^2 h$$
$$= \pi r^2 \cdot 3r + \frac{1}{3}\pi r^2 \cdot r$$
$$= 3\pi r^3 + \frac{1}{3}\pi r^3 = \frac{10}{3}\pi r^3$$
$$= \frac{10}{3}\pi(3)^3 = 90\,\pi\ \text{m}^3$$

Thus (b) is correct option.
(ii) Surface area of tank,
= SA of top + CSA of cylinder + CSA of cone

$$= \pi r^2 + 2\pi rl + \pi r\sqrt{h^2+r^2}$$
$$= \pi r^2 + 2\pi r \cdot 3r + \pi r\sqrt{r^2+r^2}$$
$$= \pi r^2 + 6\pi r^2 + \sqrt{2}\,\pi r^2$$
$$= (1+6+\sqrt{2})\pi r^2$$
$$= (7+\sqrt{2})\pi(3)^2$$
$$= 9(7+\sqrt{2})\pi\ \text{m}^2$$

Thus (b) is correct option.

(iii)Volume of cylindrical part

$$= \pi r^2 l$$
$$= \pi r^2 \cdot 3r = 3\pi r^3$$

Volume of conical part $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi r^2 r = \frac{1}{3}\pi r^3$$

Ratio of volume of cylindrical part to conical part

$$= \frac{3\pi r^3}{\frac{1}{3}\pi r^3} = 9$$

Thus (b) is correct option.

(iv)Surface area of sheet used $= 9(7+\sqrt{2})\pi\ \text{m}^2$

Total cost = Cost of sheet + Fabrication cost

$$= 2000 + 1000$$
$$= 3000 \text{ per sq. meter}$$

Total cost of tank $= 9(7+\sqrt{2})\pi \times 3000$

$$= 27000(7+\sqrt{2})\pi$$

Thus (c) is correct option.
(v) Volume of tank i.e. volume of oil $= 90\,\pi\ \text{m}^3$

Density of oil = 1050 kg per cubic meter

Weight of oil $= 90\pi \times 1050$

$$= 90 \times \frac{22}{7} \times 1050$$
$$= 90 \times 22 \times 150$$
$$= 297000\ \text{kg} = 297 \text{ Tonne}$$

Thus (c) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$, where a and b are prime numbers than verify $\text{LCM}(p,q) \times \text{HCF}(q,q) = pq$

Ans : [Sample Paper 2017]

We have $p = a^2b^3$
$= a \times a \times b \times b \times b$
and $q = a^3b = a \times a \times a \times b$
Now LCM $(p,q) = a \times a \times a \times b \times b \times b$
$= a^3b^3$
and HCF $(p,q) = a \times a \times b$
$= a^2b$

$$\text{LCM}(p,q) \times \text{HCF}(p,q) = a^3b^3 \times a^2b$$
$$= a^5b^4$$
$$= a^2b^3 \times a^3b$$
$$= pq$$

22. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

Ans : [Board Term-1 2015]

Let age of father and son be x and y respectively.

$$x + y = 40 \quad \ldots(1)$$
$$x = 3y \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$x = 30 \text{ and } y = 10$$

Ages are 30 years and 10 years.

23. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

Ans : [Board Term-2 Delhi 2016]

We have $ax^2 + 7x + b = 0 \quad (1)$
Substituting $x = \frac{2}{3}$ in above equation we obtain

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$
$$4a + 42 + 9b = 0$$
$$4a + 9b = -42 \quad (2)$$

and substituting $x = -3$ in (1) we obtain

$$9a - 21 + b = 0$$
$$9a + b = 21 \quad (3)$$

Solving (2) and (3), we get $a = 3$ and $b = -6$

or

Find the roots of $x^2 - 4x - 8 = 0$ by the method of completing square.

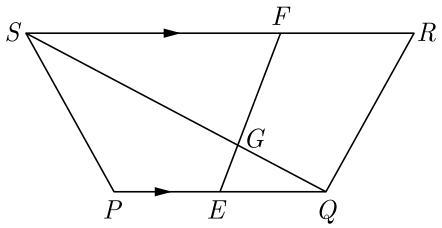
Ans : [Board Term-2, 2015]

We have $x^2 - 4x - 8 = 0$

$$x^2 - 4x + 4 - 4 - 8 = 0$$
$$(x-2)^2 - 12 = 0$$
$$(x-2)^2 = 12$$
$$(x-2)^2 = (2\sqrt{3})^2$$
$$x - 2 = \pm 2\sqrt{3}$$
$$x = 2 \pm 2\sqrt{3}$$

Thus $x = 2 + 2\sqrt{3},\ 2 - 2\sqrt{3}$

24. In the figure, $PQRS$ is a trapezium in which $PQ \,||\, RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.



Ans : [Board Term-1 2016]

In ΔGEQ and ΔGFS,

Due to vertical opposite angle,

$$\angle EGQ = \angle FGS$$

Due to alternate angle,

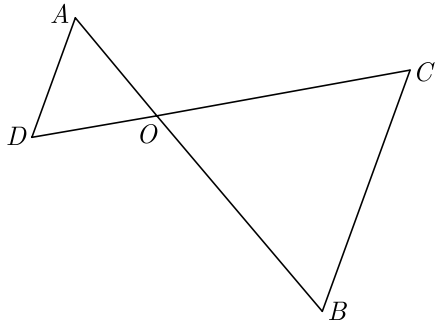
$$\angle EQG = \angle FSG$$

Thus by AA similarity we have

$$\Delta GEQ \sim GFS$$
$$\frac{EQ}{FS} = \frac{GQ}{GS}$$
$$EQ \times GS = GQ \times FS$$

or

In the given figure, $OA \times OB = OC \times OD$, show that $\angle A = \angle C$ and $\angle B = \angle D$.



Ans : [Board Term-1 2012]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OC}{OB}$$

Due to the vertically opposite angles,

$$\angle AOD = \angle COB$$

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

25. Prove that : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

Ans : [Bard Term-1 2015]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$
$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$
$$= \tan\theta$$

26. The frequency distribution of agricultural holdings in a village below :

Area of land (in hectare)	1-3	3-5	5-7	7-9	9-11	11-13
Number of families	20	45	80	55	40	12

Find the modal agricultural holding of the village.

Ans : [Board Term-1 2012]

Class 5-7 has the maximum frequency 80, therefore this is model class.

Here $l = 5, f_1 = 80, f_0 = 45,$ $h = 2,$ $f_2 = 55$

Mode, $M_o = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$

$$= 5 + \frac{80 - 45}{160 - 45 - 55} \times 2 = 5 + \frac{35 \times 2}{60}$$
$$= 6.17$$

27. Prove that $\sqrt{5}$ is an irrational number.

Ans : [Board 2020 OD Standard]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$
$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

28. A chemist has one solution which is 50 % acid and a second which is 25 % acid. How much of each should be mixed to make 10 litre of 40 % acid solution.

Ans : [Board Term-1 2015, JRTSY]

Let 50 % acids in the solution be x and 25 % of other solution be y.

Total volume in the mixture

$$x + y = 10 \quad ...(1)\ 1$$

and $$\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100} \times 10$$

$$2x + y = 16 \quad ...(2)\ 1$$

Subtracting equation (1) from (2) we have

$$x = 6$$

Substituting this value of x in equation (1) we get

c177

$$6 + y = 10$$
$$y = 4$$

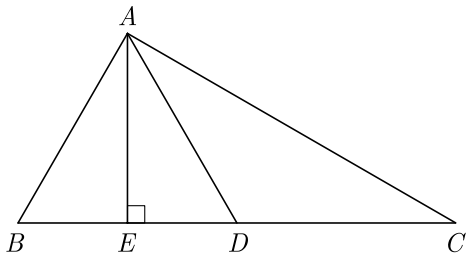
Hence, $x = 6$ and $y = 4$.

29. If in $\Delta ABC, AD$ is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.

f142



In ΔABE, using Pythagoras theorem we have

$$AB^2 = AE^2 + BE^2$$
$$= AD^2 - DE^2 + (BD - DE)^2$$
$$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$
$$= AD^2 + BD^2 - 2BD \times DE \quad ...(1)$$

In ΔAEC, we have

$$AC^2 = AE^2 + EC^2$$
$$= (AD^2 - ED^2) + (ED + DC)^2$$
$$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$$
$$= AD^2 + CD^2 + 2ED \times CD$$
$$= AD^2 + DC^2 + 2DC \times DE \quad ...(2)$$

Adding equation (1) and (2) we have

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad (BD = DC)$$
$$= 2AD^2 + 2\left(\frac{1}{2}BC\right)^2 \quad (BD = \tfrac{1}{2}BC)$$
$$= 2AD^2 + \frac{1}{2}BC^2 \quad \text{Hence Proves}$$

30. Provethat $\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A}$$
$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

h296

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \sin^2 A}}$$
$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$
$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$
$$= \frac{1}{1 - 2\cos^2 A}$$
$$= \text{RHS}$$

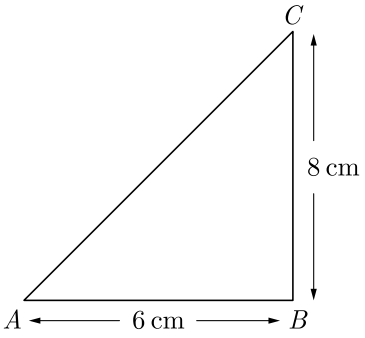
or

If in a triangle ABC right angled at B, $AB = 6$ units and $BC = 8$ units, then find the value of $\sin A \cos C + \cos A \sin C$.

Ans : [Board Term-1 2016]

As per question statement figure is shown below.

h132



We have $AC^2 = 8^2 + 6^2 = 100$

$$AC = 10 \text{ cm}$$

Now $$\sin A = \frac{BC}{AC} = \frac{8}{10};$$

$$\cos A = \frac{AB}{AC} = \frac{6}{10}$$

and $$\sin C = \frac{AB}{AC} = \frac{6}{10};$$

$$\cos C = \frac{BC}{AC} = \frac{8}{10}$$

Thus $\sin A \cos C + \cos A \sin C = \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$

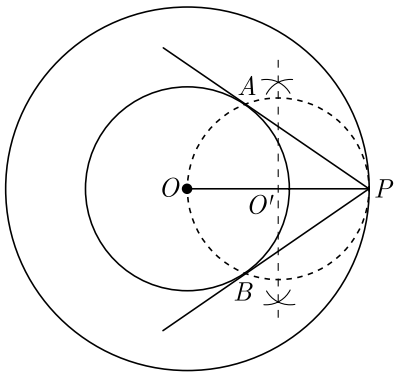
$$= \frac{64}{100} + \frac{36}{100}$$
$$= \frac{100}{100} = 1$$

31. Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

Ans : [Foreign Set I 2017]

Steps of Construction :

1. Draw a circle with radius 3 cm and centre O.
2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on the circumference of outer circle and join O to P.
4. Taking OP as diameter draw another circle which intersect the smallest circle at A and B.
5. Join A to P and B to P. AP and BP are the required tangents.



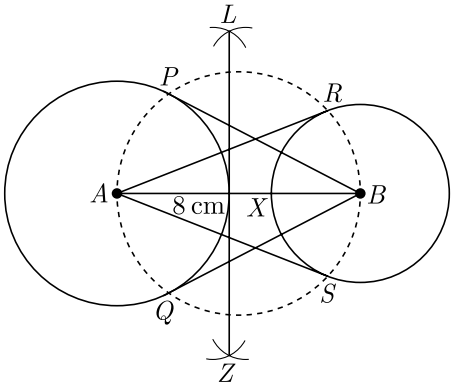
or

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

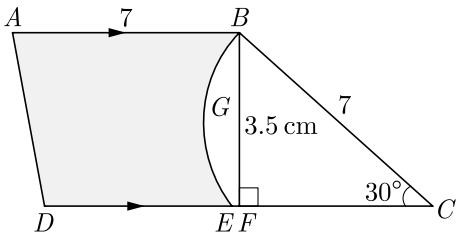
Ans : [Board Term-2 Foreign 2017, OD 2014]

Steps of Construction :

1. Draw a line segment AB of length 8 cm.
2. Draw a circle with centre A and radius 4 cm.
3. Draw another circle with centre B and radius 3 cm.
4. Taking AB as diameter draw another circle, which intersects first two circles at P and Q, and R and S.
5. Join B to P, B to Q, A to R and A to S. Thus BP, BQ, AR and AS are the required tangents.



32. Adjoining fig, $ABCD$ is a trapezium with $AB \| DC$ and $\angle BCD = 30^\circ$. Fig. $BGEC$ is a sector of a circle with centre C and $AB = BC = 7$ cm, $DE = 4$ cm and $BF = 3.5$ cm, then find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 OD Compt. 2017]

We have $AB = 7$ cm

$DE = 4$ cm, and

$BF = 3.5$ cm

Now $DC = DE + EC = 4 + 7 = 11$ cm

Area of Trapezium $ABCD$

$$\text{Area}_{\square} = \frac{1}{2}(DC + AB)(BF)$$

$$= \frac{1}{2}(11 + 7) \times 3.5 = \frac{1}{2} \times 18 \times 3.5$$

$$= 31.5 \text{ cm}^2$$

Area of circular sector,

$$\text{Area}_{\frown} = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{1}{12} \times 22 \times 7$$

$$= 12.83 \text{ cm}^2$$

Area of shaded region,

$$= \text{Area}_{\square} - \text{Area}_{\frown}$$

$$= 31.5 - 12.83 = 18.67 \text{ cm}^2$$

33. The mode of a distribution is 55 and the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class.

Ans :

Mode, $M_o = 55$

Modal class $= 45 - 60$

Frequency of the class preceding,

$f_1 = 5$

Frequency of the class succeeding the modal class,

$f_2 = 10$

Let the frequency of modal class be f.

Mode $M_o = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$

$$55 = 45 + \frac{f - 5}{2f - 5 - 10} \times 15$$

$$10 = \left(\frac{f - 5}{2f - 15}\right) \times 15$$

$$\frac{10}{15} = \frac{f - 5}{2f - 15}$$

$$\frac{2}{3} = \frac{f - 15}{2f - 15}$$

$$4f - 30 = 3f - 15$$

$$4f - 3f = -15 + 30 \Rightarrow f = 15$$

34. Find the number of terms of the AP $-12,-9,-6,.....,21$. If 1 is added to each term of this AP, then find the sum of all the terms of the AP thus obtained.

Ans : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = -12, d = -9-(-12) = 3$

$$a_n = a+(n-1)d$$
$$21 = -12+(n-1)\times 3$$
$$21+12 = (n-1)\times 3$$
$$33 = (n-1)\times 3$$
$$n-1 = 11$$
$$n = 11+1 = 12$$

e236

Now, if 1 is added to each term we have a new AP with $-12+1,-a+1,-6+1.....21+1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained AP,

$$S_{12} = \frac{12}{2}[-11+22]$$
$$= 6\times 11 = 66$$

Hence the sum of new AP is 66.

35. (i) Derive section formula.

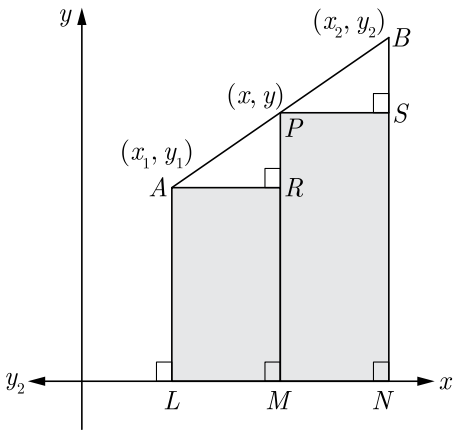
(ii) In what ratio does $(-4,6)$ divides the line segment joining the point $A(-6,4)$ and $B(3,-8)$

Ans : [Board Term-2 Delhi 2014]

(i) Section Formula : Let $A(x_1,y_1)$ and $B(x_2,y_2)$ are two points. Let $P(x,y)$ be a point on line, joining A and B, such that P divides it in the ratio $m_1:m_2$.

Now $(x,y) = \left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

g204



Proof : Let AB be a line segment joining the points. $A(x_1,y_1)$, $B(x_2,y_2)$.

Let P divides AB in the ratio $m_1:m_2$. Let P have co-ordinates (x,y).

Draw $AL, PM, PN, \perp$ to x-axis

It is clear form figure, that

$$AR = LM = OM - OL = x - x_1$$
$$PR = PM - RM = y - y_1.$$

also, $$PS = ON - OM = x_2 - x$$
$$BS = BN - SN = y_2 - y$$

Now $\Delta APR \sim \Delta PBS$ [AAA]

Thus $$\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$$

and $$\frac{AR}{PS} = \frac{AP}{PB}$$

$$\frac{x-x_1}{x_2-x} = \frac{m_1}{m_2}$$
$$m_2x - m_2x_1 = m_1x_2 - m_1x$$
$$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$$

Now $$\frac{PR}{BS} = \frac{AP}{PB}$$

$$\frac{y-y_2}{y_2-y} = \frac{m_1}{m_2}$$
$$y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

Thus co-ordinates of P are $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

(ii) Assume that $(-4,6)$ divides the line segment joining the point $A(-6,4)$ and $B(3,-8)$ in ratio $k:1$

Using section formula for x co-ordinate we have

$$-4 = \frac{k(3)-6}{k+1}$$
$$-4k-4 = 3k-6 \Rightarrow k = \tfrac{2}{7}$$

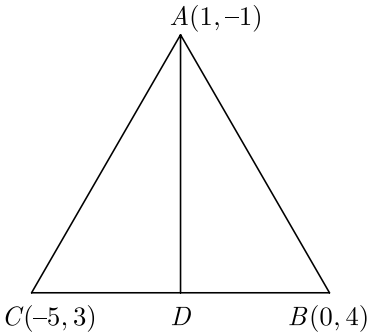
or

$(1,-1),(0,4)$ and $(-5,3)$ are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1,-1)$ the mid-point of the opposite side.

Ans : [Board Term-2, 2015]

Let the vertices of ΔABC be $A(1,-1)$, $B(0,4)$ and $C(-5,3)$. Let $D(x,y)$ be mid point of BC. Now the triangle is shown below.

g210



Using distance formula, we get

$$AB = \sqrt{(1-0)^2+(-1-4)^2} = \sqrt{1+5^2} = \sqrt{26}$$
$$BC = \sqrt{(-5-0)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$$
$$AC = \sqrt{(-5-1)^2+(3+1)^2} = \sqrt{36+16} = 2\sqrt{13}$$

Since $AB = BC \neq AC$, triangle ΔABC is isosceles.

Now, using mid-section formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$
$$y = \frac{3+4}{2} = \frac{7}{2}$$
$$D(x,y) = \left(-\frac{5}{2}, \frac{7}{2}\right)$$

Length of median AD,

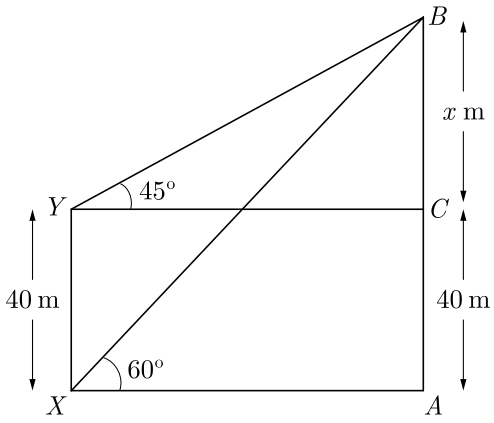
$$AD = \sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}+1\right)^2}$$
$$= \sqrt{\left(\frac{-7}{2}\right)^2+\left(\frac{9}{2}\right)^2}$$
$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2} \text{ square unit}$$

Thus length of median AD is $\frac{\sqrt{130}}{2}$ units.

36. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB.

Ans : [Board Term-2 OD 2016]

As per given in question we have drawn figure below.



In right $\Delta\, YCB$, we have

$$\tan 45^\circ = \frac{BC}{YC}$$
$$1 = \frac{x}{YC}$$
$$YC = x = XA$$

In right $\Delta\, XAB$ we have

$$\tan 60^\circ = \frac{AB}{XA}$$
$$\sqrt{3} = \frac{x+40}{x}$$
$$\sqrt{3}\,x = x+40$$
$$x\sqrt{3} - x = 40$$
$$x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$= 20(\sqrt{3}+1)$$
$$= 20\sqrt{3}+20$$

Thus height of the tower,

$$AB = x+40$$
$$= 20\sqrt{3}+20+40$$
$$= 20\sqrt{3}+60$$
$$= 20(\sqrt{3}+3)$$

In right $\Delta\, XAB$ we have,

$$\sin 60^\circ = \frac{AB}{BX}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{BX}$$
$$BX = \frac{2AB}{\sqrt{3}} = \frac{20 \times 2(\sqrt{3}+3)}{\sqrt{3}}$$
$$= 40(1+\sqrt{3})$$
$$= 40 \times 2.73 = 109.20$$