CLASS X (2020-21)
MATHEMATICS STANDARD (041)
SAMPLE PAPER-02

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

Section - I

1. What is the HCF of smallest primer number and the smallest composite number?

Ans : [Board 2018]

Smallest prime number is 2 and smallest composite number is 4. HCF of 2 and 4 is 2.

a222

or

Write one rational and one irrational number lying between 0.25 and 0.32.

Ans : [Board 2020 SQP Standard]

Given numbers are 0.25 and 0.32.

Clearly $0.30 = \frac{30}{100} = \frac{3}{10}$

a223

Thus 0.30 is a rational number lying between 0.25 and 0.32. Also 0.280280028000.....has non-terminating non-repeating decimal expansion. It is an irrational number lying between 0.25 and 0.32.

2. Find the value of k for which the system of equations $x+y-4=0$ and $2x+ky=3$, has no solution.

Ans : [Board 2020 Delhi Standard]

We have $x+y-4=0$

and $2x+ky-3=0$

Here, $\frac{a_1}{a_1} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{k}$ and $\frac{c_1}{c_2} = \frac{-4}{-3} = \frac{4}{3}$

Since system has no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$

$$k = 2 \text{ and } k \neq \frac{3}{4}$$

c236

3. If α and β are the zeroes of the polynomial x^2+2x+1, then what is the value of $\frac{1}{\alpha}+\frac{1}{\beta}$?

Ans : [Board 2020 Delhi Basic]

Since α and β are the zeros of polynomial x^2+2x+1,

Sum of zeroes, $\alpha+\beta = -\frac{2}{1} = -2$

and product of zeroes, $\alpha\beta = \frac{1}{1} = 1$

d323

Now, $$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

$$= -\frac{2}{1} = -2$$

or

If α and β are the zeroes of the polynomial $2x^2-13x+6$, then what is the value of $\alpha+\beta$?

Ans : [Board 2020 Delhi Basic]

We have $p(x) = 2x^2-13x+6$

Comparing it with ax^2+bx+c we get $a=2$, $b=-13$ and $a=6$.

Sum of zeroes $\alpha+\beta = -\frac{b}{a} = -\frac{(-13)}{2} = \frac{13}{2}$

d324

4. What is the value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP ?

Ans : [Board 2020 Delhi Standard]

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$$(x+10)-2x = (3x+2)-(x+10)$$

$$-x+10 = 2x-8$$

$$-x-2x = -8-10$$

$$-3x = -18 \Rightarrow x = 6$$

e332

or

If the first term of AP is p and the common difference is q, then what is its 10th term?

Ans : [Board 2020 Delhi St

We have $a = p$ and $d = q$

e333

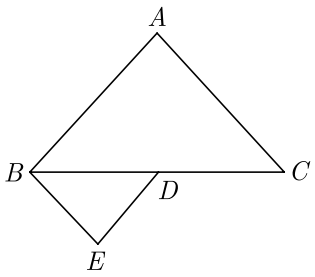
$$a_{10} = a + (10-1)\,d$$
$$= p + 9q$$

5. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ..... .

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.

f228

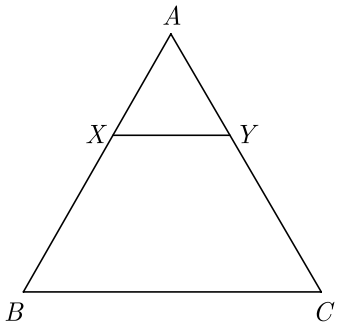


$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2} = \frac{(BC)^2}{\left(\frac{1}{2}BC\right)^2}$$
$$= \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1$$

6. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Ans : [Board Term-1 2016, 2015]

As per question we have drawn figure given below.



In this figure we have

$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9$$

Now $\frac{AX}{XB} = \frac{3}{4}$ and $\frac{AY}{YC} = \frac{5}{9}$

f103

Since $\frac{AX}{XB} \neq \frac{AY}{YC}$

Hence XY is not parallel to BC.

7. If $\sec 5A = \text{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then what is the value of A?

Ans :

We have, $\sec 5A = \text{cosec}(A + 30^\circ)$

h318

$$\sec 5A = \sec[90^\circ - (A - 30^\circ)]$$
$$\sec 5A = \sec(60^\circ - A)$$
$$5A = 60^\circ - A$$
$$6A = 60^\circ \Rightarrow A = 10^\circ$$

8. If $\tan A = \cot B$, then find the value of $(A + B)$.

Ans : [Board 2020 OD Standard]

We have $\tan A = \cot B$

h267

$$\tan A = \tan(90^\circ - B)$$
$$A = 90^\circ - B$$

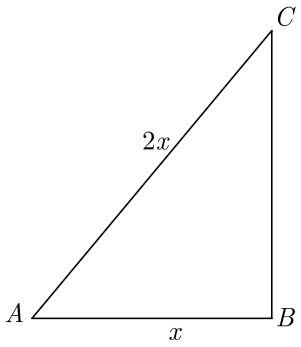
Thus $A + B = 90^\circ$

9. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal.

Ans : [Board Term-2 2015]

Let the distance between the foot of the ladder and the wall is x, then length of the ladder will be $2x$. As per given in question we have drawn figure below.

i103



In ΔABC, $\angle B = 90^o$

$$\cos A = \frac{x}{2x} = \frac{1}{2} = \cos 60^o$$
$$A = 60^o$$

10. If a line intersects a circle in two distinct points, what is it called ?

Ans : [Board Term-2, 2012]

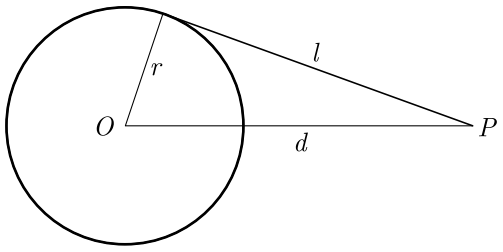
The line which intersects a circle in two distinct points is called secant.

j146

11. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

Ans : [Board Term-2, 2012]

As per the given question we draw the figure as below.



Length of the tangent, $l = \sqrt{d^2 - r^2}$

j150

$$= \sqrt{8^2 - 6^2}$$
$$= \sqrt{64 - 36}$$
$$= \sqrt{28} = 2\sqrt{3} \text{ cm.}$$

12. If circumference of a circle is 44 cm, then what will be the area of the circle?

Ans : [Board Term-2 2012]

Circumference of a circle = 44 cm

Radius of the circle $= \frac{22}{2 \times \frac{22}{7}} = 7$ cm

Area of the circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\, cm^2$

or

A steel wire when bent in the form of a square encloses an area or 121 cm². If the same wire is bent in the form of a circle, then find the circumference of the circle.

Ans : [Board Term-2 2012]

Area of square = $(\text{side})^2 = 121$ cm²

Side of square $= \sqrt{121} = 11$ cm

Parameter of square $= 4 \times 11 = 44$ cm

Circumference of the circle = Perimeter of the square

= 44 cm

13. A solid metallic cuboid 24 cm × 11 cm × 7 cm is melted and recast and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed.

Ans :

Let n be the number of cones formed.
Now, according to question,

Volume of n cones = Volume of cuboid

$$n \times \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6 = 24 \times 11 \times 7$$

$$n = \frac{24 \times 11 \times 7 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 24$$

Thus $n = 24$.

14. What is the ratio of the total surface area of the solid hemisphere to the square of its radius.

Ans : [Board Term-2, 2012]

$$\frac{\text{Total surface area of hemisphere}}{\text{Square of its radius}} = \frac{3\pi r^2}{r^2} = \frac{3\pi}{1}$$

Thus required ratio is $3\pi : 1$.

15. From the following frequency distribution, find the median class :

Cost of living index	1400-1500	1550-1700	1700-1850	1850-2000
Number of weeks	8	15	21	8

Ans : [Board Term-1, 2015]

We prepare following cumulative frequency table to find median class.

Cost of living index	**Number of weeks** f	**c.f.**
1400-1500	8	8
1550-1700	15	23
1700-1850	21	44
1850-2000	8	52

We have $N = 52$; $\frac{N}{2} = 26$

Cumulative frequency just greater than $\frac{N}{2}$ is 44 and the corresponding class is 1700-1850.
Thus median class is 1700-1850.

or

In the following frequency distribution, find the median class.

Height (in cm)	104-145	145-150	150-155	155-160	160-165	165-170
Frequency	5	15	25	30	15	10

Ans : [Board Term-1 2015]

We prepare following cumulative frequency table to find median class.

Height	**Frequency**	**c.f.**
140-145	5	5
145-150	15	20
150-155	25	45
155-160	30	75
160-165	15	90
165-170	10	100
	$N = 100$	

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 75 and the corresponding class is 155-160.
Thus median class is 155-160.

16. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. What is the lower limit of the class?

Ans :

Let x be the upper limit and y be the lower limit.
Since the mid value of the class is 10.

Hence, $\frac{x+y}{2} = 10$

$$x + y = 20 \quad \ldots(1)$$

Since width of the class is 6,

$$x - y = 6 \quad \ldots(2)$$

Solving (1) and (2), we get $y = 7$
Hence, lower limit of the class is 7.

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. The Prime Minister's Citizen Assistance and Relief in Emergency Situations Fund was created on 28 March 2020, following the COVID-19 pandemic in India. The fund will be used for combating, and containment and relief efforts against the coronavirus outbreak and similar pandemic like situations in the future.

The allotment officer is trying to come up with a method to calculate fair division of funds across various affected families so that the fund amount and amount received per family can be easily adjusted based on daily revised numbers. The total fund allotted for a village is $x^3+6x^2+20x+9$. The officer has divided the fund equally among families of the village and each family receives an amount of x^2+2x+2. After distribution, some amount is left.

(i) How many families are there in the village?
(a) $x+4$
(b) $x-3$
(c) $x-4$
(d) $x+3$

b402

(ii) If an amount of ₹1911 is left after distribution, what is value of x?
(a) 190 (b) 290
(c) 191 (d) 291

(iii) How much amount does each family receive?
(a) 24490 (b) 34860
(c) 22540 (d) 36865

(iv) What is the amount of fund allocated?
(a) Rs 72 72 759 (b) Rs 75 72 681
(c) Rs 69 72 846 (d) Rs 82 74 888

(v) How many families are there in the village?
(a) 191 (b) 98
(c) 187 (d) 195

Ans :

(i) To get number of families we divide $x^3+6x^2+20x+9$ by x^2+2x+2.

$$\begin{array}{r} x+4 \\ x^2+2x+2\overline{)\,x^3+6x^2+20x+9} \\ \underline{x^3+2x^2+2x} \\ 4x^2+18x+9 \\ \underline{4x^2+8x+8} \\ 10x+1 \end{array}$$

Number of families are $x+4$.
Thus (a) is correct option.

(ii) Amount left $= 10x+1$

$$10x+1 = 1911$$

$$x = \frac{1910}{10} = 191$$

Thus (c) is correct option.

(iii) Since, $x=191$, amount received by each family is

$$x^2+2x+2 = (191)^2+2(191)+2$$
$$= 36865$$

Thus (d) is correct option.

(iv) Since $x=191$, allotted fund,

$$x^3+6x^2+20x+9 = (x^2+2x+2)(x+4)+10x+1$$
$$= 36865(191+4)+1911$$
$$= 69,72,846$$

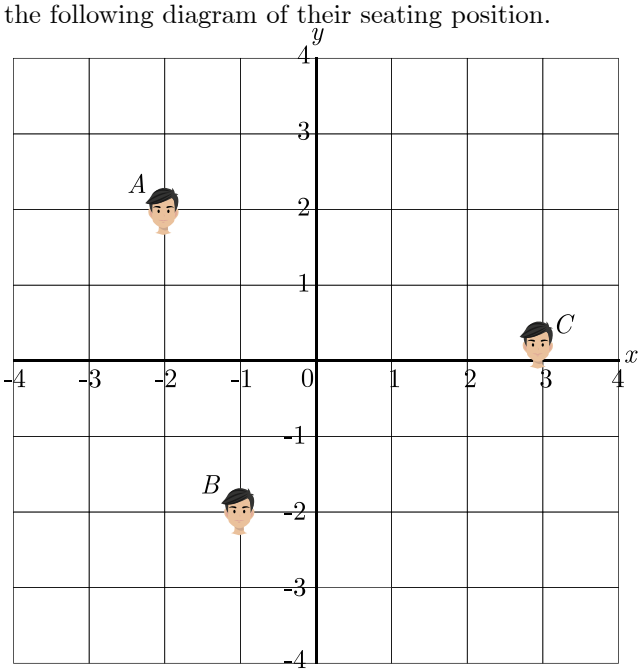
Thus (c) is correct option.

(v) No. of families $= x+4$

$$= 191+4 = 195$$

Thus (d) is correct option.

18. Ajay, Bhigu and Colin are fast friend since childhood. They always want to sit in a row in the classroom . But teacher doesn't allow them and rotate the seats row-wise everyday. Bhigu is very good in maths and he does distance calculation everyday. He consider the centre of class as origin and marks their position on a paper in a co-ordinate system. One day Bhigu make the following diagram of their seating position.



(i) What are the coordinates of point A?
(a) $(2,2)$
(b) $(2,-2)$
(c) $(-2,2)$
(d) $(-2,-2)$

g402

(ii) What is the distance of point A from origin ?
(a) 8 (b) $2\sqrt{2}$
(c) 4 (d) $4\sqrt{2}$

(iii) What is the distance between A and B ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $\sqrt{17}$ (d) $2\sqrt{5}$

(iv) What is the distance between B and C ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $2\sqrt{17}$ (d) $2\sqrt{5}$

(v) A point D lies on the line segment between points A and B such that $AD:DB=4:3$. What are the the coordinates of point D ?

(a) $\left(\frac{10}{7}, \frac{2}{7}\right)$ (b) $\left(\frac{2}{7}, \frac{7}{7}\right)$

(c) $\left(-\frac{10}{7}, -\frac{2}{7}\right)$ (d) $\left(-\frac{2}{7}, -\frac{7}{7}\right)$

Ans :

(i) It may be seen easily from figure that coordinates of point A are $(-2, 2)$.
Thus (c) is correct option.

(ii) $OA = \sqrt{(0+2)^2+(0-2)^2} = 2\sqrt{2}$
Thus (b) is correct option.

(iii) It may be seen easily from figure that coordinates of point A are $(-1, -2)$.

$$AB = \sqrt{(-2+1)^2+(2+2)^2}$$
$$= \sqrt{1+4^2} = \sqrt{17}$$

Thus (c) is correct option.

(iv) It may be seen easily from figure that coordinates of point A are $(0,\ 3)$.

$$BC = \sqrt{(-1-3)^2+(-2-0)^2}$$
$$= \sqrt{4^2+4} = 2\sqrt{5}$$

Thus (d) is correct option.

(v) We have $A(-2, 2)$ and $B(-1, -2)$

$$\frac{m}{n} = \frac{4}{3}$$
$$x = \frac{mx_2+nx_1}{m+n} = \frac{-1(4)+3(-2)}{4+3} = \frac{-10}{7}$$
$$y = \frac{my_2+hy_1}{m+n} = \frac{-2(4)+3(2)}{4+3} = \frac{-2}{7}$$

Thus (c) is correct option.

19. A hot air balloon is a type of aircraft. It is lifted by heating the air inside the balloon, usually with fire. Hot air weighs less than the same volume of cold air (it is less dense), which means that hot air will rise up or float when there is cold air around it, just like a bubble of air in a pot of water. The greater the difference between the hot and the cold, the greater the difference in density, and the stronger the balloon will pull up.

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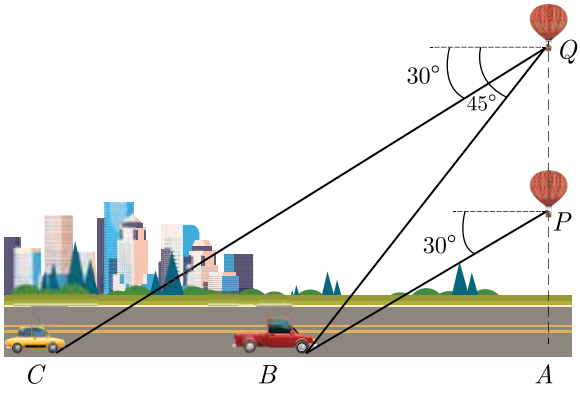


Lakshman is riding on a hot air balloon. After reaching at height x at point P, he spots a lorry parked at B on the ground at an angle of depression of 30°. The balloon rises further by 50 metres at point Q and now he spots the same lorry at an angle of depression of 45° and a car parked at C at an angle of depression of 30°.

(i) What is the relation between the height x of the balloon at point P and distance d between point A and B?
(a) $x = 3d$ (b) $d = 3x$
(c) $d^2 = 3x^2$ (d) $3d^2 = x^2$

(ii) When balloon rises further 50 metres, then what is the relation between new height y and d?
(a) $y = d + 50$ (b) $d = y$
(c) $y = \sqrt{3}\,d$ (d) $\sqrt{3}\,y = d$

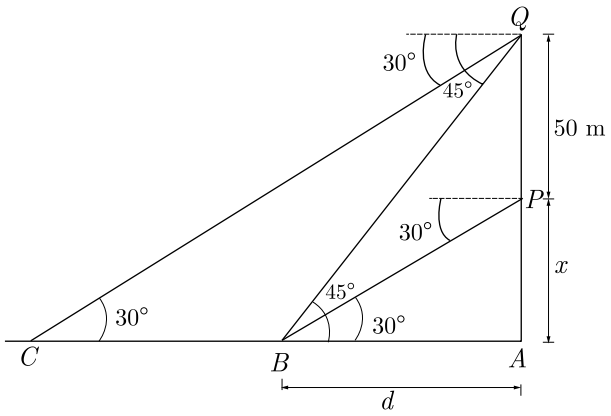
(iii) What is the new height of the balloon at point Q ?
(a) $50(\sqrt{3}+3)$ m (b) $25(\sqrt{3}+1)$ m
(c) $50(\sqrt{3}+1)$ m (d) $25(\sqrt{3}+3)$ m

(iv) What is the distance AB on the ground ?
(a) $50(\sqrt{3}+3)$ m (b) $25(3+3\sqrt{3})$ m
(c) $50(\sqrt{3}+1)$ m (d) $25(\sqrt{3}+3)$ m

(v) What is the distance AC on the ground ?
(a) $75(1+\sqrt{3})$ m (b) $25(1+\sqrt{3})$ m
(c) $50(1+\sqrt{3})$ m (d) $25(\sqrt{3}+3)$ m

Ans :

(i) We make the diagram as per given information.



In ΔAPB, $\tan 30^\circ = \frac{AP}{AB}$

$$\frac{1}{\sqrt{3}} = \frac{x}{d}$$
$$d = \sqrt{3}\,x \Rightarrow d^2 = 3x^2$$

Thus (c) is correct option.

(ii) In ΔBAQ,

$$\tan 45^\circ = \frac{AQ}{AB}$$
$$AB = AQ$$
$$d = y$$

Thus (b) is correct option.

(iii) From (i) and (ii) we have

$$d = \sqrt{3}\,x \text{ and } d = y$$

Since point Q is 50 m above point P, Thus

$$y = x + 50$$

Thus $d = x + 50$

Solving above equations we get

$$\sqrt{3}\,x = x + 50$$
$$x(\sqrt{3}-1) = 50$$
$$x = \frac{50}{(\sqrt{3}-1)} = 25(\sqrt{3}+1)$$

$$\begin{aligned} y &= x + 50 \\ &= 25(\sqrt{3}+1)+50 \\ &= 25\sqrt{3}+25+50 \\ &= 25(\sqrt{3}+3) \end{aligned}$$

Thus (d) is correct option.

(iv) The distance AB on the ground is d and which is equal to

$$d = \sqrt{3}\,x$$

or $$d = y = 25(\sqrt{3}+3)$$

Thus (d) is correct option.

(v) In $\Delta\, CAQ$,

$$\tan 30^\circ = \frac{AQ}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{AC}$$

$$= \frac{25(\sqrt{3}+3)}{AC}$$

$$\begin{aligned} AC &= 25\sqrt{3}\,(\sqrt{3}+3) \\ &= 25(3+3\sqrt{3}) \\ &= 75(1+\sqrt{3}) \end{aligned}$$

Thus (a) is correct option.

20. In two dice game, the player take turns to roll both dice, they can roll as many times as they want in one turn. A player scores the sum of the two dice thrown and gradually reaches a higher score as they continue to roll. If a single number 1 is thrown on either die, the score for that whole turn is lost. Two dice are thrown simultaneously.

(i) What is the probability of getting the sum as an even number ?

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{5}{8}$

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(ii) What is the probability of getting the sum as a prime number ?

(a) $\frac{5}{12}$ (b) $\frac{1}{6}$

(c) $\frac{7}{12}$ (d) $\frac{11}{12}$

(iii) What is the probability of getting the sum of atleast 10?

(a) $\frac{5}{12}$ (b) $\frac{5}{6}$

(c) $\frac{1}{6}$ (d) $\frac{7}{12}$

(iv) What is the probability of getting a doublet of even number ?

(a) $\frac{1}{12}$ (b) $\frac{5}{12}$

(c) $\frac{11}{12}$ (d) $\frac{7}{12}$

(v) What is the probability of getting a product of numbers greater than 16?

(a) $\frac{7}{36}$ (b) $\frac{2}{9}$

(c) $\frac{5}{18}$ (d) $\frac{11}{36}$

Ans :

(i) All possible outcome are given as below:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Number of all possible outcomes in all case,

$$n(S) = 6 \times 6 = 36$$

Favourable outcome are $\{2, 4, 6, 8, 10, 12\}$. We may get as follows

{(1, 1), (1, 3), (3, 1), (2, 2), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (2, 6), (6, 2), (3, 5), (5, 3), (4, 4), (6, 4), (4, 6), (5, 5), (6, 6)}

Thus number of favourable outcomes,

$$n(E_1) = 18$$

P(sum as an even number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

Thus (d) is correct option.

(ii) Favourable outcome are $\{2, 3, 5, 7, 11\}$, which may be as follows

{(1, 1), (1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (6, 5), (5, 6)}

Thus number of favourable outcomes,

$$n(E_1) = 15$$

P(sum as a prime number),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

Thus (a) is correct option.

(iii) Favourable outcomes are {(5, 5), (6, 4), (4, 6), (6, 5), (5, 6), (6, 6)}

Thus number of favourable outcomes,

$$n(E_3) = 6$$

P(sum of atleast 10),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Thus (c) is correct option.

(iv) Favourable outcomes are {(2, 2), (4, 4), (6, 6)}

Thus number of favourable outcomes,

$$n(E_3) = 3$$

P(doublet of even number),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

Thus (a) is correct option.

(v) Favourable outcomes are {(3, 6), (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6),}

Thus number of favourable outcomes,

$$n(E_5) = 10$$

P(product of numbers greater than 16),

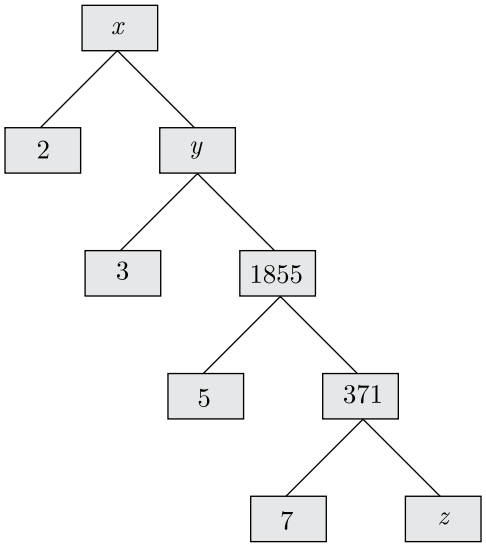
$$P(E_5) = \frac{n(E_5)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

Thus (c) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Complete the following factor tree and find the composite number x



Ans : [Board Term-1 2015, Set DDE-M]

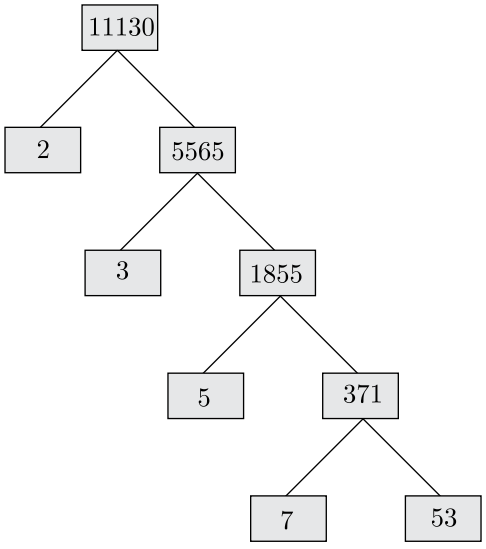
We have $z = \frac{371}{7} = 53$

$y = 1855 \times 3 = 5565$

$x = 2 \times y = 2 \times 5565 = 11130$

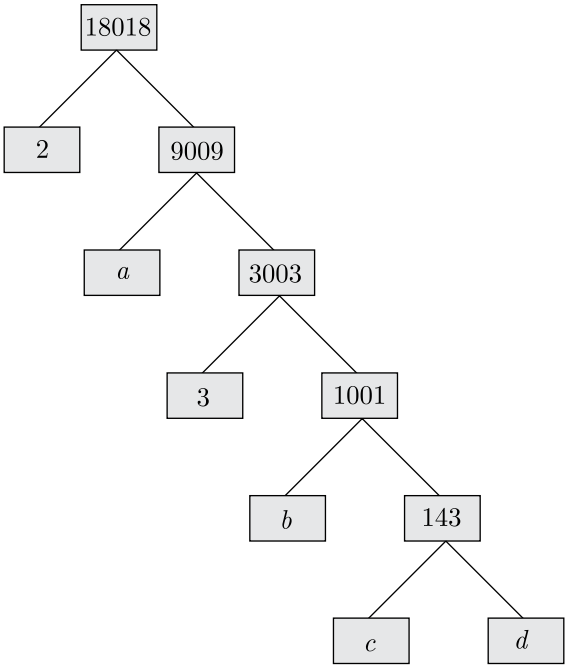
a112

Thus complete factor tree is as given below.



or

Find the missing numbers a, b, c and d in the given factor tree:



Ans : [Board Term-1 2012]

We have $a = \frac{9009}{3003} = 3$

$b = \frac{1001}{143} = 7$

a113

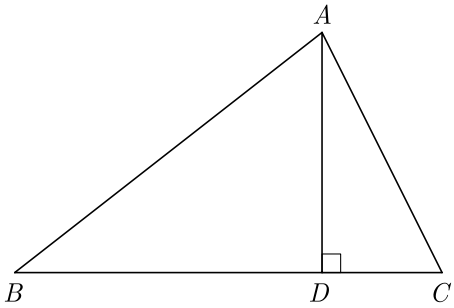
Since $143 = 11 \times 13,$

Thus $c = 11$ and $d = 13$ or $c = 13$ and $d = 11$

22. In $\Delta ABC, AD \perp BC,$ such that $AD^2 = BD \times CD.$ Prove that ΔABC is right angled at $A.$

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



We have $AD^2 = BD \times CD$

$\frac{AD}{CD} = \frac{BD}{AD}$

f114

Since $\angle D = 90^\circ$, by SAS we have

$\Delta ADC \sim \Delta BDA$

and $\angle BAD = \angle ACD;$

Since corresponding angles of similar triangles are equal

$\angle DAC = \angle DBA$

$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^\circ$

$2\angle BAD + 2\angle DAC = 180^\circ$

$\angle BAD + \angle DAC = 90^\circ$

$\angle A = 90^\circ$

Thus ΔABC is right angled at $A.$

or

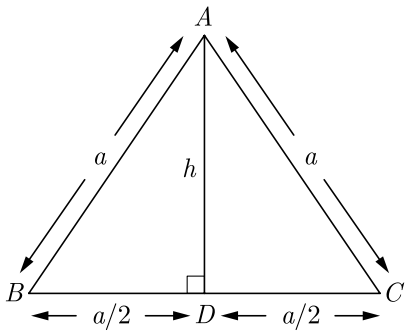
In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. [Board 2020 SQP Standard]

or

Find the altitude of an equilateral triangle when each of its side is a cm.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



In ΔABD, $a^2 = \left(\frac{a}{2}\right)^2 + h^2$

$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$

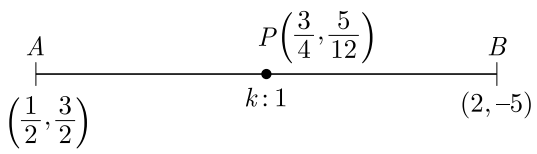
Thus $h = \frac{\sqrt{3}a}{2}$

Thus $4h^2 = 3a^2$ Hence Proved

23. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the point $A\ \left(\frac{1}{2}, \frac{3}{2}\right)$ and $(2, -5)$.

Ans : [Board Term-2 Delhi 2015]

Let P divides AB in the ratio k:1. Line diagram is shown below.



Now $\frac{k(2)+1\left(\frac{1}{2}\right)}{k+1} = \frac{3}{4}$

$8k+2 = 3k+3$

$k = \frac{1}{5}$

Thus required ratio is $\frac{1}{5}$:1 or 1:5.

24. If $\sin\phi = \frac{1}{2}$, show that $3\cos\phi - 4\cos^3\phi = 0$.

Ans :

We have $\sin\phi = \frac{1}{2}$

$\phi = 30^\circ$

Now substituting this value of θ in LHS we have

$3\cos\phi - 4\cos^3\phi = 3\cos 30^\circ - 4\cos^3 30^\circ$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

$= 0$ Hence Proved

25. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere.

Ans : [Board Term-2, 2014]

No. of spheres $= 12$

Radius of cone, $r = 1$ cm

Height of the cone $= 48$ cm

Volume of 12 spheres = Volume of cone

Let the radius of sphere be R. Let r and h be radius and height of cone.

Now $12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$

$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (1)^2 \times 48$

$R^3 = 1$

$R = 1$ cm

26. Find the mean of the following data :

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	20	35	52	44	38	31

Ans :

Let $a = 70$ be assumed mean.

C.I.	Frequency f	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-20	20	10	-3	-60
20-40	35	30	-2	-70
40-60	52	50	-1	-52
60-80	44	70	0	0
80-100	38	90	1	38
100-120	31	110	2	62
	$\sum f_i = 220$			$\sum f_i u_i = -82$

Mean, $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 70 + \frac{(-82)}{220} \times 20$

$= 70 - \frac{82}{11} = 70 - 7.45 = 62.55$

27. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Ans : [Board Term-1 2011, Set-44]

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3 \times 3 = 3^2$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5$$
$$= 150 \text{ minutes}$$

a131

The bells will toll next together after 180 minutes.

28. Solve for x and y :

$$\frac{x}{2} + \frac{2y}{3} = -1$$
$$x - \frac{y}{3} = 3$$

Ans : [Board Term-1 2015, NCERT]

We have $\frac{x}{2} + \frac{2y}{3} = -1$

$$3x + 4y = -6 \quad ...(1)$$

and $\frac{x}{1} - \frac{y}{3} = 3$

$$3x - y = 9 \quad ...(2)$$

Subtracting equation (2) from equation (1), we have

$$5y = -15 \Rightarrow y = -1$$

Substituting $y = -3$ in eq (1), we get

c142

$$3x + 4(-3) = -6$$
$$3x - 12 = -6$$
$$3x = 12 - 6 \Rightarrow x = 2$$

Hence $x = 2$ and $y = -3$.

29. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Ans : [Board Term-2 OD 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_1 = 1 + 2 + 3 + ... n$

$S_2 = 1 + 3 + 5 + ... \text{ up to } n \text{ terms}$

$S_3 = 1 + 4 + 7 + ... \text{ upto } n \text{ terms}$

e184

Now $S_n = \frac{n(n+1)}{2}$

$$S_2 = \frac{n}{2}[2 + (n-1)2]$$
$$= \frac{n}{2}[2n] = n^2$$

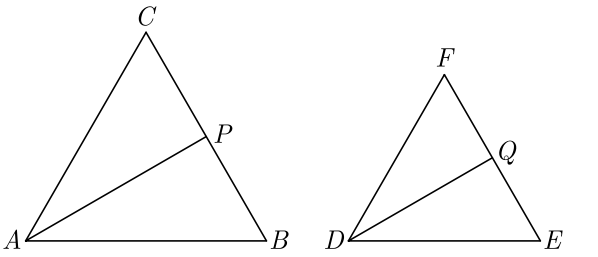
and $S_3 = \frac{n}{2}[2 + (n-1)3]$

$$= \frac{n(3n-1)}{2}$$

Now, $S_1 + S_3 = \frac{n(n+1)}{2} + \frac{n(3n-1)}{2}$

$$= \frac{n[n+1+3n-1]}{2}$$
$$= \frac{n[4n]}{2}$$
$$= 2n^2 = 2s_2$$ Hence Proved

30. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



Prove that :

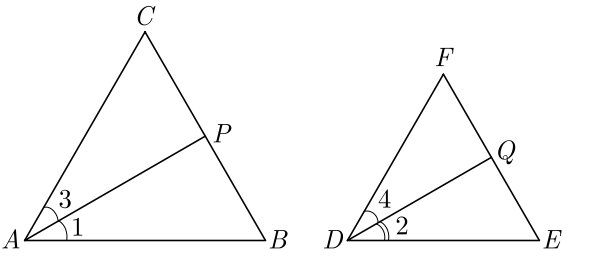
(1) $\frac{AP}{DQ} = \frac{AB}{DE}$

(2) $\Delta CAP \sim \Delta FDQ$.

f148

Ans : [Board Term-1 2016]

As per given condition we have redrawn the figure below.



(1) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$
$$2\angle 1 = 2\angle 2$$

Also $\angle B = \angle E$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE}$$ Hence Proved

(2) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D$$

and $\angle C = \angle F$

$$2\angle 3 = 2\angle 4$$
$$\angle 3 = \angle 4$$

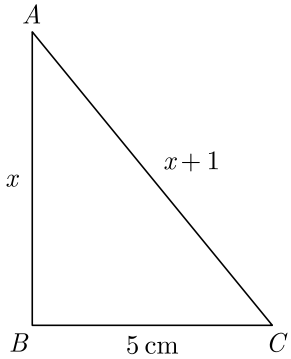
By AA similarity we have

$$\Delta CAP \sim \Delta FDQ$$

31. In ΔABC, $\angle B = 90^\circ$, $BC = 5$ cm, $AC - AB = 1$, Evaluate : $\frac{1 + \sin C}{1 + \cos C}$.

Ans : [Board Term-1 2011]

As per question we have drawn the figure given below.



We have $AC - AB = 1$

Let $AB = x$, then we have

h143

$$AC = x + 1$$

Now $AC^2 = AB^2 + BC^2$

$(x+1)^2 = x^2 + 5^2$

$x^2 + 2x + 1 = x^2 + 25$

$2x = 24$

$x = \frac{24}{2} = 12$ cm

Hence, $AB = 12$ cm and $AC = 13$ cm

Now $\sin C = \frac{AB}{AC} = \frac{12}{13}$

$\cos C = \frac{BC}{AC} = \frac{5}{13}$

Now $\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$

or

If $b\cos\theta = a$, then prove that $\text{cosec}\,\theta + \cot\theta = \sqrt{\frac{b+a}{b-a}}$.

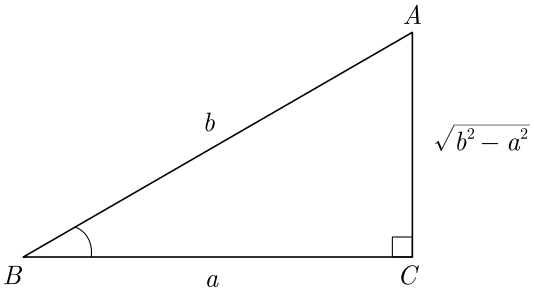
Ans : [Board Term-1 2015]

We have $b\cos\theta = a$

or, $\cos\theta = \frac{a}{b}$

h166

Now consider the triangle shown below.



$AC^2 = AB^2 - BC^2$

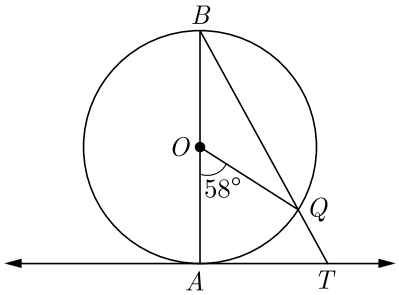
or, $\cos\theta = \frac{a}{b}$

$AC = \sqrt{b^2 - a^2}$

Now $\text{cosec}\,\theta = \frac{b}{\sqrt{b^2-a^2}}, \cot\theta = \frac{a}{\sqrt{b^2-a^2}}$

$\text{cosec}\,\theta + \cot\theta = \frac{b+a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$

32. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.



Ans : [Board Term-2, 2015]

We have $\angle AOQ = 58^\circ$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

j174

$\angle ABQ = \frac{1}{2}\angle AOQ$

$= \frac{1}{2} \times 58^\circ = 29^\circ$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90^\circ$

$\angle ABQ = \angle ABT$

Now in ΔBAT,

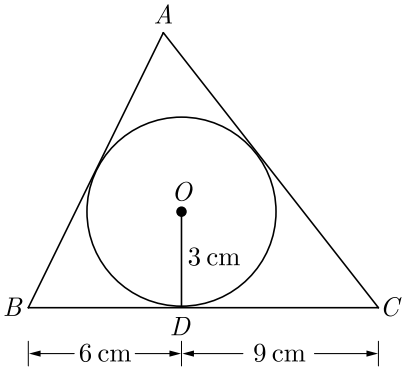
$\angle ATB = 90^\circ - \angle ABT$

$= 90^\circ - 29^\circ = 61^\circ$

Thus $\angle ATQ = \angle ATB = 61^\circ$

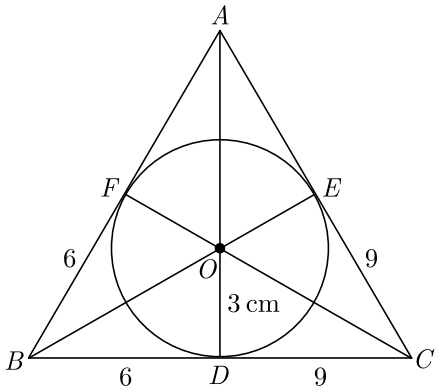
or

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC.



Ans : [Board Term-2 OD 2015]

We redraw the given circle as shown below.



Since tangents from an external point to a circle are equal,

j175

$AF = AE$

$BF = BD = 6$ cm

$CE = CD = 9$ cm

Let $AF = AE = x$

Now $AB = AF + FB = 6 + x$

$AC = AE + EC = x + 9$

$$BC = 6+9 = 15 \text{ cm}$$

Perimeter of ΔABC,

$$p = 15+6+x+9+x$$
$$= 30+2x$$

Now area, $\Delta ABC = \frac{1}{2}rp$

Here $r = 3$ is the radius of circle. Substituting all values we have

$$54 = \frac{1}{2} \times 3 \times (30+2x)$$
$$54 = 45+3x$$

or $$x = 3$$

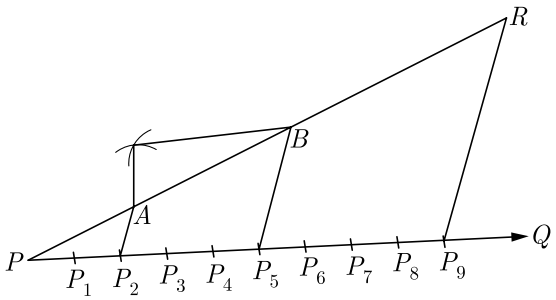
Thus $AB = 9$ cm, $AC = 12$ cm and $BC = 15$ cm.

33. Construct a triangle whose perimeter is 13.5 cm and the ratio of the three sides is 2:3:4.

Ans : [Board Term-2 2011, 2012]

Steps of Construction :

1. Draw a line segment PR of length 13.5 cm.
2. At the point P draw a ray PQ making an acute angle RPQ with PR.
3. On PQ mark $(2+3+4)$ a points $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9$ such that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = P_5P_6 = P_6P_7 = P_7P_8 = P_8P_9$.
4. Join P_9R
5. Through P_2 and P_5 draw lines P_2A and P_5B respectively parallel to P_9R intersecting PR at A and B respectively.
6. With A as centre and radius AP draw and arc. ABC is the required triangle.
7. With B as centre and radius BR draw another arc to intersect first arc.
8. Join A to C and B to C.



34. Solve for x : $\frac{1}{x+1}+\frac{2}{x+2} = \frac{4}{x+4}$ $x \neq -1, -2, -4$

Ans : [Board Term-2 OD 2016]

We have $$\frac{1}{x+1}+\frac{2}{x+2} = \frac{4}{x+4}$$

$$\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$
$$\frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$$
$$(3x+4)(x+4) = 4(x^2+3x+2)$$
$$3x^2+16x+16 = 4x^2+12x+8$$
$$x^2-4x-8 = 0$$

Now $$x = \frac{-b\sqrt{b^2+4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2-4(1)(-8)}}{2 \times 1}$$
$$= \frac{4 \pm \sqrt{16+32}}{2}$$
$$= \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$
$$= 2 \pm 2\sqrt{3}$$

Hence, $x = 2+2\sqrt{3}$ and $2-2\sqrt{3}$

or

Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board 2019 OD]

We have $$7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$

$$21y^2 - 11y - 2 = 0 \quad ...(1)$$
$$21y^2 - 14y + 3y - 2 = 0$$
$$7y(3y-2) + (3y-2) = 0$$
$$(3y-2)(7y+1) = 0$$
$$y = \frac{2}{3}, \frac{-1}{7}$$

Hence, zeros of given polynomial are,

$$y = \frac{2}{3} \text{ and } y = \frac{-1}{7}$$

Comparing the given equation with $ax^2+bx+c = 0$ we get $a = 21$, $b = -11$ and $c = -2$

Now, sum of roots, $\alpha+\beta = \frac{2}{3}+\left(-\frac{1}{7}\right)$

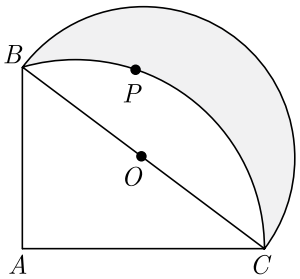
$$= \frac{2}{3}-\frac{1}{7} = \frac{11}{21}$$

Thus $\alpha+\beta = -\frac{b}{a}$ Hence verified

and product of roots, $\alpha\beta = \frac{2}{3} \times \left(-\frac{1}{7}\right) = \frac{-2}{21}$

Thus $\alpha\beta = \frac{c}{a}$ Hence verified

35. In given figure $ABPC$ is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the are of the shaded region.



Ans : [Board Term-2 SQP 2017]

Radius of the quadrant $AB = AC = 14$ cm

$$BC = \sqrt{14^2+14^2} = 14\sqrt{2} \text{ cm}$$

Radius of semicircle $= \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Area of semicircle $= \frac{1}{2}\pi(7\sqrt{2})^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 98$$
$$= 154 \text{ cm}^2$$

Area of segment $BPCO$

$$\frac{\pi r^2\theta}{360^\circ} - \frac{1}{2}r^2 = r^2\left(\frac{\pi\theta}{360^\circ} - \frac{1}{2}\right)$$
$$= 14 \times 14\left(\frac{22}{7} \times \frac{90}{360} - \frac{1}{2}\right)$$
$$= 14 \times 14\left(\frac{11}{14} - \frac{1}{2}\right)$$
$$= 14 \times 14 \times \frac{2}{7} = 56 \text{ cm}^2$$

Hence, area of shaded region is 56 cm^2.

36. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0 - 10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Ans : [Board 2019 Delhi]

Class	**Frequency** (f)	**Cumulative Frequency** (cf)
0-10	f_1	f_1
10-20	5	$f_1 + 5$
20-30	9	$f_1 + 14$
30-40	12	$f_1 + 26$
40-50	f_2	$f_1 + f_2 + 26$
50-60	3	$f_1 + f_2 + 29$
60-70	2	$f_1 + f_2 + 31$
	$N = \sum f = 40$	

Now, $f_1 + f_2 + 31 = 40$

$$f_1 + f_2 = 9$$
$$f_2 = 9 - f_1 \quad \ldots(1)$$

Since median is 32.5, which lies in 30-40, median class is 30-40.

Here $l = 30$, $\frac{N}{2} = \frac{40}{2} = 20$, $f = 12$ and $F = 14 + f_1$

Now, median $= 3.25$


n240

$$l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 32.5$$
$$30 + \left(\frac{20 - (14 + f_1)}{12}\right) \times 10 = 32.5$$
$$\left(\frac{6 - f_1}{12}\right) \times 10 = 2.5$$
$$\frac{60 - 10f_1}{12} = 2.5$$
$$60 - 10f_1 = 30$$
$$10f_1 = 30 \Rightarrow f_1 = 3$$

From equation (1), we get $f_2 = 9 - 3 = 6$

Hence, $f_1 = 3$ and $f_2 = 6$