

CLASS X (2020-21)
MATHEMATICS STANDARD (041)
SAMPLE PAPER-03

Time : 3 Hours

Maximum Marks : 80

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part-A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part-B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Scan QR Code to See Video Solutions

Part - A

Section - I

1. If $HCF(336, 54) = 6$, find $LCM(336, 54)$.

Ans : [Board 2019 OD]

$$HCF \times LCM = \text{Product of number}$$

$$6 \times LCM = 336 \times 54$$

$$LCM = \frac{336 \times 54}{6}$$

$$= 56 \times 54 = 3024$$

Thus LCM of 336 and 54 is 3024.

or

Explain why 13233343563715 is a composite number?

Ans : [Board Term-1 2016]

The number 13233343563715 ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

2. For which value(s) of p , will the lines represented by the following pair of linear equations be parallel ?

$$3x - y - 5 = 0$$

$$6x - 2y - p = 0$$

Ans :

We have, $3x - y - 5 = 0$

and $6x - 2y - p = 0$

Here, $a_1 = 3, b_1 = -1, c_1 = -5$

and $a_2 = 6, b_2 = -2, c_2 = -p$

Since given lines are parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{-1}{-2} \neq \frac{-5}{-p}$$

$$p \neq 5 \times 2 \Rightarrow p \neq 10$$

3. Find the roots of the quadratic equation $x^2 - 0.04 = 0$

Ans : [Board 2020 OD Standard]

We have $x^2 - 0.04 = 0$

$$x^2 = 0.04$$

$$x = \pm \sqrt{0.04}$$

$$x = \pm 0.2.$$

or

If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then what is the value of k ?

Ans :

We have $x^2 + kx - \frac{5}{4} = 0$

Since, $\frac{1}{2}$ is a root of the given quadratic equation, it must satisfy it.

Thus $\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{1 + 2k - 5}{4} = 0$$

$$2k - 4 = 0 \Rightarrow k = 2$$

4. Fill the two blanks in the sequence 2 , 26, so that the sequence forms an AP.

Ans : [Board 2020 SQP Standard]

Let a and b be the two numbers. AP will be 2, a , 26, b .

Now, $26 - a = a - 2$

$$2a = 28 \Rightarrow a = \frac{28}{2} = 14$$

and $b - 26 = 26 - a$

$$a + b = 52$$

$$14 + b = 52 \Rightarrow b = 38$$

Thus $a = 14$ and $b = 38$.

or

The sum of first 20 terms of the AP 1, 4, 7, 10 ... is

Ans : [Board 2020 Delhi Standard]

Given AP is 1, 4, 7, 10 ...

Here, $a = 1, d = 4 - 1 = 3$ and $n = 20$

$$S_{20} = \frac{n}{2} [2a + (n - 1) d]$$

$$= \frac{20}{2} [2 \times 1 + (20 - 1) 3]$$

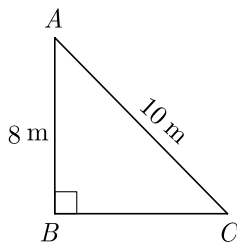
$$= 10(2 + 57) = 10 \times 59 = 590$$



5. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is m.

Ans : [Board 2020 Delhi Standard]

Let AB be the height of the window above the ground and BC be a ladder.



Here, $AB = 8$ m
and $AC = 10$ m

In right angled triangle ABC ,

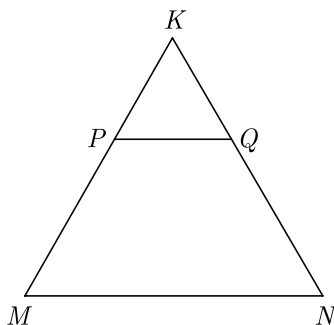
$$AC^2 = AB^2 + BC^2$$

$$10^2 = 8^2 + BC^2$$

$$BC^2 = 100 - 64 = 36$$

$$BC = 6$$
 m

6. In the figure, PQ is parallel to MN . If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4$ cm then find KQ .



Ans :

In the given figure $PQ \parallel MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN}$$

(By BPT)

$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$



$$4 \times 20.4 - 4KQ = 13KQ$$

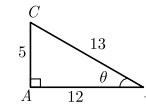
$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$

7. If $\sin \theta = \frac{5}{13}$, then the value of $\tan \theta$ is

Ans : [Board 2020 OD Basic]

From $\sin \theta = \frac{5}{13}$ we can draw the figure as given below.



Now, $\tan \theta = \frac{AC}{BC} = \frac{5}{12}$

8. If $x = 3 \sin \theta + 4 \cos \theta$ and $y = 3 \cos \theta - 4 \sin \theta$ then prove that $x^2 + y^2 = 25$.

Ans : [Board 2020 OD Basic]

We have $x = 3 \sin \theta + 4 \cos \theta$

and $y = 3 \cos \theta - 4 \sin \theta$



$$x^2 + y^2 = (3 \sin \theta + 4 \cos \theta)^2 + (3 \cos \theta - 4 \sin \theta)^2$$

$$= (9 \sin^2 \theta + 16 \cos^2 \theta + 24 \sin \theta \cos \theta) +$$

$$+ (9 \cos^2 \theta + 16 \sin^2 \theta - 24 \sin \theta \cos \theta)$$

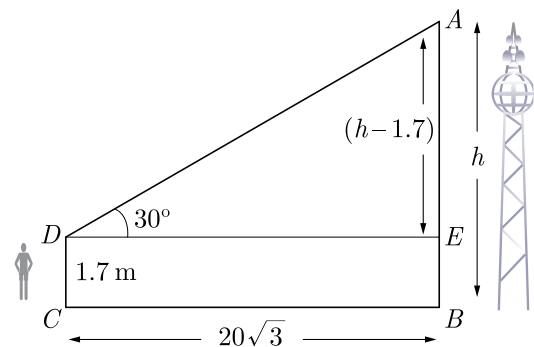
$$= 9(\sin^2 \theta + \cos^2 \theta) + 16(\sin^2 \theta + \cos^2 \theta)$$

$$= 9 + 16 = 25$$

9. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30° . Find the height of tower.

Ans : [Board Term-2 Foreign 2016]

Let height of the tower AB be h . As per given in question we have drawn figure below.



Here $AE = h - 1.7$
and $BC = DE = 20\sqrt{3}$

In $\triangle ADE$, $\angle E = 90^\circ$

$$\tan 30^\circ = \frac{h - 1.7}{20\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{h - 1.7}{20\sqrt{3}}$$

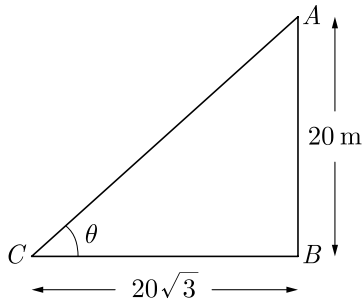
$$h - 1.7 = 20$$

or $h = 20 + 1.7 = 21.7$ m

or

In figure, a tower AB is 20 m high and BC , its shadow on the ground, is $20\sqrt{3}$ m long. find the Sun's altitude.





Ans : [Board Term-2 OD 2015]

Let the $\angle ACB$ be θ .

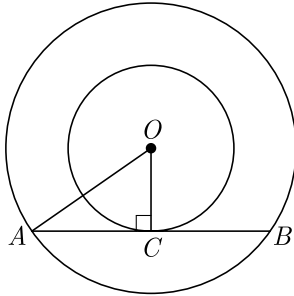
$$\tan \theta = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $\theta = 30^\circ$

10. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle.

Ans : [Board 2020 OD Basic, Foreign 2014]

As per the given question we draw the figure as below.



Here AB is the chord of large circle which touch the smaller circle at point C . We can see easily that ΔAOC is right angled triangle.

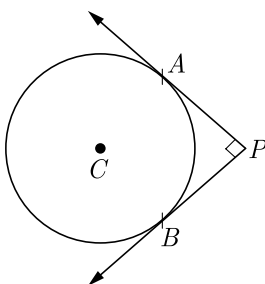
Here, $AO = 5$ cm, $OC = 3$ cm

$$\begin{aligned} AC &= \sqrt{AO^2 - OC^2} \\ &= \sqrt{5^2 - 3^2} \\ &= \sqrt{25 - 9} = \sqrt{16} = 4 \end{aligned}$$

cm

Length of chord, $AB = 8$ cm.

11. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then find the length of each tangent.



Ans : [Board Term-2, 2013]

Here tangent drawn on circle from external point P are at aright angle, $CAPB$ will be a square.

Thus $CA = AP = PB = BC = 4$ cm

Thus length of tangent is 4 cm.

12. Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of diameter 36 cm and 20 cm

Ans : [Board Term-2 2012]

Circumference of the circle,

$$2\pi r = 2\pi \times 18 + 2\pi \times 10$$

$$r = 18 + 10 = 28 \text{ cm}$$

Hence radius of given circle is 28 cm.

or

Find the diameter of a circle whose area is equal to the sum of areas of two circles of diameter 16 cm and 12 cm.

Ans : [Board Term-2 2012]

Let r be the radius of the circle. Since area of the circle is equal to the sum of areas of two circles,

$$\pi r^2 = \pi \times (8)^2 + \pi(6)^2$$

$$\pi r^2 = \pi(64 + 36)$$

$$r^2 = 100 \text{ or, } r = 10 \text{ cm}$$

Diameter of the circle = $2 \times 10 = 20$ cm.

13. The curved surface area of a cylinder is 264 m² and its volume is 924 m³. Find the ratio of its height to its diameter.

Ans : [Board Term-2 2014]

Curved Surface area of cylinder is $2\pi rh$ and volume of cylinder $\pi r^2 h$.

$$\text{Now } \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264}$$

$$\frac{r}{2} = \frac{7}{2} \Rightarrow r = 7$$

Substituting $r = 7$ in $2\pi rh = 264$ we have

$$2 \times \frac{22}{7} \times 7 \times h = 264$$

$$h = 6 \text{ m}$$

$$\text{Now } \frac{h}{2r} = \frac{6}{14} = \frac{3}{7}$$

Hence, $h : d = 3 : 7$

14. Two cubes each of volume 8 cm³ are joined end to end, then what is the surface area of resulting cuboid.

Ans : [Board Term-2 2012]

Side of the cube, $a = \sqrt[3]{8} = \sqrt{2}$ cm

Length of cuboid, $l = 4$ cm

Breadth, $b = 2$ cm

Height, $h = 2$ cm

$$\begin{aligned} \text{Surface area of cuboid} &= 2(l \times b + b \times h + h \times l) \\ &= 2(4 \times 2 + 2 \times 2 + 2 \times 4) \\ &= 2 \times 20 = 40 \text{ cm}^2 \end{aligned}$$

15. Find median of the data, using an empirical relation when it is given that Mode = 12.4 and Mean = 10.5.

Ans : [Board Term-1, 2015]

Mode, $M_o = 12.4$
 Mean, $M = 10.5$
 Median, $M_d = \frac{1}{3}M + \frac{2}{3}M_o$
 $= \frac{1}{3}(12.4) + \frac{2}{3}(10.5)$
 $= \frac{12.4}{3} + \frac{21}{3}$
 $= \frac{12.4 + 21}{3} = \frac{33.4}{3}$
 $= \frac{33.4}{3} = 11.13$



Based on this model answer the following questions:

- (i) When no barrels are produce what is a profit loss?
 (a) Rs 22000 (b) Rs 66000
 (c) Rs 11000 (d) Rs 33000
- (ii) What is the break even point ? (Zero profit point is called break even)
 (a) 10 barrels (b) 30 barrels
 (c) 20 barrels (d) 100 barrels
- (iii) What is the profit/loss if 175 barrels are produced
 (a) Profit 266200 (b) Loss 266200
 (c) Profit 240250 (d) Loss 240250
- (iv) What is the profit/loss if 400 barrels are produced
 (a) Profit Rs 466200 (b) Loss Rs 266000
 (c) Profit Rs 342000 (d) Loss Rs 342000
- (v) What is the maximum profit which can manufacturer earn?
 (a) Rs 240250 (b) Rs 480500
 (c) Rs 680250 (d) Rs 240250



16. Consider the following frequency distribution of the heights of 60 students of a class

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
Number of students	15	13	10	8	9	5

What is the upper limit of the median class in the given data?

Ans : [Board 2020 SQP Standard]

We prepare the following cumulative table

Height x (in cm)	Number of Students (f)	cf
150-155	15	15
155-160	13	28
160-165	10	38
165-170	08	46
170-175	09	55
175-180	08	63
	$N = 63$	

We have, $N = 63; \frac{N}{2} = \frac{63}{2} = 31.5$

The cumulative frequency just greater than $\frac{N}{2}$ is 38 and the corresponding class is 160-165. Thus upper limit is 165.



Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. An barrels manufacturer can produce up to 300 barrels per day. The profit made from the sale of these barrels can be modelled by the function $P(x) = -10x^2 + 3500x - 66000$ where $P(x)$ is the profit in rupees and x is the number of barrels made and sold.

Ans :

(i) When no barrels are produced, $x = 0$

$$P(x) = 0 + 0 - 66000$$

$$P(x) = -66000 \text{ Rs}$$

Thus (b) is correct option.

(ii) At break-even point $P(x) = 0$, thus

$$0 = -10x^2 + 3500x - 66000$$

$$x^2 + 350x + 6600 = 0$$

$$x^2 - 330x - 20x + 6600 = 0$$

$$x(x - 330) - 20(x + 330) = 0$$

$$(x - 330)(x - 20) = 0$$

$$x = 20, 330$$

Thus (c) is correct option.

$$(iii) P(175) = -10(175)^2 + 3500(175) - 66000 = 240250$$

Thus (c) is correct option.

$$(iv) P(400) = -10(400)^2 + 3500(400) - 66000 = -266000 \text{ Rs}$$

Thus (b) is correct option.

(v) Rearranging the given equation we have

$$P(x) = -10x^2 + 3500x - 66000$$

$$= -10(x^2 - 350x + 6600)$$

$$= -10[(x - 175)^2 - 30625 + 6600]$$

$$= -10[(x - 175)^2 - 24025]$$

$$= -10(x - 175)^2 + 240250$$

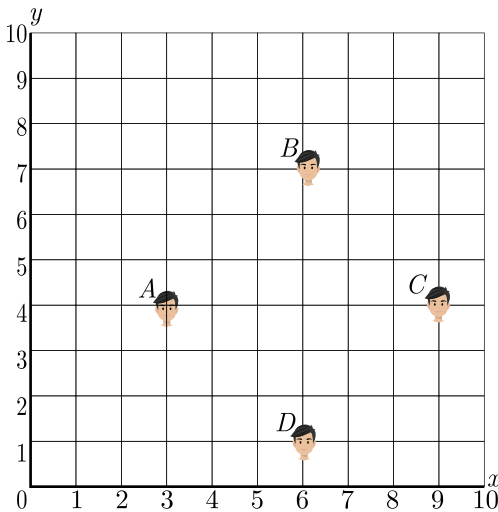
From above equation it is clear that maximum value of $P(x)$ is Rs 240250.

Thus (a) is correct option.

18. Morning assembly is an integral part of the school's schedule. Almost all the schools conduct morning assemblies which include prayers, information of latest happenings, inspiring thoughts, speech, national anthem, etc. A good school is always particular about their morning assembly schedule. Morning assembly is important for a child's development. It is essential to understand that morning assembly is not just about standing in long queues and singing prayers or national anthem, but it's something beyond just prayers. All the activities carried out in morning assembly by the school staff and students have a great influence in every point of life. The positive effects of attending school assemblies can be felt throughout life.



Have you noticed that in school assembly you always stand in row and column and this make a coordinate system. Suppose a school have 100 students and they all assemble in prayer in 10 rows as given below.



Here A, B, C and D are four friend Amar, Bharat, Colin and Dravid.

- (i) What is the distance between A and B ?
 - (a) 8
 - (b) 6
 - (c) $3\sqrt{3}$
 - (d) $2\sqrt{3}$
- (ii) What is the distance between C and D ?
 - (a) 8
 - (b) 6
 - (c) $3\sqrt{3}$
 - (d) $2\sqrt{3}$
- (iii) What is the distance between A and C ?
 - (a) 8
 - (b) 6
 - (c) $3\sqrt{3}$
 - (d) $2\sqrt{3}$
- (iv) What is the distance between D and B ?
 - (a) 8
 - (b) 6
 - (c) $3\sqrt{3}$
 - (d) $2\sqrt{3}$

- (v) These 4 friends seating arrangement make a
 - (a) square
 - (b) rhombus
 - (c) parallelogram
 - (d) rectangle

Ans :

- (i) We have $A(3, 4)$ and $B(6, 7)$.

$$AB = \sqrt{(6 - 3)^2 + (7 - 4)^2} = 2\sqrt{3}$$

Thus (d) is correct option.

- (ii) We have $C(9, 4)$ and $D(6, 1)$.

$$CD = \sqrt{(9 - 6)^2 + (4 - 1)^2} = 2\sqrt{3}$$

Thus (d) is correct option.

- (iii) We have $A(3, 4)$ and $C(9, 4)$

$$AC = \sqrt{(3 - 9)^2 + (4 - 4)^2} = 6$$

Thus (b) is correct option.

- (iv) We have $B(6, 7)$ and $D(6, 1)$.

$$BD = \sqrt{(6 - 6)^2 + (7 - 1)^2} = 6$$

Thus (b) is correct option.

- (v) We can also calculate $BC = 2\sqrt{3}$ and $AD = 2\sqrt{3}$

Here, $AB = BC = CD = AD$

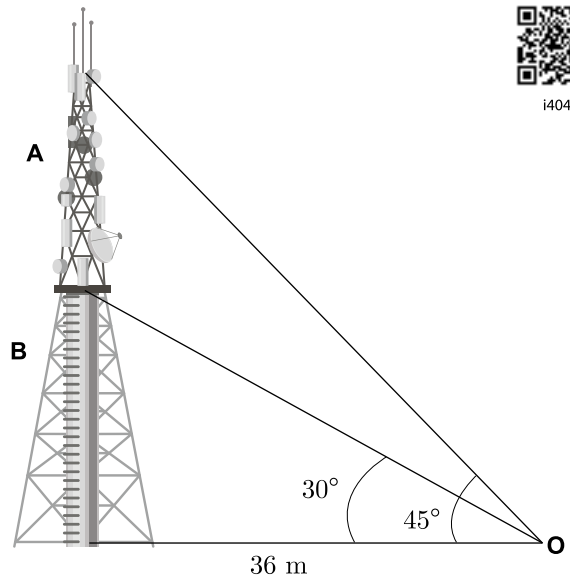
and $AC = BD$

Thus it is a square.

Thus (a) is correct option.

19. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure, including microwave dishes. They are among the tallest human-made structures. There are 2 main types: guyed and self-supporting structures.

On a similar concept, a radio station tower was built in two sections A and B . Tower is supported by wires from a point O . Distance between the base of the tower and point O is 36 m. From point O , the angle of elevation of the top of section B is 30° and the angle of elevation of the top of section A is 45° .

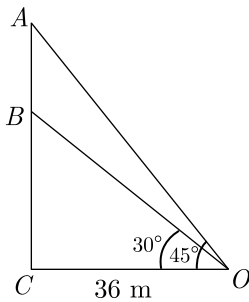


- (i) What is the height of the section B ?
 - (a) $12\sqrt{3}$ m
 - (b) $12\sqrt{2}$ m
 - (c) $8\sqrt{3}$ m
 - (d) $4\sqrt{2}$ m
- (ii) What is the height of the section A ?
 - (a) $12(2 - \sqrt{2})$
 - (b) $24(2 - \sqrt{2})$
 - (c) $12(3 - \sqrt{3})$
 - (d) $24(3 - \sqrt{3})$

- (iii) What is the length of the wire structure from the point O to the top of section A ?
 (a) $32\sqrt{2}$ m (b) $24\sqrt{3}$ m
 (c) $28\sqrt{3}$ m (d) $36\sqrt{2}$ m
- (iv) What is the length of the wire structure from the point O to the top of section B ?
 (a) $12\sqrt{3}$ m (b) $24\sqrt{3}$ m
 (c) $28\sqrt{3}$ m (d) $16\sqrt{3}$ m
- (v) What is the angle of depression from top of tower to point O ?
 (a) 30° (b) 45°
 (c) 15° (d) 75°

Ans :

(i) We make the following diagram as per given information.



In ΔBCO $\tan 30^\circ = \frac{BC}{OC}$
 $BC = OC \tan 30^\circ$
 $BC = 36 \times \frac{1}{\sqrt{3}} = 12\sqrt{3}$ m

Thus (a) is correct option.

(ii) In ΔACO ,
 $\tan 45^\circ = \frac{AC}{OC} = 1$
 Thus $AC = OC = 36$ m
 Now, $AB = AC - BC$
 $\qquad\qquad\qquad = 36 - 12\sqrt{3} = 12(3 - \sqrt{3})$ m

Thus (c) is correct option.

(iii) In ΔACO ,
 $\cos 45^\circ = \frac{OC}{OA}$
 $\frac{1}{\sqrt{2}} = \frac{36}{OA}$
 $OA = 36\sqrt{2}$ m

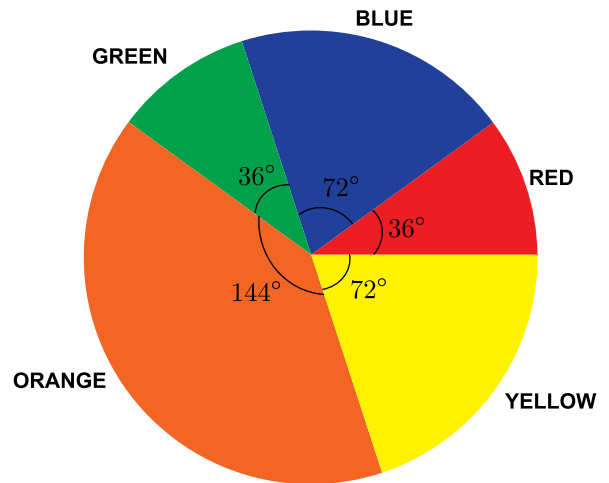
Thus (d) is correct option.

(iv) In ΔBCO ,
 $\cos 30^\circ = \frac{OC}{OB}$
 $\frac{\sqrt{3}}{2} = \frac{36}{OB}$
 $OB = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 24\sqrt{3}$ m

Thus (b) is correct option.

(v) It is clear from figure that angle of elevation from point O to top of tower is 45° . This is equal to the angle of depression from top of tower to point O . Thus (b) is correct option.

20. A survey was taken at a high school, and the results were put in a circle graph. The students were asked to list their favourite colours. The measurement of each central angle is shown. If a person is chosen at random from the school, find the probability of each response.



- (i) What is the probability of favourite colour being red ?
 (a) 0.1 (b) 0.2
 (c) 0.3 (d) 0.4
- (ii) What is the probability of favourite colour being blue or green ?
 (a) 0.1 (b) 0.2
 (c) 0.3 (d) 0.4
- (iii) What is the probability of favourite colour not being red or blue?
 (a) 0.35 (b) 0.70
 (c) 0.15 (d) 0.50
- (iv) What is the probability of favourite colour not being orange or green ?
 (a) 0.65 (b) 0.75
 (c) 0.25 (d) 0.50
- (v) What is the probability of favourite colour being red or blue?
 (a) 0.2 (b) 0.3
 (c) 0.1 (d) 0.4



Ans :

(i) Probability = $\frac{\text{Area of region}}{\text{Area of circle}}$
 $\qquad\qquad\qquad = \frac{\text{Total Angle in region}}{360^\circ}$
 $P(\text{red}) = \frac{36^\circ}{360^\circ} = \frac{1}{10} = 0.1$

Thus (a) is correct option.

(ii) $P(\text{blue or green}) = \frac{72^\circ + 36^\circ}{360^\circ} = \frac{108^\circ}{360^\circ} = \frac{3}{10} = 0.3$
 Thus (c) is correct option.

(iii)
 $P(\text{not red or blue}) = 1 - P(\text{red or blue})$
 $\qquad\qquad\qquad = 1 - \frac{36^\circ + 72^\circ}{360^\circ} = 1 - \frac{108^\circ}{360^\circ}$
 $\qquad\qquad\qquad = 1 - \frac{3}{10} = 1 - 0.3 = 0.7$

Thus (b) is correct option.

(iv) $P(\text{not orange or green}) = 1 - P(\text{orange or green})$
 $= 1 - \frac{144^\circ + 36^\circ}{360^\circ} = 1 - \frac{180^\circ}{360^\circ}$
 $= 1 - \frac{1}{2} = 0.5$

Thus (d) is correct option.

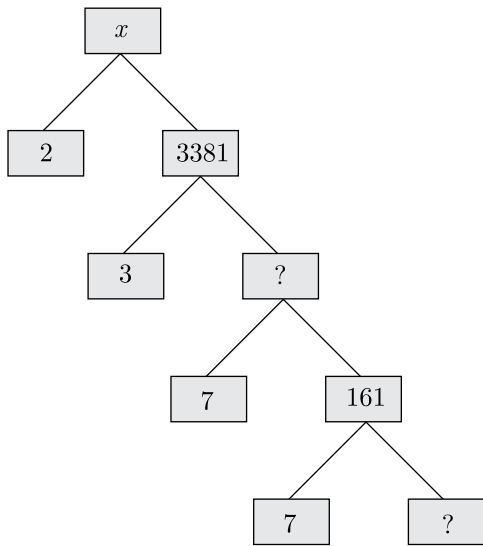
(v) $P(\text{red or blue}) = 1 - P(\text{not red or blue})$
 $= 1 - 0.7 = 0.3$

Thus (b) is correct option.

Part - B

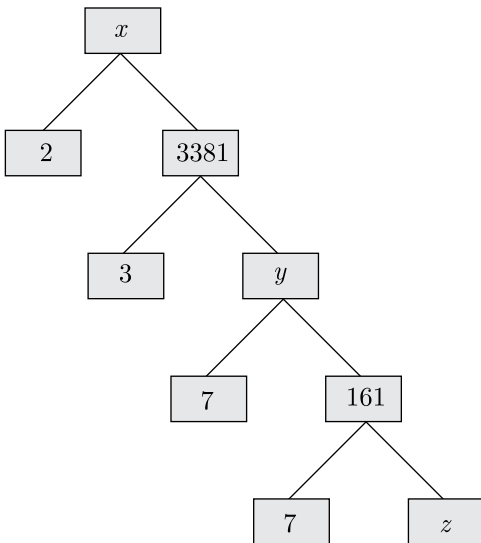
All questions are compulsory. In case of internal choices, attempt anyone.

21. Complete the following factor tree and find the composite number x .



Ans : [Board Term-1 2015, 2014]

We complete the given factor tree writing variable y and z as following.



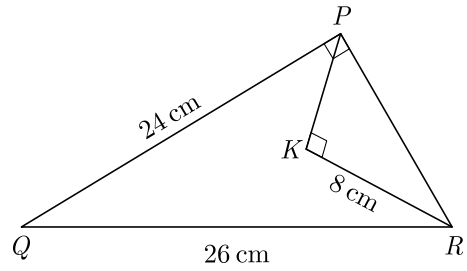
We have

$$z = \frac{161}{7} = 23$$

$$y = 7 \times 161 = 1127$$

Composite number, $x = 2 \times 3381 = 6762$

22. In the given triangle PQR , $\angle QPR = 90^\circ$, $PQ = 24$ cm and $QR = 26$ cm and in $\triangle PKR$, $\angle PKR = 90^\circ$ and $KR = 8$ cm, find PK .



Ans : [Board Term-1 2012]

In the given triangle we have

$$\angle QPR = 90^\circ$$

Thus

$$QR^2 = QP^2 + PR^2$$

$$PR = \sqrt{26^2 - 24^2}$$

$$= \sqrt{100} = 10 \text{ cm}$$

Now

$$\angle PKR = 90^\circ$$

Thus

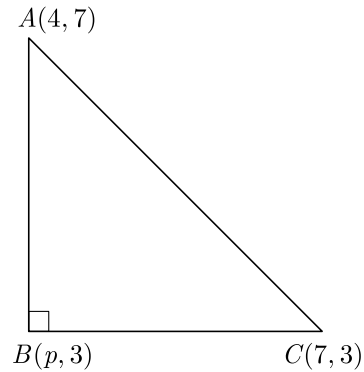
$$PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$

$$= \sqrt{36} = 6 \text{ cm}$$

23. The points $A(4, 7)$, $B(p, 3)$ and $C(7, 3)$ are the vertices of a right triangle, right-angled at B . Find the value of p .

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below. Here $\triangle ABC$ is a right angle triangle,



$$AB^2 + BC^2 = AC^2$$

$$(p - 4)^2 + (3 - 7)^2 + (7 - p)^2 + (3 - 3)^2$$

$$= (7 - 4)^2 + (3 - 4)^2$$

$$(p - 4)^2 + (-4)^2 + (7 - p)^2 + 0 = (3)^2 + (-4)^2$$

$$p^2 - 8p + 16 + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

$$(p - 4)(p - 7) = 0$$

$$p = 7 \text{ or } 4$$

or

Show that the points (a, a) , $(-a, -a)$ and $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of an equilateral triangle.

Ans : [Board Term-2 Foreign 2015]

Let $A(a, a)$, $B(-a, -a)$ and $C(-\sqrt{3}a, \sqrt{3}a)$.

Now $AB = \sqrt{(a+a)^2 + (a+a)^2}$
 $= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}a$



g125

$BC = \sqrt{(-a+\sqrt{3}a)^2 + (-a-\sqrt{3}a)^2}$
 $= \sqrt{a^2 - 2\sqrt{3}a^2 + 3a^2 + a^2 + 2\sqrt{3}a^2 + 3a^2}$
 $= 2\sqrt{2}a$

$AC = \sqrt{(a+\sqrt{3}a)^2 + (a-\sqrt{3}a)^2}$
 $= \sqrt{a^2 + 2\sqrt{3}a^2 + 3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2}$
 $= 2\sqrt{2}a$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

24. Find the value of θ , if,

$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4; \theta \leq 90^\circ$

Ans : [Board Term-1 2015]

We have $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

$\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$

$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin^2\theta} = 4$

$\frac{\cos\theta(2)}{\cos^2\theta} = 4$

$\frac{1}{\cos\theta} = 2$

$\cos\theta = \frac{1}{2}$

$\cos\theta = \cos 60^\circ$

Thus $\theta = 60^\circ$.

or

Prove that : $-1 + \frac{\sin A \sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$

Ans : [Board Term-1 2012]

$-1 + \frac{\sin A \sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$

$\frac{\sin A \sin(90^\circ - A)}{\cot(90^\circ - A)} = 1 - \sin^2 A$

$\frac{\sin A \cos A}{\tan A} = \cos^2 A$

$\frac{\sin A \cos A}{\frac{\sin A}{\cos A}} = \cos^2 A$

$\frac{\cos A}{\sin A} \sin A \cos A = \cos^2 A$

$\cos^2 A = \cos^2 A$ Hence Proved.

25. A cylinder and a cone have base radii 5 cm and 3 cm respectively and their respective heights are 4 cm and 8 cm. Find the ratio of their volumes.

Ans : [Board Term-2 2012]

Volume of cylinder,

$\pi r^2 h = \pi(5)^2 \times 4 \text{ cm}^3$
 $= 100\pi \text{ cm}^3$



m124

Volume of cone,

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 3^2 \times 8$
 $= 24\pi$

Required ratio = $100\pi : 24\pi$

= $25 : 6$.

26. Find the mode of the following frequency distribution.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	8	10	10	16	12	6	7

Ans : [Board 2019 Delhi]

Class 30-40 has the maximum frequency 16, therefore this is modal class.

We have $l = 30, f_0 = 10, f_1 = 16, f_2 = 12, h = 10$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$
 $= 30 + \left(\frac{16 - 10}{2 \times 16 - 10 - 12}\right) \times 10$
 $= 30 + \left(\frac{6}{32 - 22}\right) \times 10$
 $= 30 + \left(\frac{6}{10}\right) \times 10$
 $= 30 + 6 = 36$



h159



n169

27. Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Ans :

Finding prime factor of given number we have,

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

HCF(16, 36) = $2 \times 2 = 4$

LCM (16, 36) = $2^4 \times 3^2$

= $16 \times 9 = 144$



a132

Check :

$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$

or, $4 \times 144 = 16 \times 36$

$576 = 576$

Thus LHS = RHS

or

Find the HCF and LCM of 510 and 92 and verify that $\text{HCF} \times \text{LCM} = \text{Product of two given numbers}$.

Ans : [Board Term-1 2011]

Finding prime factor of given number we have,

$92 = 2^2 \times 23$

$510 = 30 \times 17 = 2 \times 3 \times 5 \times 17$

HCF (510, 92) = 2

LCM (510, 92) = $2^2 \times 23 \times 3 \times 5 \times 17$

= 23460

$$\begin{aligned} \text{HCF } (510, 92) \times \text{LCM } (510, 92) \\ = 2 \times 23460 = 46920 \end{aligned}$$



Product of two numbers = $510 \times 92 = 46920$

Hence, $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$

28. In an election contested between A and B , A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B . If there were 1,8000 persons on the electoral roll. How many votes for B .

Ans : [Board Term-1 2012, Set-56]

Let x and y be the no. of votes for A and B respectively.

The no. of persons who did not vote is $18000 - x - y$.

We have $x = 2(18000 - x - y)$
 $3x + 2y = 36000$... (1)

and $(18000 - x - y) = 2(x - y)$
 or $3x - y = 18000$... (2)

Subtracting equation (2) from equation (1),
 $3y = 18000$
 $y = 6000$



Hence vote for B is 6000.

29. The 14th term of an AP is twice its 8th term. If the 6th term is -8 , then find the sum of its first 20 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a , common difference be d . n th term be a_n and sum of n term be S_n .

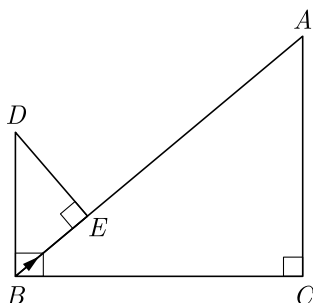
Here, $a_{14} = 2a_8$ and $a_6 = -8$
 Now $a + 13d = 2(a + 7d)$
 $a + 13d = 2a + 14d$
 $a = -d$... (1)

and $a_6 = -8$
 $a + 5d = -8$... (2)

Solving (1) and (2), we get
 $a = 2, d = -2$

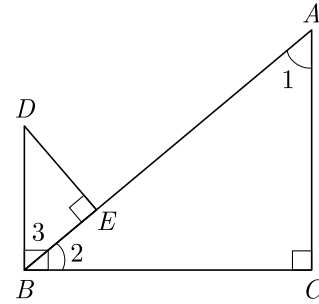
Now $S_{20} = \frac{20}{2} [2 \times 2 + (20 - 1)(-2)]$
 $= 10 [4 + 19 \times (-2)]$
 $= 10 (4 - 38)$
 $= 10 \times (-34) = -340$

30. In the given figure, $DB \perp BC, DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Ans : [Board Term-1 2011]

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In $\Delta ABC, \angle C = 90^\circ$, thus

$$\angle 1 + \angle 2 = 90^\circ$$

But we have been given,

$$\angle 2 + \angle 3 = 90^\circ$$

Hence $\angle 1 = \angle 3$

In ΔABC and ΔBDE ,

$$\angle 1 = \angle 3$$

and $\angle ACB = \angle DEB = 90^\circ$

Thus by AA similarity we have

$$\Delta ABC \sim \Delta BDE$$

Thus $\frac{AC}{BC} = \frac{BE}{DE}$. Hence Proved



31. Prove that : $(\cot \theta - \operatorname{cosec} \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

Ans : [Board Term-1 2015]

$$\cot \theta - \operatorname{cosec} \theta = \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}$$

$$(\cot \theta - \operatorname{cosec} \theta)^2 = \left(\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} \right)^2$$

$$= \left(\frac{\cos \theta - 1}{\sin \theta} \right)^2$$

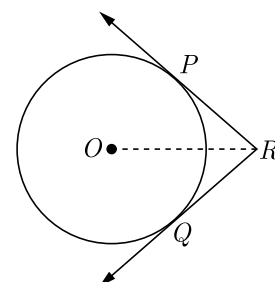
$$= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \quad [[\sin^2 \theta + \cos^2 \theta = 1]]$$

$$= \frac{(1 - \cos \theta)^2}{(1 - \cos^2 \theta)}$$

$$= \frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}$$

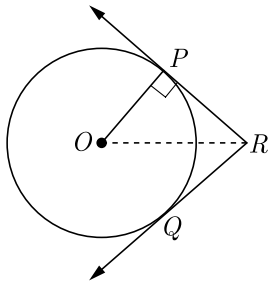
$$= \frac{1 - \cos \theta}{1 + \cos \theta} \quad \text{Hence Proved}$$

32. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O . If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Ans : [Board Term-2 OD 2015]

We redraw the given figure by joining O to P as shown below.



$$\begin{aligned} \angle PRO &= \frac{1}{2} \angle PRQ \\ &= \frac{120^\circ}{2} = 60^\circ \end{aligned}$$



Here $\triangle OPR$ is right angle triangle, thus

$$\begin{aligned} \angle POR &= 90^\circ - \angle PRO \\ &= 90^\circ - 60^\circ = 30^\circ \end{aligned}$$

Now $\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$

$$OR = 2PR = PR + PR$$

Since $PR = QR$,

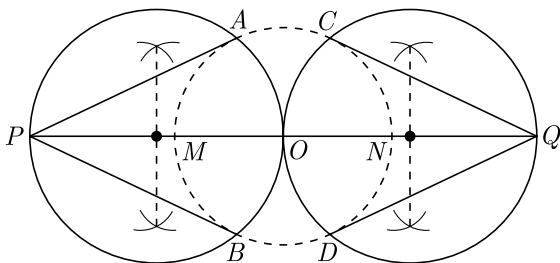
$$OR = PR + QR \quad \text{Hence Proved}$$

- 33.** Draw a circle of radius of 3 cm. Take two points P and Q one of its diameter extended on both sides, each at a distance of 7 cm on opposite sides of its centre. Draw tangents to the circle from these two points.

Ans : [Board Term-2 Foreign 2017]

Steps of Construction :

1. Draw a circle with centre O and radius 3 cm.
2. Draw its diameter MON and extend it to both the sides to P and Q . Such that $OP = OQ = 7$ cm.
3. Taking diameters as OP and OQ draw two circles each of which intersects the first circle at the points A, B and C, D respectively.
4. Join PA, PB, QC and QO to get the required tangents



or

Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

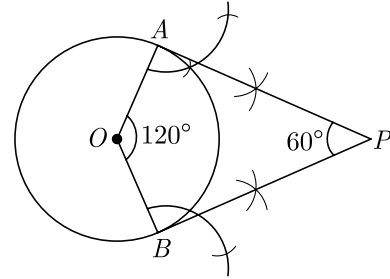
Ans : [Board Term-2 Foreign 2015, OD 2016]

Steps of Construction :

1. Draw a circle with centre O and radius 6 cm.
2. Draw two radii OA and OB inclined to each

other at an angle of 120° .

3. Draw $AP \perp OA$ at A and $BP \perp OB$ at B , which meet at P .
4. PA and PB are the required tangents inclined to each other an angle of 60° .



- 34.** Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

Ans : [Board 2019 OD]

We have $x^2 + px + 16 = 0$... (1)

If this equation has equal roots, then discriminant $b^2 - 4ac$ must be zero.

i.e., $b^2 - 4ac = 0$... (2)

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1, b = p$ and $c = 16$

Substituting above in equation (2) we have

$$p^2 - 4 \times 1 \times 16 = 0$$

$$p^2 = 64 \Rightarrow p = \pm 8$$

When $p = 8$, from equation (1) we have

$$x^2 + 8x + 16 = 0$$

$$x^2 + 2 \times 4x + 4^2 = 0$$

$$(x + 4)^2 = 0 \Rightarrow x = -4, -4$$

Hence, roots are -4 and -4 .

When $p = -8$ from equation (1) we have

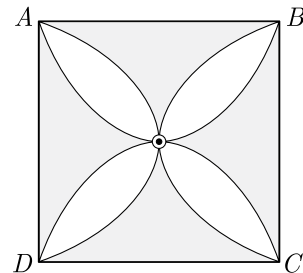
$$x^2 - 8x + 16 = 0$$

$$x^2 - 2 \times 4x + 4^2 = 0$$

$$(x - 4)^2 = 0 \Rightarrow x = 4, 4$$

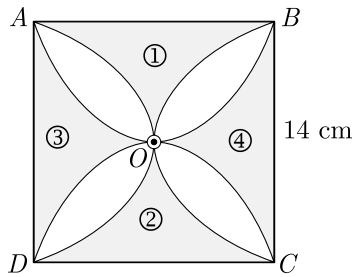
Hence, the required roots are either $-4, -4$ or $4, 4$

- 35.** In fig. $ABCD$ is a square of side 14 cm. Semi-circle are drawn with each side of square as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 Delhi 2016]

We have redrawn the given figure as shown below.



If we subtract area of two semicircle AOD and COB , from square $ABCD$ we will get area of part 1 and part 2.

Area of square = $14 \times 14 = 196 \text{ cm}^2$

Radius of semicircle = $\frac{14}{2} = 7 \text{ cm}$

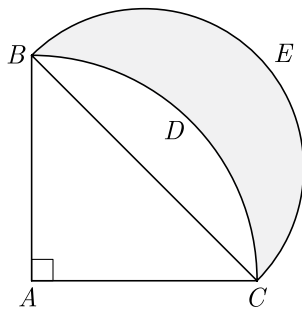
Area of semicircle $AOB + DOC$
 $= \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$

So, area of each of two shaded part
 $196 - 154 = 42 \text{ cm}^2$

Hence, area of four shaded parts is 84 cm^2 .

or

As $ABDC$ is a quadrant of a circle of radius 28 cm and a semi-circle BEC is drawn with BC as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 SQP 2017]

As ABC is a quadrant of the circle, $\angle BAC$ will be 90° .

In ΔABC , $BC^2 = AC^2 + AB^2$
 $= (28)^2 + (28)^2 = 2 \times (28)^2$
 $BC = 28\sqrt{2} \text{ cm}$

Radius of semi-circle drawn on BC ,
 $= \frac{28\sqrt{2}}{2} = 14\sqrt{2}$

Area of semi-circle = $\frac{1}{2}\pi(14\sqrt{2})^2$
 $= \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 \times 2$
 $= 616 \text{ cm}^2$

Area of $\Delta ABC = \frac{1}{2} \times 28 \times 28 = 392 \text{ cm}^2$

Area of quadrant = $\frac{1}{4} \times \frac{22}{7} \times 28 \times 28$
 $= 616 \text{ cm}^2$

Area of the shaded region
 $= \text{Area of semi-circle} + \text{area of } \Delta - \text{Area of quadrant}$
 $= 616 + 392 - 616 = 392 \text{ cm}^2$.

36. Monthly expenditures on milk in 100 families of a housing society are given in the following frequency distribution :

Monthly expenditure (in Rs.)	0-175	175-350	350-525	525-700	700-875	875-1050	1050-1125
Number of families	10	14	15	21	28	7	5

Find the mode and median for the distribution.

Ans : [Board Term-1 2016]

We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-175	10	10
157-350	14	24
350-525	15	39
525-700	21	60
700-875	28	88
875-1050	7	95
1050-1225	5	100
	$N = 100$	

We have $N = 100 ; \frac{N}{2} = 50$
 Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 525-700. Thus median class is 525-700.

Median,
 $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$
 $= 525 + \frac{50 - 39}{21} \times 175$
 $= 525 + \frac{11}{21} \times 175$
 $= 525 + 91.6$
 $= 616.6$

Class 700-875 has the maximum frequency 28, therefore this is modal class.

Here $l = 700, f_0 = 21, f_1 = 28, f_2 = 7, h = 175$

Mode,
 $M_o = l + h \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$
 $= 700 + \left(\frac{28 - 21}{2 \times 28 - 21 - 7}\right) \times 175$
 $= 700 + \frac{7}{28} \times 175$
 $= 700 + 43.75$
 $= 743.75$

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