

CLASS X (2020-21)
MATHEMATICS STANDARD (041)
SAMPLE PAPER-04

Time : 3 Hours

Maximum Marks : 80

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part-A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part-B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.



Part - A

Section - I

1. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a + b)$.

Ans : [Board Term-1 2014]

Here a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. The least prime factor of $(a + b)$ would be 2.



or

What is the HCF of the smallest composite number and the smallest prime number?

Ans : [Board Term-1 OD 2018]

The smallest prime number is 2 and the smallest composite number is $4 = 2^2$. Hence, required HCF is $(2^2, 2) = 2$.



2. The 2 digit number which becomes $\frac{5}{6}$ th of itself when its digits are reversed. If the difference in the digits of the number being 1, what is the two digits number?

Ans :

If the two digits are x and y , then the number is $10x + y$.

Now $\frac{5}{6}(10x + y) = 10y + x$



Solving, we get $44x + 55y$

$$\frac{x}{y} = \frac{5}{4}$$

Also $x - y = 1$. Solving them, we get $x = 5$ and $y = 4$. Therefore, number is 54.

3. What are the values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots?

Ans :

We have $2x^2 - kx + k = 0$
Comparing with $ax^2 + bx + c = 0$ we $a = 2$, $b = -k$ and $c = k$.

For equal roots, the discriminant must be zero.

Thus $b^2 - 4ac = 0$

$$(-k)^2 - 4(2)k = 0$$

$$k^2 - 8k = 0$$

$$k(k - 8) = 0 \Rightarrow k = 0, 8$$

Hence, the required values of k are 0 and 8.



or

If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then show that $a = c$.

Ans :

If one root is α , then the other $\frac{1}{\alpha}$.

Product of roots, $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{c}{a} \Rightarrow a = c$$

4. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.

Ans : [Board 2020 Delhi Standard]

Given, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$.

Common difference,

$$\begin{aligned} d_1 &= (a^2 + b^2) - (a - b)^2 \\ &= (a^2 + b^2) - (a^2 + b^2 - 2ab) \\ &= a^2 + b^2 - a^2 - b^2 + 2ab \\ &= 2ab \end{aligned}$$

and $d_2 = (a + b)^2 - (a^2 + b^2)$

$$\begin{aligned} &= a^2 + b^2 + 2ab - a^2 - b^2 \\ &= 2ab \end{aligned}$$

Since, $d_1 = d_2$, thus, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.

or

Find the sum of all 11 terms of an AP whose middle term is 30.



Ans : [Board 2020 OD Standard]

In an AP with 11 terms, the middle term is $\frac{11+1}{2} = 6^{\text{th}}$ term.

Now, $a_6 = a + 5d = 30$

Thus, $S_{11} = \frac{11}{2}[2a + 10d]$

$$= 11(a + 5d)$$

$$= 11 \times 30 = 330$$



5. In ΔABC , $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm, then $\angle B = \dots\dots\dots$

Ans : [Board 2020 OD Standard]

We have $AB = 6\sqrt{3}$ cm,

$AC = 12$ cm and

$BC = 6$ cm

Now $AB^2 = 36 \times 3 = 108$

$AC^2 = 144$

and $BC^2 = 36$

It can be easily observed that above values satisfy Pythagoras theorem,

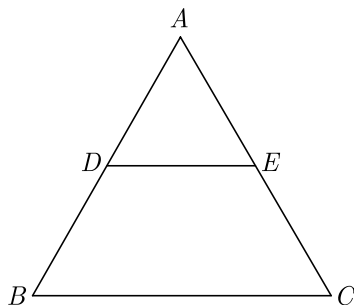
$$AB^2 + BC^2 = AC^2$$

$$108 + 36 = 144 \text{ cm}$$

Thus $\angle B = 90^\circ$



6. In given figure $DE \parallel BC$. If $AD = 3c$, $DB = 4c$ cm and $AE = 6$ cm then find EC .



Ans : [Board Term-1 2016]

In the given figure $DE \parallel BC$, thus

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{3}{4} = \frac{6}{EC}$$

$EC = 8$ cm



7. The value of the $(\tan^2 60^\circ + \sin^2 45^\circ)$ is

Ans : [Board 2020 OD Basic]

$$\tan^2 60^\circ + \sin^2 45^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3 + \frac{1}{2} = \frac{7}{2}$$



8. Evaluate $\sin^2 60^\circ - 2 \tan 45^\circ - \cos^2 30^\circ$

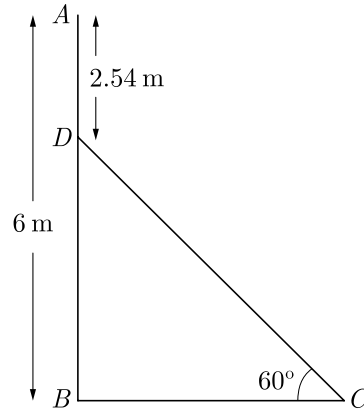
Ans : [Board 2019 OD]



$$\sin^2 60^\circ - 2 \tan 45^\circ - \cos^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{3}{4} - 2 - \frac{3}{4} = -2$$

9. In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If $AD = 2.54$ m, find the length of ladder. (use $\sqrt{3} = 1.73$)



Ans : [Board Term-2 Delhi 2016]

We have $AD = 2.54$ m

$DB = 6 - 2.54 = 3.46$ m

In ΔBCD , $\angle B = 90^\circ$

$$\sin 60^\circ = \frac{BD}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

$$DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46}{1.73} = 4$$

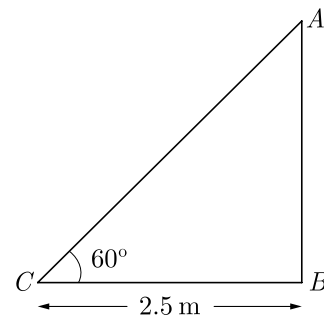
Thus length of ladder is 4 m.

or

A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Ans : [Board Term-2 2011]

As per given in question we have drawn figure below.



In ΔACB with $\angle C = 60^\circ$, we get

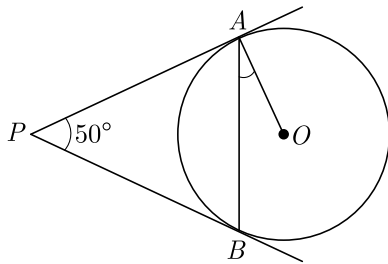
$$\cos 60^\circ = \frac{2.5}{AC}$$

$$\frac{1}{2} = \frac{2.5}{AC}$$



$$AC = 2 \times 2.5 = 5 \text{ m}$$

10. In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$. Write the measure of $\angle OAB$.



Ans : [Board Term-2 Delhi 2015]

We have $\angle APB = 50^\circ$

$$\angle PAB = \angle PBA = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

Here OA is radius and AP is tangent at A , since radius is always perpendicular to tangent at point of contact, we have

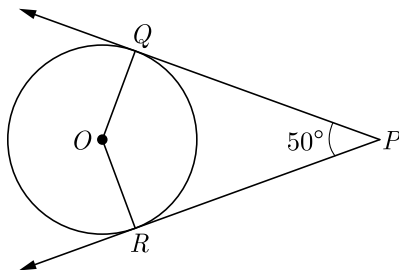
$$\angle OAP = 90^\circ$$

Now $\angle OAB = \angle OAP - \angle PAB$
 $= 90^\circ - 65^\circ = 25^\circ$



j148

11. In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 50^\circ$. Then find $\angle OQR$.



Ans : [Board Term-2 Delhi 2012, 2015]

We have $\angle QPR = 50^\circ$ (Given)

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\begin{aligned} \angle QOR + \angle QPR &= 180^\circ \\ \angle QOR &= 180^\circ - \angle QPR \\ &= 180^\circ - 50^\circ = 130^\circ \end{aligned}$$



j152

From ΔOQR we have

$$\begin{aligned} \angle OQR &= \angle ORQ = \frac{180^\circ - 130^\circ}{2} \\ &= \frac{50^\circ}{2} = 25^\circ \end{aligned}$$

12. If the circumference of a circle increases from 4π to 8π , then what about its area ?

Ans : [Board Term-2 Delhi 2013]

Circumference of the circle

$$2\pi r = 4\pi \text{ cm or } r = 2 \text{ cm.}$$

Increased circumference



i121

$$2\pi R = 8\pi \text{ cm or } R = 4 \text{ cm.}$$

Area of the 1st circle

$$\pi r^2 = \pi \times (2)^2 = 4\pi \text{ cm}$$

Area of the new circle

$$\pi R^2 = \pi(4)^2 = 16\pi = 4 \times 4\pi$$

Area of the new circle = 4 times the area of first circle.

or

If the radius of the circle is 6 cm and the length of an arc 12 cm. Find the area of the sector.

Ans : [Board Term-2 2014]

Area of the sector = $\frac{1}{2} \times (\text{length of the corresponding arc}) \times \text{radius}$

$$\begin{aligned} &= \frac{1}{2} \times l \times r = \frac{1}{2} \times 12 \times 6 \\ &= 36 \text{ cm}^2 \end{aligned}$$



h122

13. A rectangular sheet paper $40 \text{ cm} \times 22 \text{ cm}$ is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder.

Ans : [Board Term-2 Foreign 2014]

Here, $h = 40 \text{ cm}$, circumference = 22 cm

$$2\pi r = 22$$

$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} = 3.5 \text{ cm}$$



m102

14. The radius of sphere is $r \text{ cm}$. It is divided into two equal parts. Find the whole surface of two parts.

Ans : [Board Term-2 2012]

Whole surface of each part

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Total surface of two parts

$$= 2 \times 3\pi r^2 = 6\pi r^2$$



m106

15. Consider the following distribution :

M a r k s Obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
N u m b e r o f students	63	58	55	51	48	42

(i) Calculate the frequency of the class 30 - 40.

(ii) Calculate the class mark of the class 10 - 25.

Ans : [Board Term-1, 2014]

Class Interval	c.f.	f
0-10	63	5
10-20	58	3
20-30	55	4
30-40	51	3
40-50	48	6
50-60	42	42

(i) Frequency of the class 30 - 40 is 3.



n157

(ii) Class mark of the class : $10 - 25 = \frac{10 + 25}{2}$
 $= \frac{35}{2} = 17.5$

16. For finding the popular size of readymade garments, which central tendency is used?

Ans :

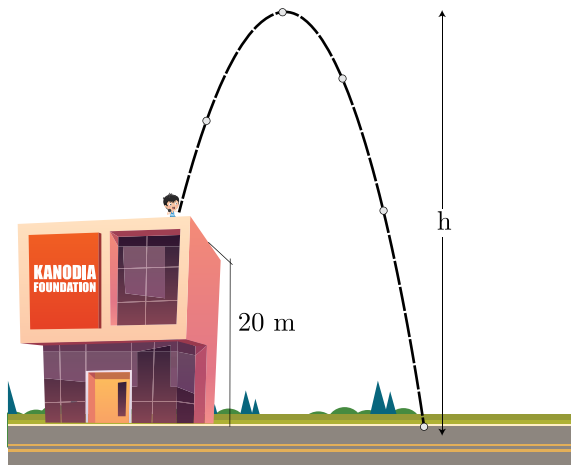
For finding the popular size of ready made garments, mode is the best measure of central tendency.



Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. Lavanya throws a ball upwards, from a rooftop, which is 20 m above from ground. It will reach a maximum height and then fall back to the ground. The height of the ball from the ground at time t is h , which is given by $h = -4t^2 + 16t + 20$.



(i) What is the height reached by the ball after 1 second?

- (a) 64 m
- (b) 128 m
- (c) 32 m
- (d) 20 m



(ii) What is the maximum height reached by the ball?

- (a) 54 m
- (b) 44 m
- (c) 36 m
- (d) 18 m

(iii) How long will the ball take to hit the ground?

- (a) 4 seconds
- (b) 3 seconds
- (c) 5 seconds
- (d) 6 seconds

(iv) What are the two possible times to reach the ball at the same height of 32 m?

- (a) 1 and 3 seconds
- (b) 1 and 4 seconds
- (c) 1 and 2 seconds
- (d) 1 and 5 seconds

(v) Where is the ball after 5 seconds ?

- (a) at the ground
- (b) rebounds
- (c) at highest point
- (d) fall back

Ans :

(i) Height is given by,

$$h = -4t^2 + 16t + 20$$

At $t = 1$ second,

$$h = -4(1)^2 + 16(1) + 20 = 32 \text{ m}$$

Thus (c) is correct option.

(ii) Rearranging the given equation, by completing the square,

$$\begin{aligned} h &= -4(t^2 - 4t - 5) \\ &= -4(t^2 - 4t + 4 - 4 - 5) \\ &= -4[(t - 2)^2 - 9] \\ &= -4(t - 2)^2 + 36 \end{aligned}$$

Height is maximum, at $t = 2$, thus

$$h_{\text{max}} = 0 + 36 = 36 \text{ m}$$

Thus (c) is correct option.

(iii) When ball hits the ground, $h = 0$, thus

$$\begin{aligned} -4t^2 + 16t + 20 &= 0 \\ t^2 - 4t - 5 &= 0 \\ (t - 5)(t + 1) &= 0 \end{aligned}$$

Thus $t = 5$ or $t = -1$. Since, time cannot be negative, the $t = 5$ seconds is correct answer.

Thus (c) is correct option.

(iv) Since,

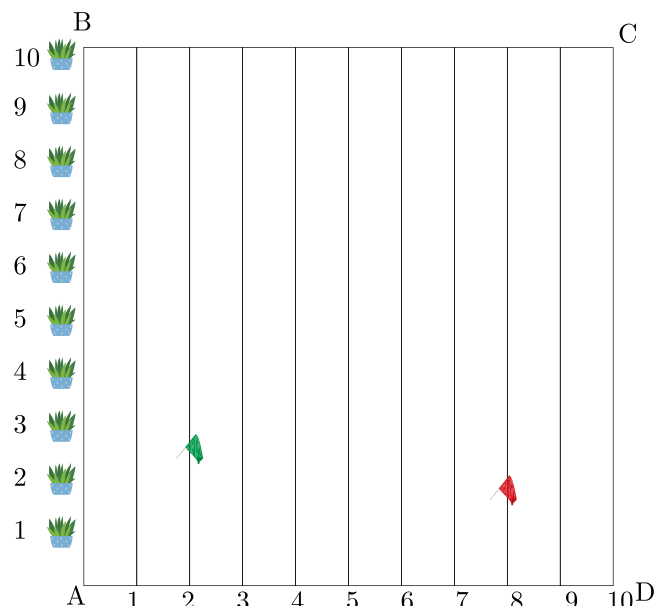
$$\begin{aligned} h &= -4t^2 + 16t^2 + 20 \\ 32 &= -4t^2 + 16t^2 + 20 \\ 8 &= -t^2 + 4t^2 + 5 \\ t^2 - 4t + 3 &= 0 \\ t^2 + 3t - t + 3 &= 0 \\ (t - 1)(t - 3) &= 0 \Rightarrow t = 3, 1 \end{aligned}$$

Thus (a) is correct option.

(v) From (iii) at $t = 5$ we have $h = 0$. Thus it will hit ground, then after that ball will rebound.

Thus (b) is correct option.

18. To conduct sports day activities, in a rectangular shaped school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AB , as shown in figure. Nishtha runs $\frac{1}{4}$ th the distance AB on the 2nd line and posts a green flag. Suman runs $\frac{1}{5}$ th the distance AB on the 8th line and posts a red flag.



- (i) What is the position of green flag ?
 (a) (2, 25) (b) (25, 4)
 (c) (25,2) (d) (4, 25)
- (ii) What is the position of red flag ?
 (a) (20, 4) (b) (8, 20)
 (c) (20, 8) (d) (4, 20)
- (iii) What is the distance between both the flags?
 (a) $\sqrt{51}$ (b) $3\sqrt{3}$
 (c) $\sqrt{61}$ (d) $2\sqrt{3}$
- (iv) What is the distance of red flag from point A ?
 (a) $4\sqrt{29}$ (b) $2\sqrt{29}$
 (c) $8\sqrt{15}$ (d) $16\sqrt{3}$
- (v) If Rakhi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?
 (a) (20, 4) (b) (22.5, 5)
 (c) (4, 20) (d) (5, 22.5)



Ans :

(i) $\frac{1}{4}$ th of the AD corresponds to $y = \frac{100}{4} = 25$ and 2nd line corresponds to $x = 2$. Thus coordinates of green flag point are (2, 25).

Thus (a) is correct option.

(ii) $\frac{1}{5}$ th of the AD corresponds to $y = \frac{100}{5} = 20$ and 8th line corresponds to $x = 8$. Thus coordinates of red point are (8, 20).

Thus (b) is correct option.

$$\begin{aligned} \text{(iii)} \quad d &= \sqrt{(8-2)^2 + (20-25)^2} \\ &= \sqrt{6^2 + 5^2} = \sqrt{61} \end{aligned}$$

Thus (c) is correct option.

$$\text{(iv)} \quad d = \sqrt{8^2 + 20^2} = 4\sqrt{29}$$

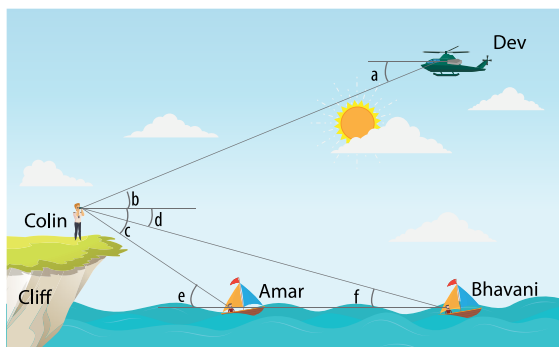
Thus (a) is correct option.

(v) Mid point of green flag and red flag

$$= \left(\frac{2+8}{2}, \frac{25+20}{2} \right) = (5, 22.5)$$

Thus (d) is correct option.

19. Navy officer Mr. Colin is tasked with planning a coup on the enemy at a certain date. Currently he is inspecting the area standing on top of the cliff. Agent Dev is on a chopper in the sky. When Mr. Colin looks down below the cliff towards the sea, he has Bhawani and Amar in boats positioned to get a good vantage point. Bhawani boat is behind the Amar boat.



Following angle have been measured :

From Colin to Bhawani : 30°

From Dev to Colin : 60°

From Amar to Colin : 60°

- (i) Which of the following is a pair of angle of elevation?
 (a) $(\angle a, \angle e)$ (b) $(\angle b, \angle e)$
 (c) $(\angle c, \angle d)$ (d) $(\angle a, \angle f)$
- (ii) Which of the following is a pair of angle of depression?
 (a) $(\angle a, \angle e)$ (b) $(\angle b, \angle e)$
 (c) $(\angle c, \angle d)$ (d) $(\angle a, \angle f)$
- (iii) If angle of elevation of Amar to Colin is 60° , what is the distance of Amar boat from the base of hill ?
 (a) $\frac{\sqrt{3}h}{2}$ (b) $\frac{h}{\sqrt{3}}$
 (c) $\frac{2h}{\sqrt{3}}$ (d) $\sqrt{3}h$
- (iv) If angle of depression of Colin to Bhawani is 30° , what is the distance of Amar boat from the Bhawani boat?
 (a) $\frac{\sqrt{3}h}{2}$ (b) $\frac{h}{\sqrt{3}}$
 (c) $\frac{2h}{\sqrt{3}}$ (d) $\sqrt{3}h$
- (v) If angle of depression of Dev to Colin is 60° , what is the height of Dev from base of hill ?
 (a) h (b) $2h$
 (c) $3h$ (d) $4h$

Ans :

(i) The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight). In our case clearly $(\angle b, \angle e, \angle f)$ are angle of depression.

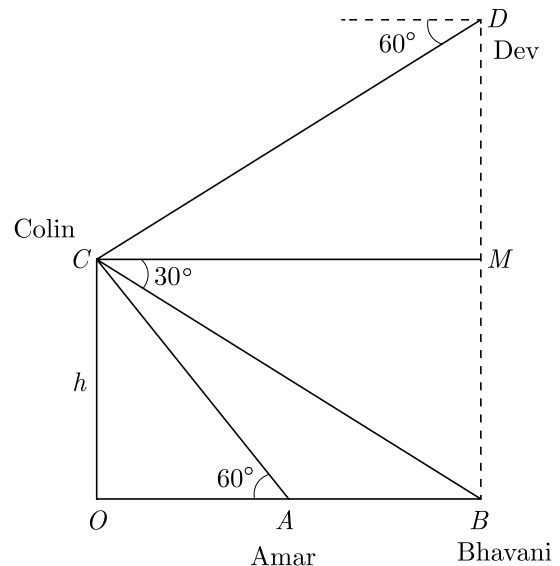
Thus (b) is correct option.

(ii) If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression. In our case clearly $(\angle a, \angle c, \angle d)$ are angle of depression.

Thus (c) is correct option.

(iii) We make the figure as given below.

Here $\angle OAC = 60^\circ$ is angle of elevation.



$$\frac{h}{OA} = \tan 60^\circ = \sqrt{3}$$

$$OA = \frac{h}{\sqrt{3}}$$

Thus (b) is correct option.

(iv) Here $\angle OBC = \angle MCB = 30^\circ$

$$\frac{h}{OB} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$OB = \sqrt{3}h$$

$$AB = OB - OA$$

$$= \sqrt{3}h - \frac{h}{\sqrt{3}} = \frac{1}{\sqrt{3}}(3h - h) = \frac{2h}{\sqrt{3}}$$

Thus (c) is correct option.

(v) Here $\angle DCM = 60^\circ$

Now,
$$\frac{DM}{CM} = \tan 60^\circ = \sqrt{3}$$

$$DM = \sqrt{3} CM$$

But
$$CM = OB = \sqrt{3}h$$

Thus
$$DM = \sqrt{3} \cdot \sqrt{3}h = 3h$$

Height of Dev from Bhawani,

$$= DB = DM + MB = 3h + h = 4h$$

Thus (d) is correct option.

20. A game at a stall in new year carnival involves spinning a wheel first as a first step to complete the game with certain rules. If the wheel stops at a particular number, then the player is allowed to roll a 6 faced unbiased dice.

Rules of Game:

1. If the wheel stops at a particular number, then the player is allowed to roll a unbiased dice.
2. If the wheel stops at any other number, player get to try again and only one extra try allowed.
If player reach the next stage and roll a dice, he may get a prize depending on the number on dice.



(i) What is the probability of getting an even number on the wheel?

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{8}$ (d) $\frac{1}{16}$



(ii) If getting an odd number on the wheel allows a player to roll the die, then what is the probability of his rolling the die ?

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(iii) If the player is allowed to roll the dice and getting a number greater than 4 entitles him to get prize, what is the probability of his winning the prize?

- (a) $\frac{3}{4}$ (b) $\frac{1}{6}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(iv) If getting a square number on the wheel allows a player to roll the dice, then what is the probability of his rolling the dice ?

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(v) If the player is allowed to roll the die and getting a prime number on die entitles him to get prize, then what is the probability of his winning the prize?

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

Ans :

(i) Total outcomes $n(S) = 8$
Favourable outcome are $\{2, 4, 6, 8\}$, therefore

$$n(E) = 4$$

Probability of getting an even number on the wheel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Thus (b) is correct option.

(ii) Total outcomes $n(S) = 8$
Favourable outcome are $\{3, 5, 7, 9\}$, therefore

$$n(E) = 4$$

Probability of getting an odd number on the wheel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Thus (b) is correct option.

(iii) Dice has total six outcome, thus total outcome

$$n(S) = 6$$

Number greater than 4 on dice are $\{5, 6\}$, therefore

$$n(E) = 2$$

Probability of getting a number greater than 4,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Thus (c) is correct option.

(iv) Total outcomes $n(S) = 8$
Favourable outcome are $\{4, 9\}$, therefore

$$n(E) = 2$$

Probability of getting a square number on the wheel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{4}$$

Thus (a) is correct option.

(v) Dice has total six outcome, thus total outcome

$$n(S) = 6$$

Prime number on dice are $\{2, 3, 5\}$, therefore

$$n(E) = 3$$

Probability of getting a prime number on dice,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Thus (b) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

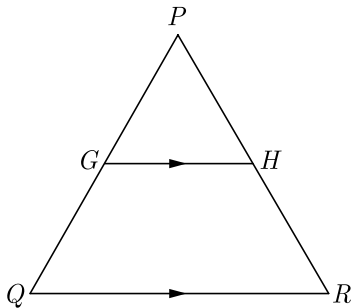
Ans : [Board Term-1 2016-17 Set; 193RQTQ, 2015, DDE-E]

A prime number (or a prime) is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1 , involve 5 itself. However, 6 is composite because it is the product of two numbers (2×3) that are both smaller than 6. Every composite number can be written as the product of two or more (not necessarily distinct) primes.

$$\begin{aligned} 3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\ &= 4(909 + 1) \\ &= 4(910) \\ &= 2 \times 2 \times (10 \times 7 \times 13) \\ &= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\ &= \text{a composite number} \end{aligned}$$



22. In the given figure, G is the mid-point of the side PQ of $\triangle PQR$ and $GH \parallel QR$. Prove that H is the mid-point of the side PR or the triangle PQR .



Ans : [Board Term-1 2012]

Since G is the mid-point of PQ we have

$$\begin{aligned} PG &= GQ \\ \frac{PG}{GQ} &= 1 \end{aligned}$$



We also have $GH \parallel QR$, thus by BPT we get

$$\begin{aligned} \frac{PG}{GQ} &= \frac{PH}{HR} \\ 1 &= \frac{PH}{HR} \end{aligned}$$

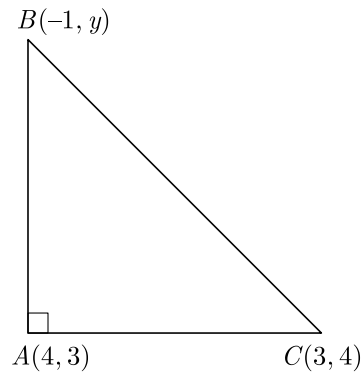
$$PH = HR. \quad \text{Hence proved.}$$

Hence, H is the mid-point of PR .

23. If $A(4,3)$, $B(-1,y)$, and $C(3,4)$ are the vertices of a right triangle ABC , right angled at A , then find the value of y .

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below.



$$\begin{aligned} \text{Now} \quad AB^2 + AC^2 &= BC^2 \\ (4+1)^2 + (3-y)^2 + (4-3)^2 &= (3+1)^2 + (4-y)^2 \\ (5)^2 + (3-y)^2 + (-1)^2 + (1)^2 &= (4)^2 + (4-y)^2 \\ 25 + 9 - 6y + y^2 + 1 + 1 &= 16 + 16 - 8y + y^2 \\ 36 + 2y - 32 &= 0 \\ 2y + 4 &= 0 \\ y &= -2 \end{aligned}$$

24. If $\sin \theta - \cos \theta = \frac{1}{2}$, then find the value of $\sin \theta + \cos \theta$.

Ans : [Board Term-1 2013]

$$\text{We have} \quad \sin \theta - \cos \theta = \frac{1}{2}$$

Squaring both sides, we get

$$(\sin \theta - \cos \theta)^2 = \left(\frac{1}{2}\right)^2$$

$$\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$1 - 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$2 \sin \theta \cos \theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\begin{aligned} \text{Again, } (\sin \theta + \cos \theta)^2 &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\ &= 1 + 2 \sin \theta \cos \theta \\ &= 1 + \frac{3}{4} = \frac{7}{4} \end{aligned}$$

$$\text{Thus} \quad \sin \theta + \cos \theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$$

25. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere.

Ans : [Board Term-2 2012]

Here diameter of sphere is equal to the radius of hemisphere which is 6 cm.

$$\begin{aligned} \text{Diameter of sphere} &= \text{Radius of hemisphere} \\ &= 6 \text{ cm} \end{aligned}$$

$$\text{Radius of sphere} = 3 \text{ cm}$$

$$\begin{aligned} \text{Volume,} \quad V &= \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3^3 \text{ cm}^3 \\ &= 113.14 \text{ cm}^3. \end{aligned}$$

or

If the total surface area of a solid hemisphere is 462 cm^2 , find its volume. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 OD 2014]

Total surface area of hemisphere,

$$3\pi r^2 = 462 \text{ cm}^2$$

$$\frac{22r^2}{7} = \frac{462}{3}$$

$$r^2 = \frac{462 \times 7}{22 \times 3} = 49$$

$$r = 7 \text{ cm.}$$

Volume of hemisphere,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{2156}{3} = 718.67 \text{ cm}^3.$$



26. The data regarding marks obtained by 48 students of a class in a class test is given below. Calculate the modal marks of students.

Marks obtained	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
Number of students	1	0	2	0	0	10	25	7	2	1

Ans : [Board Term-1, 2015]

Class 30-35 has the maximum frequency 25, therefore this is modal class.

Now $l = 30, f_1 = 25, f_0 = 10, f_2 = 7, h = 5$

$$\text{Mode } M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 30 + \frac{25 - 10}{50 - 10 - 7} \times 5$$

$$= 30 + 2.27 \text{ or } 32.27 \text{ approx.}$$

or

Find the value of λ , if the mode of the following data is 20 :

15, 20, 25, 18, 13, 15, 25, 15, 18, 17, 20, 25, 20, λ , 18.

Ans : [Board Term-1, 2015]

First we prepare the following table as discrete frequency distribution.

x_i	f_i
13	1
15	3
17	1
18	3
20	3
λ	1
25	3



Frequency of 20 must be highest to be mode of the frequency distribution, $\lambda = 20$.

27. The HCF of 65 and 117 is expressible in the form $65m - 117$. Find the value of m . Also find the LCM of 65 and 117 using prime factorization method.

Ans : [Board Term-1 2011, Set-40]

Finding prime factor of given number we have.

$$117 = 13 \times 3 \times 3$$

$$65 = 13 \times 5$$

$$\text{HCF}(117, 65) = 13$$

$$\text{LCM}(117, 65) = 13 \times 5 \times 3 \times 3 = 585$$

$$\text{HCF} = 65m - 117$$

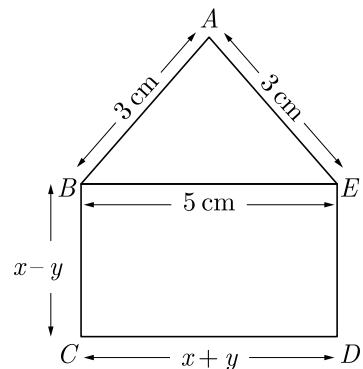
$$13 = 65m - 117$$

$$65m = 117 + 13 = 130$$

$$m = \frac{130}{65} = 2$$



28. In the figure below $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to DC . If the perimeter of $ABCDE$ is 21 cm, find the values of x and y .



Ans : [Board Term-1 2011]

Since $BC \parallel DE$ and $BE \parallel CD$ with $BC \perp DC$, $BCDE$ is a rectangle.

$$BE = CD,$$

$$x + y = 5 \tag{1}$$

and

$$DE = BE = x - y$$

Since perimeter of $ABCDE$ is 21,

$$AB + BC + CD + DE + EA = 21$$

$$3 + x - y + x + y + x - y + 3 = 21$$

$$6 + 3x - y = 21$$

$$3x - y = 15$$

Adding equations (1) and (2), we get

$$4x = 20 \tag{2}$$

$$x = 5$$

Substituting the value of x in (1), we get

$$y = 0$$

Thus $x = 5$ and $y = 0$.

or

Solve for x and y :

$$\frac{x+1}{2} + \frac{y-1}{3} = 9 ; \frac{x-1}{3} + \frac{y+1}{2} = 8.$$

Ans : [Board Term-1 2011, Set-52]

We have $\frac{x+1}{2} + \frac{y-1}{3} = 9$

$$3(x+1) + 2(y-1) = 54$$



$$3x + 3 + 2y - 2 = 54$$

$$3x + 2y = 53$$

and

$$\frac{x-1}{3} + \frac{y+1}{2} = 8$$

$$2(x-1) + 3(y+1) = 48$$

$$2x - 2 + 3y + 3 = 48$$

$$2x + 3y = 47$$

Multiplying equation (1) by 3 we have

$$9x + 6y = 159$$

Multiplying equation (2) by 2 we have

$$4x + 6y = 94$$

Subtracting equation (4) from (3) we have

$$5x = 65$$

or

$$x = 13$$

Substitute the value of x in equation (2),

$$2(13) + 3y = 47$$

$$3y = 47 - 26 = 21$$

$$y = \frac{21}{3} = 7$$

Hence, $x = 13$ and $y = 7$

29. If the sum of the first n terms of an AP is $\frac{1}{2}[3n^2 + 7n]$, then find its n^{th} term. Hence write its 20th term.

Ans : [Board Term-2 Delhi 2015]

Let the first term be a , common difference be d , n th term be a_n and sum of n term be S_n .

Sum of n term, $S_n = \frac{1}{2}[3n^2 + 7n]$

Sum of 1 term, $S_1 = \frac{1}{2}[3 \times (1)^2 + 7(1)]$

$$= \frac{1}{2}[3 + 7] = \frac{1}{2} \times 10 = 5$$

Sum of 2 term, $S_2 = \frac{1}{2}[3(2)^2 + 7 \times 2]$

$$= \frac{1}{2}[12 + 14] = \frac{1}{2} \times 26 = 13$$

Now

$$a_1 = S_1 = 5$$

$$a_2 = S_2 - S_1 = 13 - 5 = 8$$

$$d = a_2 - a_1 = 8 - 5 = 3$$

Now, AP is 5, 8, 11,

n^{th} term, $a_n = a + (n-1)d$

$$= 5 + (n-1)3$$

$$= 5 + (20-1)(3)$$

$$= 5 + 57$$

$$= 62$$

Hence,

$$a_2 = 62$$

or

In an AP, if the 12th term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a , common difference be d , n th term be a_n and sum of n term be S_n .

$$a_{12} = a + 11d = -13 \quad \dots(1)$$



c146

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now

$$S_4 = 2[2a + 3d] = 24$$

$$2a + 3d = 12 \quad \dots(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$(2a + 22d) - (2a + 3d) = -26 - 12$$

$$19d = -38$$

$$d = -2$$

Substituting the value of d in (1) we get

$$a + 11 \times -2 = -13$$

$$a = -13 + 22$$

$$a = 9$$

Now,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

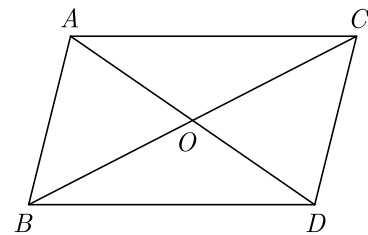
$$S_{10} = \frac{10}{2}(2 \times 9 + 9 \times -2)$$

$$= 5 \times (18 - 18) = 0$$

Hence, $S_{10} = 0$

30. In the given figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC . AD and BC intersect at O .

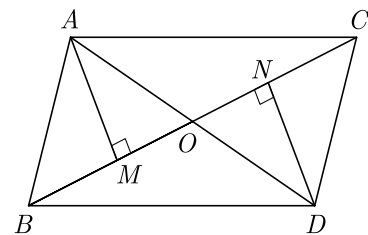
Prove that $\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AO}{DO}$.



Ans :

[Board 2020 OD Std, 2016, 2011]

As per given condition we have redrawn the figure below. Here we have drawn $AM \perp BC$ and $DN \perp BC$.



In $\triangle AOM$ and $\triangle DON$,

$$\angle AOM = \angle DON$$

(Vertically opposite angles)

$$\angle AMO = \angle DNO = 90^\circ \text{ (Construction)}$$

or,

$$\triangle AOM \sim \triangle DON \text{ (By AA similarity)}$$

Thus

$$\frac{AO}{DO} = \frac{AM}{DN} \quad \dots(1)$$

Now,

$$\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$$

$$= \frac{AM}{DN} = \frac{AO}{DO} \text{ From equation (1)}$$



e189



e188



f150

31. Prove that : $\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Ans : [Board Term-1 2013]

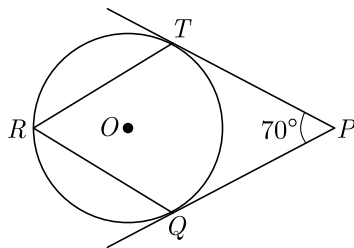
We have

$$\begin{aligned} \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} &= \operatorname{cosec}^2\theta \left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1} \right] \\ &= \operatorname{cosec}^2\theta \left[\frac{\sin\theta}{1 - \sin\theta} - \frac{\sin\theta}{1 + \sin\theta} \right] \\ &= \frac{1}{\sin^2\theta} \sin\theta \left[\frac{(1 + \sin\theta) - (1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} \right] \\ &= \frac{1}{\sin\theta} \left[\frac{2\sin\theta}{1 - \sin^2\theta} \right] \\ &= \frac{2}{\cos^2\theta} = 2\sec^2\theta \end{aligned}$$



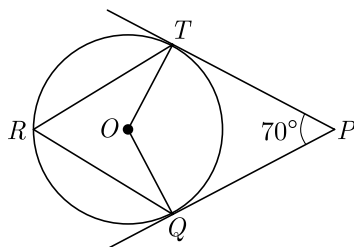
Hence Proved

32. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P . If $\angle TPQ = 70^\circ$, find $\angle TRQ$.



Ans : [Board Term-2 Foreign 2015]

We redraw the given figure by joining O to T and Q as shown below.



Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $\angle TOQ = 180^\circ - \angle TPQ$
 $= 180^\circ - 70^\circ = 110^\circ$

Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

$$\begin{aligned} \angle TRQ &= \frac{1}{2} \angle TOQ \\ &= \frac{1}{2} \times 110^\circ = 55^\circ \end{aligned}$$

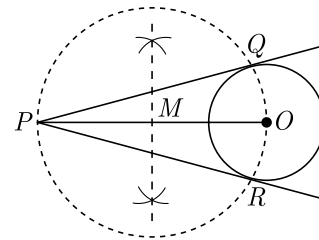
33. Draw a circle of radius 3 cm. From a point P , 7 cm away from centre draw two tangents to the circle. Measure the length of each tangent.

Ans : [Board Term-2 Foreign 2015]

Steps of Construction :

1. Draw a line segment PO of length 7 cm.
2. Draw a circle with centre O and radius 3 cm.

3. Draw a perpendicular bisector of PO . Let M be the mid-point of PO .
4. Taking M as centre and OM as radius draw a circle. Let this circle intersects the given circle at the point Q and R .
5. Join PQ and PR . On measuring we get $PQ = PR = 6.3$ cm.



34. Find the values of k for which the equation $(3k + 1)^2 + 2(k + 1)x + 1$ has equal roots. Also find the roots.

Ans : [Board Term-2, 2014]

We have $(3k + 1)^2 + 2(k + 1)x + 1$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (3k + 1), B = 2(k + 1), C = 1$$

If roots are equal, $B^2 - 4AC = 0$

$$[2(k + 1)]^2 - 4(3k + 1)(1) = 0$$

$$4(k^2 + 2k + 1) - (12k + 4) = 0$$

$$4k^2 + 8k + 4 - 12k - 4 = 0$$

$$4k^2 - 4k = 0$$

$$4k(k - 1) = 0$$

$$k = 0, 1.$$

Substituting $k = 0$, in the given equation,

$$x^2 + 2x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = -1$$

Again substituting $k = 1$, in the given equation,

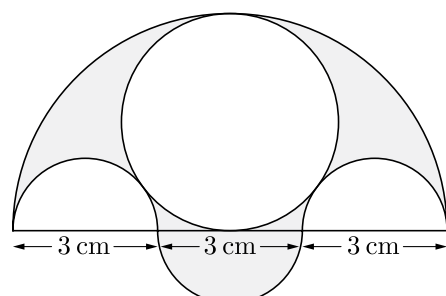
$$4x^2 + 4x + 1 = 0$$

$$(2x + 1)^2 = 0$$

or, $x = -\frac{1}{2}$

Hence, roots = $-1, -\frac{1}{2}$

35. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



Ans :

Area of shaded region

$$\begin{aligned}
 &= \text{Area of semicircle with } d = 9 \text{ cm} \\
 &+ \text{Area of semicircle with } d = 3 \text{ cm} \\
 &\quad - 2 \times \text{area of semicircle with } d = 3 \text{ cm} \\
 &\quad \quad - \text{area of circle with } d = 4.5 \text{ cm} \\
 &= \frac{1}{2} \times \pi \times \left(\frac{9}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 \\
 &\quad - 2 \times \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - \pi \times \left(\frac{4.5}{2}\right)^2 \\
 &= \frac{\pi}{8} [(9)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2] \\
 &= \frac{\pi}{8} [4(4.5)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2] \\
 &= \frac{\pi}{8} [2(4.5)^2 - (3)^2] = \frac{\pi}{8} [2(3 \times 1.5)^2 - (3)^2] \\
 &= \frac{\pi(3)^2}{8} [2(1.5)^2 - 1] = \frac{9\pi}{8} [4.5 - 1] \\
 &= \frac{9 \times 22}{8 \times 7} \times 3.5 = \frac{99}{8} = 12.375 \text{ cm}^2
 \end{aligned}$$



Thus area of shaded region is 12.375 cm²

36. If the mean of the following frequency distribution is 91, and sum of frequency is 150, find the missing frequency x and y :

Class	0- 30	30- 60	60- 90	90- 120	120- 150	150- 180
Frequency	12	21	x	52	y	11

Ans : [Board Term-1 2012]

We prepare following table to find mean.

Class	x_i (Class marks)	f_i	$f_i x_i$
0-30	15	12	180
30-60	45	21	945
60-90	75	x	$75x$
90-120	105	52	5460
120-150	135	y	$135y$
150-180	165	11	1815
Total		$\sum f =$ $x + y + 96 = 150$	$\sum f x =$ $8400 + 75x + 135y$

$$\begin{aligned}
 96 + x + y &= 150 \\
 x + y &= 54 \quad \dots(1)
 \end{aligned}$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$91 = \frac{8400 + 75x + 135y}{150}$$

$$13650 = 8,400 + 75x + 135y$$

$$75x + 135y = 5250$$

$$5x + 9y = 350 \quad \dots(2)$$



Solving equation (1) and (2) we get $x = 34$ and $y = 20$

or

Find the median of the following data :

Profit (in lakh of rupee)	Number of shops
More than of equal to 5	30
More than of equal to 10	28
More than of equal to 15	16
More than of equal to 20	14
More than of equal to 25	10
More than of equal to 30	7
More than of equal to 35	3

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Class	f	$c.f.$
5-10	2	2
10-15	12	14
15-20	2	16
20-25	4	20
25-30	3	23
30-35	4	27
35-40	3	30
Total	$\sum f = 30 = N$	

We have $N = 30 ; \frac{N}{2} = 15$
 Cumulative frequency just greater than $\frac{N}{2}$ is 16 and the corresponding class is 15-20. Thus median class is 15-20.

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$\text{Now, } l = 15, N = 30, F = 14, f = 2, h = 5$$

$$\begin{aligned}
 \text{Median, } M_d &= l + \left(\frac{\frac{N}{2} - F}{f}\right)h \\
 &= 15 + \left(\frac{15 - 14}{2}\right) \times 5 \\
 &= 15 + 2.5 = 17.5
 \end{aligned}$$



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