CLASS X (2020-21)
MATHEMATICS STANDARD (041)
SAMPLE PAPER-07

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Scan QR Code to See Video Soutions

Part - A

Section - I

1. What is the condition for the decimal expansion of a rational number to terminate? Explain with the help of an example.

Ans : [Board Term-1 2016]

The decimal expansion of a rational number terminates, if the denominator of rational number can be expressed as 2^m5^n where m and n are non negative integers and p and q both co-primes.

a148

e.g. $\frac{3}{10} = \frac{3}{2^1 \times 5^1} = 0.3$

2. Find a quadratic polynomial, whose zeroes are -3 and 4 ?

Ans :

b234

We have $\alpha = -3$ and $\beta = 4$.

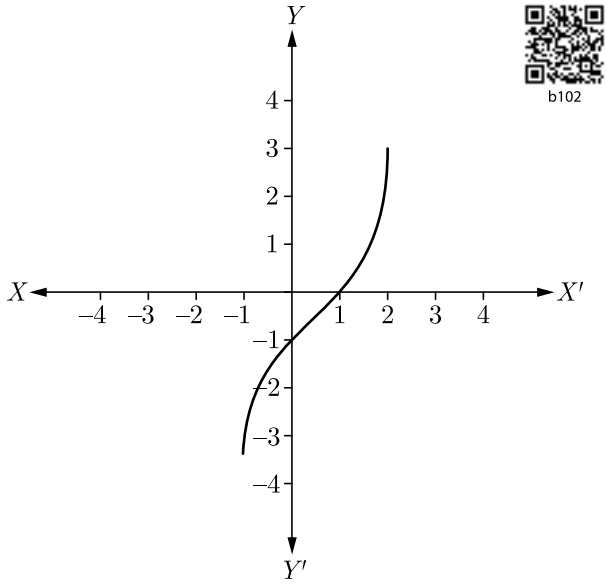
Sum of zeros $\alpha + \beta = -3 + 4 = 1$

Product of zeros, $\alpha \cdot \beta = -3 \times 4 = -12$

So, the quadratic polynomial is

$$x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - 1 \times x + (-12)$$
$$= x^2 - x - 12$$
$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

3. In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.



b102

Ans : [Board Term-1 2013]

The graph intersects x-axis at one point $x = 1$. Thus the number of zeroes of $p(x)$ is 1.

4. Find the roots of the quadratic equation $\sqrt{3}\,x^2 - 2x - \sqrt{3} = 0$

Ans : [Board Term-2, 2012, 2011]

We have
$$\sqrt{3}\,x^2 - 2x - \sqrt{3} = 0$$
$$\sqrt{3}\,x^2 - 3x + x - \sqrt{3} = 0$$
$$\sqrt{3}\,x(x - \sqrt{3}) + 1(x - \sqrt{3}) = 0$$
$$(x - \sqrt{3})(\sqrt{3}\,x + 1) = 0$$

Thus $x = \sqrt{3}, \frac{-1}{\sqrt{3}}$

d103

or

Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.

Ans : [Board Term-2, 2016]

We have $kx^2 - 14x + 8 = 0$

Let one root be α and other root be 6α.

Sum of roots, $\alpha + 6\alpha = \frac{14}{k}$

$$7\alpha = \frac{14}{k} \text{ or } \alpha = \frac{2}{k} \quad ...(1)$$

Product of roots , $\alpha(6\alpha) = \frac{8}{k}$ or $6\alpha^2 = \frac{8}{k}$ ...(2)

Solving (1) and (2), we obtain

d104

$$6\left(\frac{2}{k}\right)^2 = \frac{8}{k}$$
$$6 \times \frac{4}{k^2} = \frac{8}{k}$$
$$\frac{3}{k^2} = \frac{1}{k}$$
$$3k = k^2$$
$$3k - k^2 = 0$$
$$k[3-k] = 0$$
$$k = 0 \text{ or } k = 3$$

Since $k = 0$ is not possible, therefore $k = 3$.

5. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$ an AP? Give reason.

Ans : [Board Term-2 2015]

Let common difference be d then we have

e106

$$d = a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$$
$$d = a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$$
$$d = a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3$$

As common difference are not equal, the given series is not in AP.

or

What is the next term of an AP $\sqrt{7}$, $\sqrt{28}$, $\sqrt{63}, ...$?

Ans : [Board Term-2 Foreign 2014]

Let the first term of an AP be a and common difference be d.

Here, $a = \sqrt{7}$, $a + d = \sqrt{28}$

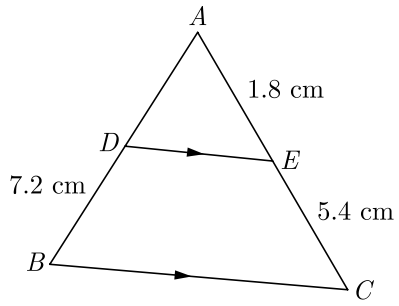
$$d = \sqrt{28} - \sqrt{7} = 2\sqrt{7} - \sqrt{7}$$
$$= \sqrt{7}$$

e107

$$\text{Next term} = \sqrt{63} + \sqrt{7}$$
$$= 3\sqrt{7} + \sqrt{7} = 4\sqrt{7}$$
$$= \sqrt{7 \times 16}$$
$$= \sqrt{112}$$

6. In Figure, $DE \,||\, BC$. Find the length of side AD, given that $AE = 1.8$ cm, $BD = 7.2$ cm and $CE = 5.4$ cm.

f237



Ans : [Board 2019 OD]

Since $DE \,||\, BC$ we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the values, we get

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$
$$AD = \frac{1.8 \times 7.2}{5.4} = \frac{12.96}{5.4} = 2.4 \text{ cm}$$

7. If the mid-point of the line segment joining the points $A(3,4)$ and $B(k,6)$ is $P(x,y)$ and $x + y - 10 = 0$, find the value of k.

Ans : [Board 2020 OD Standard]

If $P(x,y)$ is mid point of $A(3,4)$ and $B(k,6)$, then we have

$$\frac{3+k}{2} = x \text{ and } y = \frac{4+6}{2} = \frac{10}{2} = 5$$

Substituting above value in $x + y - 10 = 0$ we have

g269

$$\frac{3+k}{2} + 5 - 10 = 0$$
$$\frac{3+k}{2} = 5$$
$$3 + k = 10 \Rightarrow k = 10 - 3 = 7$$

8. The ordinate of a point A on y-axis is 5 and B has co-ordinates $(-3,1)$. Find the length of AB.

Ans : [Board Term-2 2014]

We have $A(0,5)$ and $B(-3,1)$.

g102

Distance between A and B,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-3-0)^2 + (1-5)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25} = 5$$

9. If $\sec\theta \cdot \sin\theta = 0$, then find the value of θ.

Ans : [Board Term-1 2016]

We have $\sec\theta \cdot \sin\theta = 0$

h102

$$\frac{1}{\cos\theta} \cdot \sin\theta = 0$$
$$\frac{\sin\theta}{\cos\theta} = 0$$
$$\tan\theta = 0 = \tan 0^\circ$$

Thus $\theta = 0^\circ$

10. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A + B$.

Ans : [Board Term-1 2016]

We have

h110

$$\sin A = \cos B$$
$$\sin A = \sin(90^\circ - B)$$
$$A = 90^\circ - B$$
$$A + B = 90^\circ$$

11. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is ..... .

Ans : [Board 2020 Delh

We have $\tan(A+B) = \sqrt{3}$

h253

$= \tan 60^\circ$

Hence, $A + B = 60^\circ$ ...(1)

Again, $\tan(A - B) = \frac{1}{\sqrt{3}}$

$= \tan 30^\circ$

$A - B = 30^\circ$ ...(2)

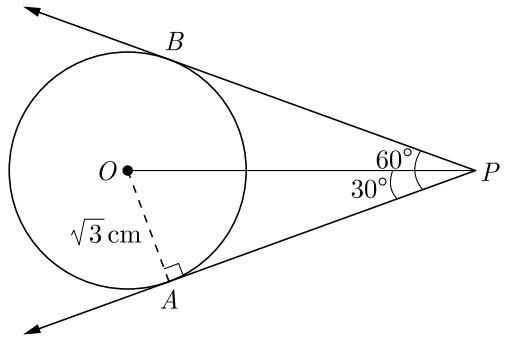
Adding equation (1) and (2) we get

$2A = 90^\circ \Rightarrow A = 45^\circ$

12. Two tangents making an angle of 60° between them are drawn to a circle of radius $\sqrt{3}$ cm, then find the length of each tangent.

Ans : [Board, Term-2, 2013]

As per the given question we draw the figure as below.



Since, $\tan\theta = \frac{OA}{AP}$

So. $\tan 30^\circ = \frac{OA}{AP}$

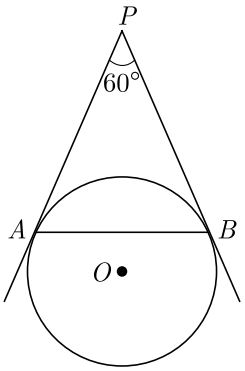
$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$

$AP = \sqrt{3} \times \sqrt{3} = 3$ cm.

j157

or

In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Ans : [Board Term-2 Delhi 2016]

Since length of 2 tangents drawn from an external point to a circle are equal, we have

$PA = PB$

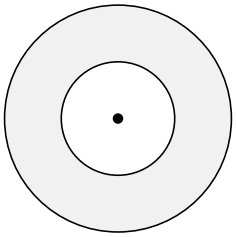
Thus $\angle PAB = \angle PBA = 60^\circ$

Hence ΔPAB is an equilateral triangle.

Therefore $AB = PA = 5$ cm.

j158

13. Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, find the area of the shaded ring shaped region in square cm.



I103

Ans : [Board Term-2 2012]

Area of circle $= \pi r^2$

Area of the shaded region $= \pi(2)^2 - \pi(1)^2$

$= 4\pi - \pi = 3\pi$ sq cm

or

The diameter of two circle with centre A and B are 16 cm and 30 cm respectively. If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C.

Ans : [Board Term-2 2012]

Let the radius of circle with centre C be r.

According to question we have,

$\pi(8)^2 + \pi(15)^2 = \pi r^2$

$64\pi + 225\pi = \pi r^2$

$289\pi = \pi r^2$

$r^2 = 289$ or $R = 17$ cm

Circumference of circle

$2\pi r = 2\pi \times 17$

$= 34\pi$ cm

I104

14. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, find the ratio of their volumes.

Ans : [Board Term-2 2012]

$$\frac{\text{Volume of } 1^{\text{st}} \text{cylinder}}{\text{Volume of } 2^{\text{nd}} \text{cylinder}} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

$$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$$

$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$

$= 20 : 27$

m110

15. Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median in 45.5.

Ans : [Board Term-1 2015]

Mode, $M = 50.5$

Median, $M_d = 45.5$

Now $3M_d = M_o + 2M$

$3 \times 45.5 = 50.5 + 2M$

Mean, $M = \frac{136.5 - 50.5}{2} = 43$

Hence mean is 43.

n160

16. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.

Ans : [Board Term-2 2016]

Total number of outcomes,

$$n(S) = 48$$

Favourable outcomes are 4, 9, 16, 25, 36 and 49.

No. of favourable outcomes,

o151

$$n(E) = 6$$

P(perfect square number),

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{48} \text{ or } = \frac{1}{8}$$

or

20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is a multiple of 3 or 7.

Ans : [Board Term-2 Foreign 2016]

Total number of cases,

o152

$$n(S) = 20$$

Favourable outcome,

$$E = \{3,6,7,9,12,14,15,18\}$$

Number of favourable cases,

$$n(E) = 8$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. The Republic Day parade, first held in 1950, has been a yearly ritual since. The parade marches from the Rashtrapati Bhawan along the Rajpath in New Delhi. Several regiments of the army, navy, and air force, along with their bands, march to India Gate. The parade is presided over by the President of India, who is the Commander-in-Chief of the Indian Armed Forces. As he unfurls the tricolour, the national anthem is played. The regiments of the Armed Forces then start their march past. Prestigious awards like Kirti Chakra, Ashok Chakra, Paramvir Chakra and Vir Chakra are given out by the President. Nine to twelve different regiments of the Indian Army, in addition to the Navy and Air Force march toward India Gate along with their bands. Contingents of paramilitary forces and other civil forces also participate in the parade.

On 71th republic day parade, captain RS Meel is planing for parade of following two group:

(a) First group of Army troops of 624 members behind an army band of 32 members.

(b) Second group of CRPF troops with 468 soldiers behind the 228 members of bikers.

These two groups are to march in the same number of columns. This sequence of soldiers is followed by different states Jhanki which are showing the culture of the respective states.

(i) What is the maximum number of columns in which the army troop can march?

a403

(a) 8 (b) 16

(c) 4 (d) 32

(ii) What is the maximum number of columns in which the CRPF troop can march?

(a) 4 (b) 8

(c) 12 (d) 16

(iii) What is the maximum number of columns in which total army troop and CRPF troop together can march past?

(a) 2 (b) 4

(c) 6 (d) 8

(iv) What should be subtracted with the numbers of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop?

(a) 4 Soldiers and 4 Bikers

(b) 4 Soldiers and 2 Bikers

(c) 2 Soldiers and 4 Bikers

(d) 2 Soldiers and 2 Bikers

(v) What should be added with the numbers of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop?

(a) 4 Soldiers and 4 Bikers

(b) 12 Soldiers and 12 Bikers

(c) 6 Soldiers and 6 Bikers

(d) 12 Soldiers and 6 Bikers

Ans :

(i) We will find the HCF (624, 32) = 16

Thus (b) is correct option.

(ii) We will find the HCF (228, 468) = 12.

Thus (c) is correct option.

According to the question, we have to find out

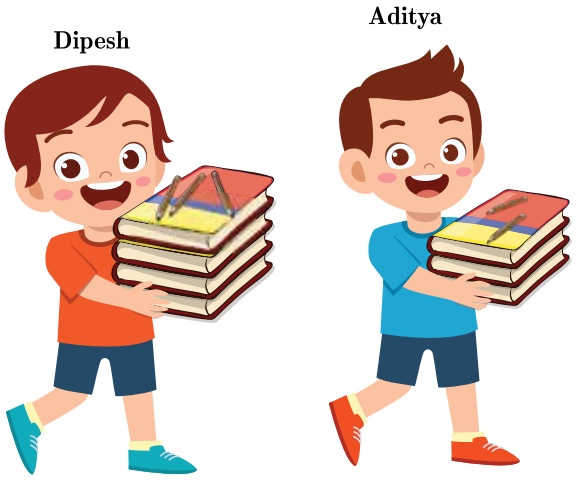
HCF(624, 32, 228, 468) = 4

(iii) Alternatively we can find,

HCF (16, 12) = 4

Thus (b) is correct option.
(iv) Maximum number of column of army troop is 16. But 228 and 468 are not divisible by 16. If we subtract 4 from 228 and 468, both(224 and 464) are divisible by 16.
Thus (a) is correct option.
(v) Maximum number of column of army troop is 16. But 228 and 468 are not divisible by 16. If we add 12 in 228 and 468, both(240 and 480) are divisible by 16.
Thus (b) is correct option.

18. Dipesh bought 3 notebooks and 2 pens for Rs. 80. His friend Ramesh said that price of each notebook could be Rs. 25. Then three notebooks would cost Rs.75, the two pens would cost Rs. 5 and each pen could be for Rs. 2.50. Another friend Amar felt that Rs. 2.50 for one pen was too little. It should be at least Rs. 16. Then the price of each notebook would also be Rs.16.



Aditya also bought the same types of notebooks and pens as Dipesh. He paid 110 for 4 notebooks and 3 pens.

(i) Whether the estimation of Ramesh and Amar is applicable for Aditya?
(a) Ramesh's estimation is wrong but Amar's estimation is correct.
(b) Ramesh's estimation is correct but Amar's estimation is wrong.
(c) Both estimation are correct.
(b) Ramesh's estimation is wrong but Amar's estimation is also wrong.

(ii) Let the cost of one notebook be x and that of pen be y. Which of the following set describe the given problem ?
(a) $2x+3y=80$ and $3x+4y=110$
(b) $3x+2y=80$ and $4x+3y=110$
(c) $2x+3y=80$ and $4x+3y=110$
(d) $3x+2y=80$ and $3x+4y=110$

(iii) What is the exact cost of the notebook?
(a) Rs 10
(b) Rs 20
(c) Rs 16
(d) Rs 24

c403

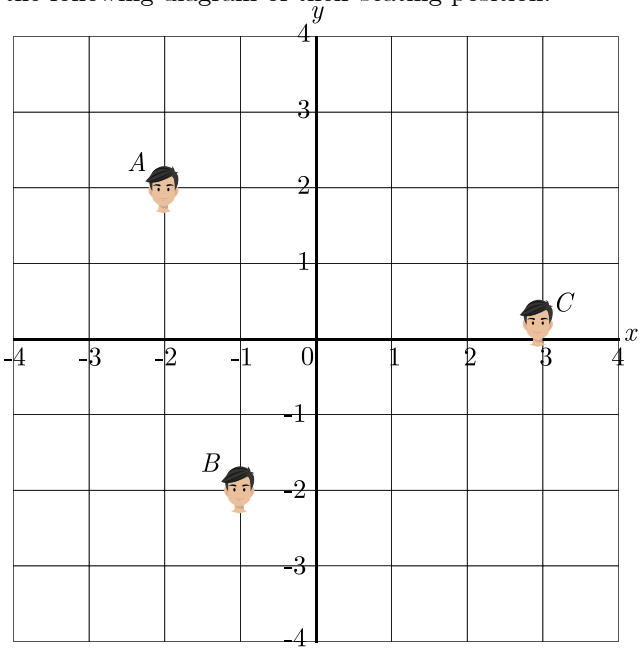
(iv) What is the exact cost of the pen?
(a) Rs 10 (b) Rs 20
(c) Rs 16 (d) Rs 24

(v) What is the total cost if they purchase the same type of 15 notebooks and 12 pens.
(a) Rs 410 (b) Rs 200
(c) Rs 420 (d) Rs 240

Ans :

(i) Consider the prices mentioned by Ramesh.
If the price of one notebook is Rs. 25 and the price of one pen is Rs. 2.50 then,
The cost of 4 notebooks would be : $4\times25=100$ Rs
And the cost for 3 pens would be : $3\times2.5=7.5$ Rs
Aditya should have paid $100+7.5=107.5$ Rs.
But he paid Rs. 110, thus Ramesh's estimation is wrong.
Now, consider the prices mentioned by Amar.
The cost of 4 notebooks, if one is for Rs.16, would be : $4\times16=64$ Rs
And the cost for 3 pens, if one is for Rs. 16, would be : $3\times16=64$ Rs
Aditya should have paid $64+48=112$ Rs but this is more than the price he paid.
Therefore, Amar's estimation is also wrong.
Thus (d) is correct option.
(ii) According to the statement, we have
$3x+2y=80$ and $4x+3y=110$
Thus (b) is correct option.
(iii) Solving $3x+2y=80$ and $4x+3y=110$ we get $x=20$ and $y=10$
Thus cost of 1 notebook is 20 Rs and cost of 1 pen is 10 Rs
Thus (b) is correct option.
(iv) Cost of 1 pen = Rs. 10
Thus (a) is correct option.
(v) Total cost $15\times20+12\times10=420$ Rs
Thus (c) is correct option.

19. Ajay, Bhigu and Colin are fast friend since childhood. They always want to sit in a row in the classroom . But teacher doesn't allow them and rotate the seats row-wise everyday. Bhigu is very good in maths and he does distance calculation everyday. He consider the centre of class as origin and marks their position on a paper in a co-ordinate system. One day Bhigu make the following diagram of their seating position.



(i) What are the coordinates of point A?
(a) $(2, 2)$
(b) $(2, -2)$
(c) $(-2, 2)$
(d) $(-2, -2)$

g402

(ii) What is the distance of point A from origin ?
(a) 8 (b) $2\sqrt{2}$
(c) 4 (d) $4\sqrt{2}$

(iii) What is the distance between A and B ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $\sqrt{17}$ (d) $2\sqrt{5}$

(iv) What is the distance between B and C ?
(a) $3\sqrt{19}$ (b) $3\sqrt{5}$
(c) $2\sqrt{17}$ (d) $2\sqrt{5}$

(v) A point D lies on the line segment between points A and B such that $AD : DB = 4 : 3$. What are the the coordinates of point D ?
(a) $\left(\frac{10}{7}, \frac{2}{7}\right)$ (b) $\left(\frac{2}{7}, \frac{7}{7}\right)$
(c) $\left(-\frac{10}{7}, -\frac{2}{7}\right)$ (d) $\left(-\frac{2}{7}, -\frac{7}{7}\right)$

Ans :

(i) It may be seen easily from figure that coordinates of point A are $(-2, 2)$.
Thus (c) is correct option.

(ii) $OA = \sqrt{(0+2)^2 + (0-2)^2} = 2\sqrt{2}$
Thus (b) is correct option.

(iii) It may be seen easily from figure that coordinates of point A are $(-1, -2)$.

$$AB = \sqrt{(-2+1)^2 + (2+2)^2}$$
$$= \sqrt{1+4^2} = \sqrt{17}$$

Thus (c) is correct option.

(iv) It may be seen easily from figure that coordinates of point A are $(0,\ 3)$.

$$BC = \sqrt{(-1-3)^2 + (-2-0)^2}$$
$$= \sqrt{4^2+4} = 2\sqrt{5}$$

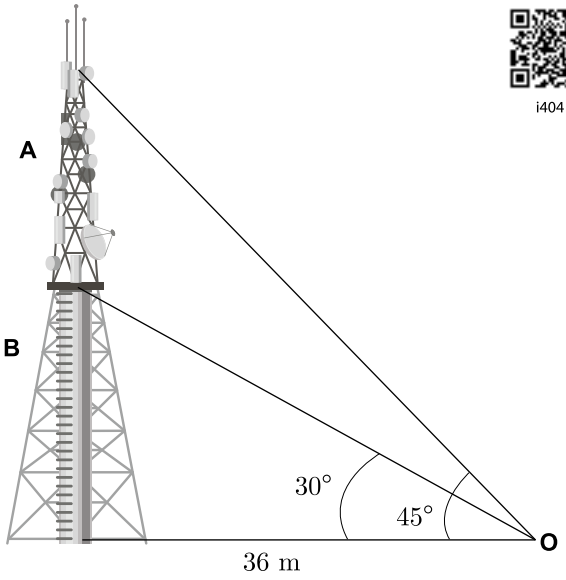
Thus (d) is correct option.

(v) We have $A(-2, 2)$ and $B(-1, -2)$

$$\frac{m}{n} = \frac{4}{3}$$
$$x = \frac{mx_2 + nx_1}{m+n} = \frac{-1(4) + 3(-2)}{4+3} = \frac{-10}{7}$$
$$y = \frac{my_2 + hy_1}{m+n} = \frac{-2(4) + 3(2)}{4+3} = \frac{-2}{7}$$

Thus (c) is correct option.

20. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure, including microwave dishes. They are among the tallest human-made structures. There are 2 main types: guyed and self-supporting structures.
On a similar concept, a radio station tower was built in two sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 m. From point O, the angle of elevation of the top of section B is 30° and the angle of elevation of the top of section A is 45°.



i404

(i) What is the height of the section B ?
(a) $12\sqrt{3}$ m (b) $12\sqrt{2}$ m
(c) $8\sqrt{3}$ m (d) $4\sqrt{2}$ m

(ii) What is the height of the section A ?
(a) $12(2-\sqrt{2})$ (b) $24(2-\sqrt{2})$
(c) $12(3-\sqrt{3})$ (d) $24(3-\sqrt{3})$

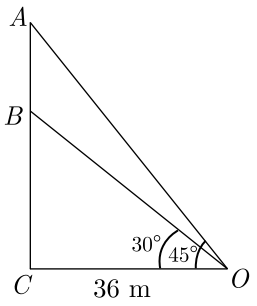
(iii) What is the length of the wire structure from the point O to the top of section A ?
(a) $32\sqrt{2}$ m (b) $24\sqrt{3}$ m
(c) $28\sqrt{3}$ m (d) $36\sqrt{2}$ m

(iv) What is the length of the wire structure from the point O to the top of section B ?
(a) $12\sqrt{3}$ m (b) $24\sqrt{3}$ m
(c) $28\sqrt{3}$ m (d) $16\sqrt{3}$ m

(v) What is the angle of depression from top of tower to point O ?
(a) 30° (b) 45°
(c) 15° (d) 75°

Ans :

(i) We make the following diagram as per given information.



In ΔBCO $\tan 30° = \frac{BC}{OC}$

$$BC = OC \tan 30°$$
$$BC = 36 \times \frac{1}{\sqrt{3}} = 12\sqrt{3} \text{ m}$$

Thus (a) is correct option.

(ii) In ΔACO,

$$\tan 45^\circ = \frac{AC}{OC} = 1$$

Thus $AC = OC = 36\text{ m}$

Now, $AB = AC - BC$

$= 36 - 12\sqrt{3} = 12(3-\sqrt{3})$ m

Thus (c) is correct option.

(iii) In ΔACO,

$$\cos 45^\circ = \frac{OC}{OA}$$

$$\frac{1}{\sqrt{2}} = \frac{36}{OA}$$

$$OA = 36\sqrt{2}\text{ m}$$

Thus (d) is correct option.

(iv) In ΔBCO,

$$\cos 30^\circ = \frac{OC}{OB}$$

$$\frac{\sqrt{3}}{2} = \frac{36}{OB}$$

$$OB = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 24\sqrt{3}\text{ m}$$

Thus (b) is correct option.

(v) It is clear from figure that angle of elevation from point O to top of tower is 45°. This is equal to the angle of depression from top of tower to point O.

Thus (b) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

Ans : [Board Term-1 2012]

We have $\frac{m}{n} + \frac{n}{m} = \frac{m^2+n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn}$ (1)

Sum of zeroes $m + n = -\frac{11}{3}$

Product of zeroes $mn = \frac{-4}{3}$

Substituting in (1) we have

$$\frac{m}{n} + \frac{n}{m} = \frac{(m+n)^2 - 2mn}{mn}$$

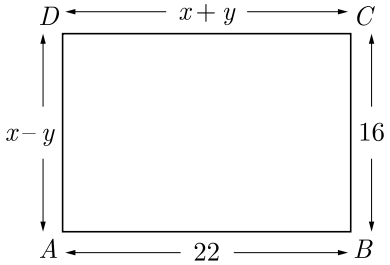
$$= \frac{(-\frac{11}{3})^2 - \frac{-4}{3} \times 2}{\frac{-4}{3}}$$

$$= \frac{121 + 4 \times 3 \times 2}{-4 \times 3}$$

or $\frac{m}{n} + \frac{n}{m} = \frac{-145}{12}$

22. In the figure given below, $ABCD$ is a rectangle. Find the values of x and y.

Ans : [Board Term-1 2012, Set-30]



From given figure we have

$x + y = 22$ ...(1)

and $x - y = 16$ ...(2)

Adding (1) and (2), we have

$2x = 38$

$x = 19$

Substituting the value of x in equation (1), we get

$19 + y = 22$

$y = 22 - 19 = 3$

Hence, $x = 19$ and $y = 3$.

23. Find the roots of the following quadratic equation :

$(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

Ans : [Board Term-2 2012]

We have $(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

$x^2 + 3x - x - 3 = 3x - 1$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

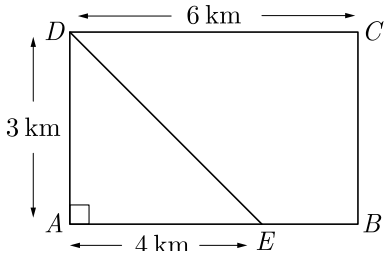
$(x-2)(x+1) = 0$

Thus $x = 2, -1$

24. In a rectangle $ABCD$, E is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Ans : [Board Term-1 2016]

As per given condition we have drawn the figure below.



We have $AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4$ km

In right triangle ADE,

$DE^2 = (3)^2 + (4)^2 = 25$

Thus $DE = 5$ km

25. Show that : $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}=1$

Ans : [Board 2020 OD Standard]

$$\text{LHS} = \frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$$
$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(90^\circ-45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(90^\circ-30^\circ+\theta)}$$
$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(60^\circ+\theta)}$$
$$= \frac{1}{1} = 1 = \text{RHS}$$

h286

or

If $\cos(40^\circ + x) = \sin 30^\circ$, find the value of x.

Ans : [Board Term-1 2015]

We have

$$\cos(40^\circ - x) = \sin 30^\circ$$
$$\cos(40^\circ + x) = \sin(90^\circ - 60^\circ)$$
$$\cos(40^\circ + x) = \cos 60^\circ$$
$$40^\circ + x = 60^\circ$$
$$x = 60^\circ - 40^\circ = 20^\circ$$

h135

Thus $x = 20^\circ$.

26. Find the unknown values in the following table :

Class Interval	**Frequency**	**Cumulative Frequency**
0-10	5	5
10-20	7	x_1
20-30	x_2	18
30-40	5	x_3
40-50	x_4	30

Ans : [Board Term-1 2016]

We have $x_1 = 5 + 7 = 12$

$x_2 = 18 - x_1 = 18 - 12 = 6$

$x_3 = 18 + 5 = 23$

and $x_4 = 30 - x_3 = 30 - 23 = 7$

n177

or

Calculate the median from the following data :

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	5	15	30	8	2

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Marks	**No. of students**	**c.f.**
0-10	5	5
10-20	15	20
20-30	30	50
30-40	8	58
40-50	2	60
	$N = 60$	

We have $N = 60$; $\frac{N}{2} = 30$

Cumulative frequency just greater than $\frac{N}{2}$ is 50 and the corresponding class is 20-30. Thus median class is 20-20.

Now $l = 20,\ f = 30,\ F = 20,\ h = 10$

Median, $M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right) \times h$

$$= 20 + \left(\frac{30-20}{30}\right) \times 10$$
$$= 20 + \frac{100}{30} = 20 + \frac{10}{3}$$
$$= 20 + 3.33$$

Thus $Md = 23.33$

n178

27. If α and β are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$

Ans : [Board 2020 Delhi Basic]

We have $p(x) = x^2 - 4x - 5$

Comparing it by $ax^2 + bx + c$, we get $a = 1,\ b = -4$ and $c = -5$

Since, given α and β are the zeroes of the polynomial,

Sum of zeroes, $\alpha + \beta = -\dfrac{b}{a} = \dfrac{-(-4)}{1} = 4$

and product of zeroes, $\alpha\beta = \dfrac{c}{a} = \dfrac{-5}{1} = -5$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$= (4)^2 - 2(-5)$$
$$= 16 + 10 = 26$$

d294

or

Find the quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. Hence find the zeroes.

Ans : [Board 2020 OD Basic]

Sum of zeroes $\alpha + \beta = -3$ ...(1)

and product of zeroes $\alpha\beta = 2$

Thus quadratic equation is

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$
$$x^2 - (-3)x + 2 = 0$$
$$x^2 + 3x + 2 = 0$$

d296

Thus quadratic equation is $x^2 + 3x + 2 = 0$.

Now above equation can be written as

$$x^2 + 2x + x + 2 = 0$$
$$x(x+2) + (x+2) = 0$$
$$(x+2)(x+1) = 0$$

Hence, zeroes are -2 and -1.

28. Show that the sum of all terms of an AP whose first term is a, the second term is b and last term is c, is equal to $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$

Ans : [Board 2020 OD Standard]

Given, first term, $A = a$

and second term $A_2 = b$

Common difference, $D = b - a$

Last term, $A_n = c$

e310

$$A + (n-1)\,d = c$$
$$a + (n-1)(b-a) = c$$
$$(b-a)(n-1) = c - a$$
$$n - 1 = \frac{c-a}{b-a}$$
$$n = \frac{c-a}{b-a} + 1$$
$$= \frac{c-a+b-a}{b-a}$$
$$n = \frac{b+c-2a}{b-a}$$

Now sum of all terms

$$S_n = \frac{n}{2}[A + A_n] = \frac{(b+c-2a)}{2(b-a)}[a+c]$$
$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$ Hence Proved

or

If in an AP, the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m+n)$ terms is $-(m+n)$.

Ans : [Board 2020 OD Standard]

Let 1st term of series be a and common difference be d, then we have

e311

$$S_m = n$$

and $$S_n = m$$

$$\frac{m}{2}[2a + (m-1)\,d] = n \quad ...(1)$$

$$\frac{n}{2}[2a + (n-1)\,d] = m \quad ...(2)$$

Subtracting we have

$$a(m-n) + \frac{d}{2}[m(m-1) - n(n-1)] = n - m$$
$$2a(m-n) + d[m^2 - n^2 - (m-n)] = 2(n-m)$$
$$2a(m-n) + d(m-n)[(m+n) - 1] = 2(n-m)$$
$$2a + d[(m+n) - 1] = -2$$

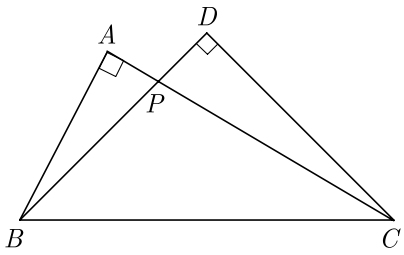
Now, $$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)\,d]$$
$$= \frac{m+n}{2}(-2)$$
$$= -(m+n)$$

29. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

Ans : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In ΔBAP and ΔCDP we have

$$\angle BAP = \angle CDP = 90^\circ$$

f243

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

By AA similarity we have

$$\Delta BAP \sim \Delta CDP$$

Therefore $$\frac{BP}{PC} = \frac{AP}{PD}$$

$$AP \times PC = BP \times PD$$ Hence Proved

30. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Ans : [Board 2020 SQP Standard]

Given, $\sin\theta + \cos\theta = \sqrt{3}$

h290

Squaring above equation, we have

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$
$$1 + 2\sin\theta\cos\theta = 3$$
$$2\sin\theta\cos\theta = 3 - 1 = 2$$
$$\sin\theta\cos\theta = 1$$

Now, $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$

Substituting value of $\sin\theta\cos\theta$ we have

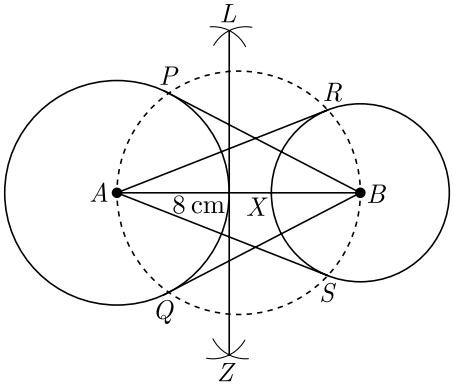
$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

31. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

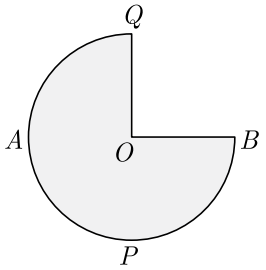
Ans : [Board Term-2 Foreign 2017, OD 2014]

Steps of Construction :

1. Draw a line segment AB of length 8 cm.
2. Draw a circle with centre A and radius 4 cm.
3. Draw another circle with centre B and radius 3 cm.
4. Taking AB as diameter draw another circle, which intersects first two circles at P and Q, and R and S.
5. Join B to P, B to Q, A to R and A to S. Thus BP, BQ, AR and AS are the required tangents.

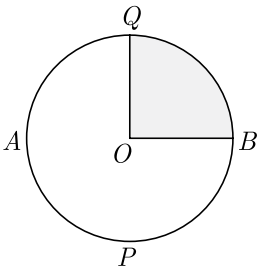


32. In fig. APB and AQP are semi-circle, and $AO = OB$. If the perimeter of the figure is 47 cm, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 Delhi 2015]

We have redrawn the given figure as shown below;



Let r be the radius of given circle. It is given that perimeter of given figure is 47 cm.

$$2\pi r - \frac{1}{4}(2\pi r) + 2r = 47$$

$$\frac{3\pi r}{2} + 2r = 47$$

$$r\left(\frac{3}{2} \times \frac{22}{7} + 2\right) = 47$$

$$r\left(\frac{33}{7} + 2\right) = 47$$

$$r = \frac{47 \times 7}{47} = 7 \text{ cm}$$

Now, area of shaded region

$$A = \text{area of circle} - \frac{1}{4} \text{ area of circle}$$

$$= \frac{3}{4} \text{ area of circle}$$

$$= \frac{3}{4}\pi r^2 = \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{3}{2} \times 77 = 115.5 \text{ cm}^2$$

33. Find the mean of the following distribution by step deviation method :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	13	20	15	7	5

Ans : [Board Term-1 2011]

Class	x_i **(Class Marks)**	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
0-10	5	-3	5	-15
10-20	15	-2	13	-26
20-30	25	-1	20	-20
30-40	35	0	15	0
40-50	45	1	7	7
50-60	55	2	5	10
	Total		$\sum f_i = 65$	$\sum f_i u_i = -44$

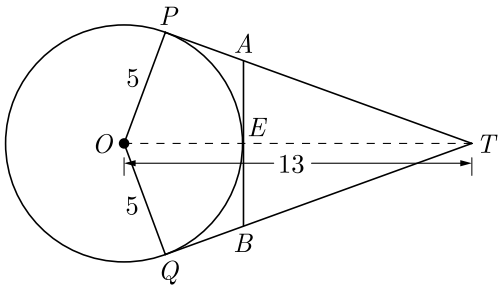
Let assumed mean, $a = 35$ and given $h = 10$.

Mean, $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 35 + \frac{-44}{65} \times 10$$

$$= 35 - 6.76 = 28.24$$

34. In figure O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Ans : [Board Term-2 Delhi 2016]

Here ΔOPT is right angled triangle because PT is tangent on radius OP.

Thus $$PT = \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25} = 12 \text{ cm}$$

and $$TE = OT - OE$$

$$= 13 - 5 = 8 \text{ cm}$$

Since length of tangents from an external point to a circle are equal,

Let $$PA = AE = x$$

Here ΔAET is right angled triangle because AB is tangent on radius OE.

In ΔAET, $$TA^2 = TE^2 + EA^2$$

$$(TP - PA)^2 = 8^2 + x^2$$

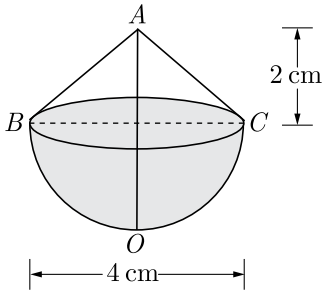
$$(12 - x)^2 = 64 + x^2$$

$$144 - 24x + x^2 = 64 + x^2$$
$$24x = 144 - 64 = 80$$
or, $$x = 3.3 \text{ cm.}$$
Thus $$AB = 2 \times x = 2 \times 3.3 = 6.6 \text{ cm.}$$

35. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and toy. (Use $\pi = 3.14$)

Ans : [Board Term-2 2012]

Let BOC is a hemisphere and ABC is a cone. As per question the figure is shown below.



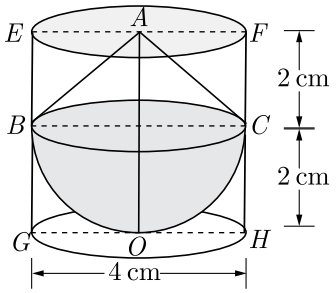
m215

Radius of hemisphere is equal to the radius of cone which is $\frac{4}{2} = 2$ cm.

Height of cone, $h = 2$ cm

$$\text{Volume of toy} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$
$$\frac{1}{3}\pi r^2(2r + h) = \frac{1}{3} \times 3.14 \times 2 \times 2(2 \times 2 + 2)$$
$$= \frac{1}{3} \times 3.14 \times 4 \times 6$$
$$= 25.12 \text{ cm}^3$$

Let right circular cylinder $EFGH$ circumscribe the given solid toy.



Radius of cylinder $= 2$ cm

Height of cylinder $= 4$ cm

Volume of right circular cylinder
$$\pi r^2 h = 3.14 \times (2)^2 \times 4 \text{ cm}^3$$
$$= 50.24 \text{ cm}^3$$

Difference of two volume
$$= \text{Volume of cylinder} - \text{Volume of toy}$$
$$= 50.24 - 25.12 = 25.12 \text{ cm}^3.$$

36. A box contains 90 discs which are numbered 1 to 90. If one disc is drawn at random from the box, find the probability that it bears
(i) a two digit number,
(ii) number divisible by 5.

Ans : [Board Term-2 Foreign 2017]

Total number of discs in the box are 90.
Thus we have 90 possible outcomes.
$$n(S) = 90$$
(i) a two digit number,
Discs with two digit number are 10, 11, ...89 and 90 which are 81 numbers.
No. of favourable outcomes,

o258

$$n(E_1) = 81$$
P(a disc with two digit number)
$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$
(ii) number divisible by 5
The numbers divisible by 5 between 1 to 90 are 5, 10, 15 ..... 85 and 90 which are 18 numbers.
No. of favourable outcomes,
$$n(E_2) = 18$$
P (a disc with a number divisible by 5)
$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{18}{90} = \frac{1}{5}$$

or

Two different dice are thrown together. Find the probability that the numbers obtained have
(i) even sum, and
(ii) even product.

Ans :

There are 36 possible outcomes of rolling two dices.
$$n(S) = 36$$
(i) even sum
Favourable outcome are (1, 3), (1, 5), (1, 1), (2, 2), (2, 4), (2, 6), (3, 1) (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3) (5, 5) (6, 2), (6, 4) and (6, 6).
Number of favourable outcomes,

o259

$$n(E_1) = 18$$
P(even sum),
$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{18}{36} = \frac{1}{2} \text{ or } 0.5$$
(ii) even product
Favourable outcome are (1, 2), (1, 4), (1, 6), (2, 1), (2,2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2) (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) and (6, 6).
Number of favourable outcomes
$$n(E_2) = 27$$
P(have a product less than 16),
$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{27}{36} = \frac{3}{4} = 0.75$$
Probability of getting even product is $\frac{3}{4}$ or 0.75.