CLASS X (2020-21)
MATHEMATICS STANDARD (041)
SAMPLE PAPER-08

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :

1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :

1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A

Section - I

1. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

Ans : [Board Term-1 2016]

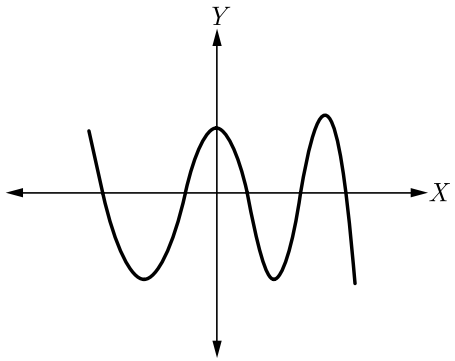
Since $\frac{1}{7} \times \frac{7}{100} = \frac{1}{100} = 0.01.$

a149

Thus smallest rational number is $\frac{7}{100}$

2. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows.

b209



The number of zeroes of $p(x)$ is ..... .

Ans : [Board 2020 SQP Standard]

The graph of the given polynomial $p(x)$ crosses the x-axis at 5 points. So, number of zeroes of $p(x)$ is 5.

or

If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other then the value of k is .....

Ans : [Board 2020 SQP Standard]

We have $(k-1)x^2 - 10x + 3 = 0$

Let one root be α, then another root will be $\frac{1}{\alpha}$

Now $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} = \frac{3}{(k-1)}$

$1 = \frac{3}{(k-1)}$

$k - 1 = 3 \Rightarrow k = 4$

b210

3. If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Ans : [Board 2009]

We have $p(x) = 3x^2 - kx - 6$

Sum of the zeroes $= 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

b104

Thus $3 = -\frac{(-k)}{3} \Rightarrow k = 9$

or

If -1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

Ans :

We have $f(x) = x^2 - 7x - 8$

Let other zero be k, then we have

b105

Sum of zeroes, $-1 + k = -\left(\frac{-7}{1}\right) = 7$

or $k = 8$

4. If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k.

Ans : [Board Term-2 Foreign-2, 2017]

We have $6x^2 - x - k = 0$

Substituting $x = \frac{2}{3}$, we get

d105

$6\left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$

$6 \times \frac{4}{9} - \frac{2}{3} - k = 0$

$\frac{8}{3} - \frac{2}{3} - k = 0$

$$\frac{8-2}{3}-k=0$$

$$2-k=0$$

Thus $k=2$.

or

Find the value(s) of k if the quadratic equation $3x^2-k\sqrt{3}\,x+4=0$ has real roots.

Ans : [SQP 2017]

If discriminant $D=b^2-4ac$ of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $3x^2-k\sqrt{3}\,x+4=0$

Comparing with $ax^2+bx+c=0=0$ we get

$$a=3, b=-k\sqrt{3} \text{ and } c=4$$


d106

For real roots $b^2-4ac \geq 0$

$$\left(-k\sqrt{3}\right)^2-4\times 3\times 4 \geq 0$$

$$3k^2-48 \geq 0$$

$$k^2-16 \geq 0$$

$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

5. If the common difference of an AP is -6, find $a_{16}-a_{12}$.

Ans : [Board Term-2 2014]

Let the first term of an AP be a and common difference be d.

Now $d=-6$


e108

$$a_{16}=a+(16-1)(-6)=a-90$$

$$a_{12}=a+(12-1)(-6)=a-66$$

$$a_{16}-a_{12}=(a-90)-(a-66)$$

$$=a-90-n+66$$

$$=-24$$

or

For what value of k will the consecutive terms $2k+1$, $3k+3$ and $5k-1$ form an AP?

Ans : [Board Term-2 Foreign 2016]

If x, y and z are in AP then we have

$$y-x=z-y$$

Thus if $2k+1,\ 3k+3,\ 5k-1$ are in AP then

$$(5k-1)-3k+3=(3k+3)-(2k+1)$$

$$5k-1-3k-3\ \ 3k+3-2k-1$$

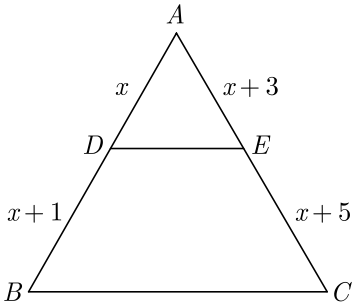
$$2k-4=k+2$$


e109

$$2k-k=4+2$$

$$k=6$$

6. In $\Delta ABC, DE||BC$, find the value of x.



Ans : [Board Term-1 2016]

In the given figure $DE||BC$, thus


f101

$$\frac{AD}{DB}=\frac{AE}{EC}$$

$$\frac{x}{x+1}=\frac{x+3}{x+5}$$

$$x^2+5x=x^2+4x+3$$

$$x=3$$

7. Write the coordinates of a point P on x-axis which is equidistant from the points $A(-2,0)$ and $B(6,0)$.

Ans : [Board 2019 OD]

Since it is equidistant from the points $A(-2,0)$ and $B(6,0)$ then


270

$$AP=BP$$

$$AP^2=BP^2$$

Using distance formula we have

$$[(x-(-2)]^2+(0-0)^2=(x+6)^2+(0-0)^2$$

$$(x+2)^2=(x+6)^2$$

$$x^2+4x+4=x^2+12x+36$$

$$8x=-32$$

$$x=-4$$

Hence, required point P is $(-4,0)$.

Alternative :

You may easily observe that both point $A(-2,0)$ and $B(6,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-2,0)$ and $B(6,0)$.

$$x=\frac{-2+6}{2}=2$$

8. Find the perpendicular distance of $A(5,12)$ from the y-axis.

Ans : [Board Term-2 2011]

Perpendicular from point $A(5,12)$ on y-axis touch it at $(0,12)$.

Distance between $(5,12)$ and $(0,12)$ is,


g103

$$d=\sqrt{(0-5)^2+(12-12)^2}$$

$$=\sqrt{25}$$

$$=5 \text{ units.}$$

9. If $\tan 2A=\cot(A+60^\circ)$, find the value of A where $2A$ is an acute angle.

Ans : [Board Term-1 2016]

We have $\tan 2A=\cot(A+60^\circ)$


h104

$$\cot(90^\circ-2A)=\cot(A+60^\circ)$$

$$90^\circ-2A=A+60^\circ$$

$$3A=30^\circ \Rightarrow A=10^\circ$$

10. If $\cos A=\frac{2}{5}$, find the value of $4+4\tan^2 A$.

Ans : [Board SQP 2018]


h113

$$4+4\tan^2 A=4(1+\tan^2 A)$$

$$4\sec^2 A=\frac{4}{\cos^2 A}$$

$$=\frac{4}{\left(\frac{2}{5}\right)^2}=4\times\frac{25}{4}=25$$

11. If $\sin A = \frac{1}{2}$, then what is the value of $\cot A$?

Ans :

We have $\sin A = \frac{1}{2}$

h309

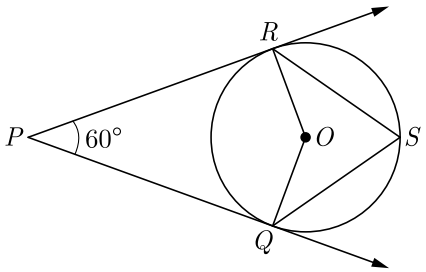
$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{2}$$

Now, $\text{Base} = \sqrt{2^2 - 1^2} = \sqrt{3}$

So, $\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Hence, the required value of $\cot A$ is $\sqrt{3}$.

12. In the given figure, find $\angle QSR$.



Ans : [Board Term-2, 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

j159

Thus $\angle ROQ + \angle RPQ = 180^\circ$

$$\angle ROQ = 180^\circ - 60^\circ = 120^\circ$$

We know that angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus $\angle QSR = \frac{1}{2}\angle ROQ = \frac{1}{2} \times 120^\circ$

$$= 60^\circ$$

13. What is the name of a line which intersects a circle at two distinct points?

Ans : [Board Term-2 2012]

A line intersecting the circle at two distinct points is called a secant.

l107

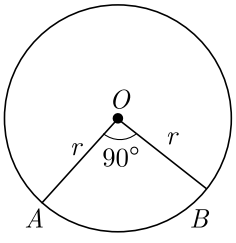
or

What is the perimeter of a sector of a circle whose central angle is 90° and radius is 7 cm?

Ans : [Board Term-2 2012]

As per question the digram is shown below.

l108



Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$= 2 \times 7 + 2 \times \frac{22}{7} \times 7 \times \frac{90}{360}$$

$$14 + 11 = 25 \text{ cm}$$

14. Volume of two spheres are in the ratio 64 : 27, find the ratio of their surface areas.

Ans : [Board Term-2 2012]

$$\frac{\text{Volume of I}^{st}\text{sphere}}{\text{Volume of II}^{nd}} = \frac{64}{27}$$

m111

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

Ratio of their surface areas,

$$\frac{2\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

15. Find the mean of first odd multiples of 5.

Ans : [Board Term-1 2012]

The first five odd multiples of 5, according to the problem are : 5, 15, 25, 35, 45

n161

$$\text{Mean} = \frac{5 + 15 + 25 + 35 + 45}{5}$$

$$= \frac{125}{5} = 25$$

16. What is the probability that a non-leap year has 53 Mondays ?

Ans : [Board Term-2, 2015]

There are 365 days in a non-leap year.

365 days = 52 weeks + 1 day

One day can be M, T, W, Th, F, S, S i.e. total 7 possible outcomes and only one favourable outcome.

Thus $n(S) = 7$ and $n(E) = 1$

o153

P(53 Mondays in non-leap year)

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$$

or

Two different dice are tossed together. Find the probability that the product of the number on the top of the dice is 6.

Ans : [Board Term-2 OD 2015]

Total number of possible outcomes,

$$n(S) = 6 \times 6 = 36$$

Product of 6 are (1, 6), (2, 3), (6, 1) and (3, 2).

Number of favourable outcomes,

$$n(E) = 4$$

o154

Total number of chances

$$n(S) = 6 \times 6 = 36$$

P(Product of 6)

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. Lavanya wants to organize her birthday party. She is very happy on her birthday. She is very health conscious, thus she decided to serve fruits only in her birthday party.

She has 36 apples and 60 bananas at home and decided to serve them. She wants to distribute fruits among guests. She does not want to discriminate among guests, so she decided to distribute fruits equally among all.

(i) How many maximum guests Shalvi can invite?
(a) 12 (b) 120
(c) 6 (d) 180

(ii) How many apples and bananas will each guest get?
(a) 3 apple 5 banana
(b) 5 apple 3 banana
(c) 2 apple 4 banana
(d) 4 apple 2 banana

a404

(iii) Lavanya decide to add 42 mangoes also. In this case how many maximum guests Lavanya can invite ?
(a) 12 (b) 120
(c) 6 (d) 180

(iv) How many total fruits will each guest get?
(a) 6 apple 5 banana and 6 mangoes
(b) 6 apple 10 banana and 7 mangoes
(c) 3 apple 5 banana and 7 mangoes
(d) 3 apple 10 banana and 6 mangoes

(v) If Lavanya decide to add 3 more mangoes and remove 6 apple in total fruits, in this case how many maximum guests Lavanya can invite ?
(a) 12 (b) 30
(c) 15 (d) 24

Ans :

(i) In this case we need to calculate HCF (36, 60) =12. Thus fruits will be equally distributed among 12 guests.
Thus (a) is correct option.

(ii) Out of 36 apples, each guest will get $\frac{36}{12} = 3$ apples and out of 60 bananas, each guest will get $\frac{60}{5} = 12$ bananas.
Thus (a) is correct option.

(iii) In this case we need to calculate HCF (36, 42, 60) =6.
Thus fruits will be equally distributed among 6 guests.
Thus (c) is correct option.

(iv) Out of 36 apples, each guest will get $\frac{36}{6} = 6$ apples and out of 42 mangoes, each guest will get $\frac{42}{6} = 7$ mangoes, out of 60 bananas, each guest will get $\frac{60}{6} = 10$ bananas. Thus each guest will get $6 + 7 + 12 = 25$ fruits.
Thus (b) is correct option.

(v) Now Lavanya has 30 apples, 60 bananas, and 45 mangoes. HCF (30, 45, 60) = 15. Thus Lavanya can invite 15 guest.
Thus (c) is correct option.

18. Mr. RK Agrawal is owner of a famous amusement park in Delhi. The ticket charge for the park is Rs 150 for children and Rs 400 for adult.

Generally he does not go to park and it is managed by team of staff. One day Mr Agrawal decided to random check the park and went there. When he checked the cash counter, he found that 480 tickets were sold and Rs 134500 was collected.

(i) Let the number of children visited be x and the number of adults visited be y. Which of the following is the correct system of equations that model the problem ?
(a) $x + y = 480$ and $3x + 8y = 2690$
(b) $x + 2y = 480$ and $3x + 4y = 2690$
(c) $x + y = 480$ and $3x + 4y = 2690$
(d) $x + 2y = 480$ and $3x + 8y = 2690$

(ii) How many children visited the park ?
(a) 250
(b) 500
(c) 230
(d) 460

c404

(iii) How many adults visited the park?
(a) 250 (b) 500
(c) 230 (d) 460

(iv) How much amount collected if 300 children and 350 adults visited the park?
(a) Rs 225400 (b) Rs 154000
(c) Rs 112500 (d) Rs 185000

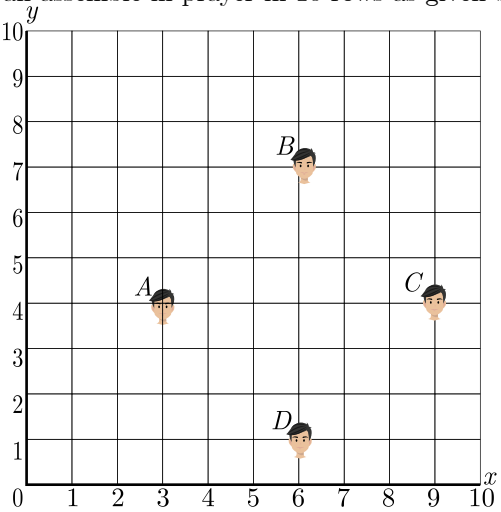
(v) One day total visited children and adults together is 750 and the total amount collected is Rs 212500. What are the number of children and adults visited the park ?
(a) (700, 800) (b) (350, 400)
(c) (800, 700) (d) (400, 350)

Ans :

(i) Since 480 people visited, we obtain $x+y=480$.
Collected amount is Rs 134500 thus
$150x+400y=134500 \Rightarrow 3x+8y=2690$
Thus (a) is correct option.
(ii) Solving the equations $x+y=480$ and $3x+8y=2690$ we get $x=230$ and $y=250$
Number of children attended = 230
Number of adults attended = 250
Thus (c) is correct option.
(iii) Number of adults visited the park = 250
Thus (a) is correct option.
(iv) Amount $=150\times300+400\times350=185000$ Rs
Thus (d) is correct option.
(v) Solving the equations $x+y=750$ and $150x+400y=212500 \Rightarrow 3x+8y=4250$ we have $x=350$ and $y=400$
i.e Number of children = 350
Number of adults = 400.
Thus (b) is correct option.

19. Morning assembly is an integral part of the school's schedule. Almost all the schools conduct morning assemblies which include prayers, information of latest happenings, inspiring thoughts, speech, national anthem, etc. A good school is always particular about their morning assembly schedule. Morning assembly is important for a child's development. It is essential to understand that morning assembly is not just about standing in long queues and singing prayers or national anthem, but it's something beyond just prayers. All the activities carried out in morning assembly by the school staff and students have a great influence in every point of life. The positive effects of attending school assemblies can be felt throughout life.

Have you noticed that in school assembly you always stand in row and column and this make a coordinate system. Suppose a school have 100 students and they all assemble in prayer in 10 rows as given below.



Here A, B, C and D are four friend Amar, Bharat, Colin and Dravid.

(i) What is the distance between A and B ?
(a) 8 (b) 6
(c) $3\sqrt{3}$ (d) $2\sqrt{3}$

(ii) What is the distance between C and D ?
(a) 8 (b) 6
(c) $3\sqrt{3}$ (d) $2\sqrt{3}$

(iii) What is the distance between A and C ?
(a) 8 (b) 6
(c) $3\sqrt{3}$ (d) $2\sqrt{3}$

(iv) What is the distance between D and B ?
(a) 8 (b) 6
(c) $3\sqrt{3}$ (d) $2\sqrt{3}$

(v) These 4 friends seating arrangement make a
(a) square (b) rhombus
(c) parallelogram (d) rectangle

Ans :

(i) We have $A(3,4)$ and $B(6,7)$.

$$AB = \sqrt{(6-3)^2+(7-4)^2} = 2\sqrt{3}$$

Thus (d) is correct option.
(ii) We have $C(9,4)$ and $D(6,1)$.

$$CD = \sqrt{(9-6)^2+(4-1)^2} = 2\sqrt{3}$$

Thus (d) is correct option.
(iii) We have $A(3,4)$ and $C(9,4)$

$$AC = \sqrt{(3-9)^2+(4-4)^2} = 6$$

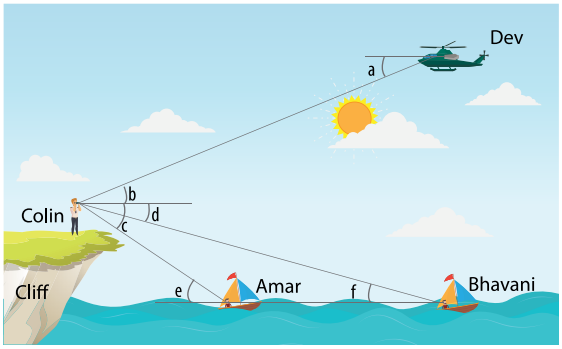
Thus (b) is correct option.
(iv) We have $B(6,7)$ and $D(6,1)$.

$$BD = \sqrt{(6-6)^2+(7-1)^2} = 6$$

Thus (b) is correct option.
(v) We can also calculate $BC=2\sqrt{3}$ and $AD=2\sqrt{3}$
Here, $AB = BC = CD = AD$
and $AC = BD$
Thus it is a square.
Thus (a) is correct option.

20. Navy officer Mr. Colin is tasked with planning a coup on the enemy at a certain date. Currently he is inspecting the area standing on top of the cliff. Agent Dev is on a chopper in the sky. When Mr. Colin looks down below the cliff towards the sea, he has Bhawani and Amar in boats positioned to get a good vantage point. Bhawani boat is behind the Amar boat.

i403



Following angle have been measured :
From Colin to Bhawani : 30°
From Dev to Colin : 60°
From Amar to Colin : 60°

(i) Which of the following is a pair of angle of elevation?
(a) $(\angle a, \angle e)$ (b) $(\angle b, \angle e)$
(c) $(\angle c, \angle d)$ (d) $(\angle a, \angle f)$

(ii) Which of the following is a pair of angle of depression?
(a) $(\angle a, \angle e)$ (b) $(\angle b, \angle e)$
(c) $(\angle c, \angle d)$ (d) $(\angle a, \angle f)$

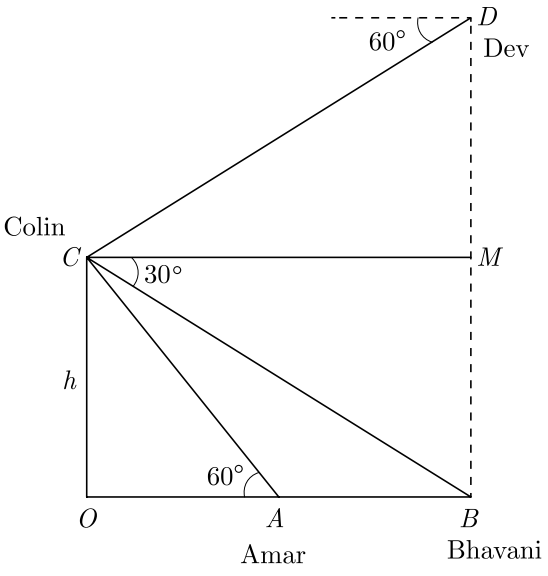
(iii) If angle of elevation of Amar to Colin is 60°, what is the distance of Amar boat from the base of hill ?
(a) $\frac{\sqrt{3}\,h}{2}$ (b) $\frac{h}{\sqrt{3}}$
(c) $\frac{2h}{\sqrt{3}}$ (d) $\sqrt{3}\,h$

(iv) If angle of depression of Colin to Bhawani is 30°, what is the distance of Amar boat from the Bhawani boat?
(a) $\frac{\sqrt{3}\,h}{2}$ (b) $\frac{h}{\sqrt{3}}$
(c) $\frac{2h}{\sqrt{3}}$ (d) $\sqrt{3}\,h$

(v) If angle of depression of Dev to Colin is 60°, what is the height of Dev from base of hill ?
(a) h (b) $2h$
(c) $3h$ (d) $4h$

Ans :

(i) The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight). In our case clearly$(\angle b, \angle e, \angle f)$ are angle of depression.
Thus (b) is correct option.
(ii) If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression. In our case clearly $(\angle a, \angle c, \angle d)$ are angle of depression.
Thus (c) is correct option.
(iii) We make the figure as given below.
Here $\angle OAC = 60^\circ$ is angle of elevation.



$$\frac{h}{OA} = \tan 60^\circ = \sqrt{3}$$

$$OA = \frac{h}{\sqrt{3}}$$

Thus (b) is correct option.
(iv) Here $\angle OBC = \angle MCB = 30^\circ$

$$\frac{h}{OB} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$OB = \sqrt{3}\,h$$

$$AB = OB - OA$$

$$= \sqrt{3}\,h - \frac{h}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}(3h - h) = \frac{2h}{\sqrt{3}}$$

Thus (c) is correct option.
(v) Here $\angle DCM = 60^\circ$

Now, $\frac{DM}{CM} = \tan 60^\circ = \sqrt{3}$

$$DM = \sqrt{3}\,CM$$

But $CM = OB = \sqrt{3}\,h$

Thus $DM = \sqrt{3} \cdot \sqrt{3}\,h = 3h$

Height of Dev from Bhawani,

$$= DB = DM + MB = 3h + h = 4h$$

Thus (d) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.
Ans : [Board Term-1 2012]

We have $f(x) = 2x^2 - 7x + 3$

Sum of zeroes $p + q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

b114

Since, $(p+q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p+q)^2 - 2pq$

$$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$$

Hence $p^2 + q^2 = \frac{37}{4}$.

or

Find the value of k if -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.
Ans : [Board Term-1 2012]

We have $p(x) = kx^2 - 4x + k$
Since, -1 is a zero of the polynomial, then

b116

$$p(-1) = 0$$

$$k(-1)^2 - 4(-1) + k = 0$$

$$k + 4 + k = 0$$

$$2k + 4 = 0$$

$$2k = -4$$

Hence, $k = -2$

22. Solve : $99x+101y=499,\quad 101x+99y=501$

Ans : [Board Term-1 2012, Set-55]

We have $99x+101y=499$ ...(1)

$101x+99y=501$ ...(2)

Adding equation (1) and (2), we have

$200x+200y=1000$

$x+y=5$ ...(3)

Subtracting equation (2) from equation (1), we get

$-2x+2y=-2$

$x-y=1$ ...(4)

Adding equations (3) and (4), we have

$2x=6 \Rightarrow x=3$


c136

Substituting the value of x in equation (3), we get

$3+y=5 \Rightarrow y=2$

or

Solve the following system of linear equations by substitution method:

$2x-y=2$

$x+3y=15$

Ans : [Board Term-1 2012]

We have $2x-y=2$ ...(1)

$x+3y=15$ ...(2)

From equation (1), we get $y=2x-2$ ...(3)

Substituting the value of y in equation (2),

$x+6x-6=15$


c137

or, $7x=21 \Rightarrow x=3$

Substituting this value of x in (3), we get

From equation (1), we have

$y=2\times 3-2=4$

$x=3$ and $y=4$

23. Find the roots of the following quadratic equation :

$\frac{2}{5}x^2-x-\frac{3}{5}=0$

Ans : [Board Term-2, 2012]

We have $\frac{2}{5}x^2-x-\frac{3}{5}=0$


d112

$\frac{2x^2-5x-3}{5}=0$

$2x^2-5x-3=0$

$2x^2-6x+x-3=0$

$2x(x-3)+1(x-3)=0$

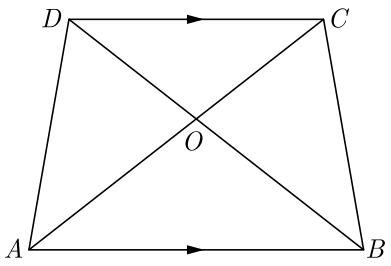
$(2x+1)(x-3)=0$

Thus $x=-\frac{1}{2},\ 3$

24. $ABCD$ is a trapezium in which $AB \| CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO}=\frac{CO}{DO}$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In ΔAOB and ΔCOD, $AB \| CD$,


f126

Thus due to alternate angles

$\angle OAB = \angle DCO$

and $\angle OBA = \angle ODC$

By AA similarity we have

$\Delta AOB \sim \Delta COD$

For corresponding sides of similar triangles we have

$\frac{AO}{CO}=\frac{BO}{DO}$

$\frac{AO}{BO}=\frac{CO}{DO}$. Hence Proved

25. Evaluate : $\frac{5\cos^2 60^\circ+4\cos^2 30^\circ-\tan^2 45^\circ}{\sin^2 30^\circ+\cos^2 60^\circ}$

Ans : [Board Term-1 2013]

$$\frac{5\cos^2 60^\circ+4\cos^2 30^\circ-\tan^2 45^\circ}{\sin^2 30^\circ+\cos^2 60^\circ}$$


h136

$$=\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{\sqrt{3}}{2}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}$$

$$=\frac{\frac{5}{4}+3-1}{\frac{1}{4}+\frac{1}{4}}$$

$$=\frac{\frac{5}{4}+2}{\frac{1}{2}}=\frac{\frac{13}{4}}{\frac{1}{2}}=\frac{13}{2}$$

26. Find the sum of the lower limit of the median class and the upper limit of the modal class :

Classes	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3
Cumulative Frequency	1	4	9	18	25	28

We have $N=28$; $\frac{N}{2}=\frac{28}{2}=14$

Cumulative frequency just greater than $\frac{N}{2}$ is 18 and the corresponding class is 40 - 50. Thus median class is 40-50.


n179

Lower limit is 40 and upper limit is 5. Their sum is $= 40 + 50 = 90$

27. If α and β are the zeroes of the polynomial $f(x) = 5x^2 - 7x + 1$ then find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

Ans : [Board 2020 OD Basic]

Since, α and β are the zeroes of the quadratic polynomial $f(x) = 5x^2 - 7x + 1$,

Sum of zeros, $\alpha + \beta = -\left(\frac{-7}{5}\right) = \frac{7}{5}$ ...(1)

Product of zeros, $\alpha\beta = \frac{1}{5}$ ...(2)

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{7}{5}\right)^2 - 2 \times \frac{1}{5}}{\frac{1}{5}}$

$= \frac{49 - 2\times 5}{5} = \frac{39}{5}$

d300

28. The 17^{th} term of an AP is 5 more than twice its 8^{th} term. If 11^{th} term of AP is 43, then find its n^{th} term.

Ans : [Board 2020 OD Basic]

Let a be the first term and d be the common difference.

n^{th} term of an AP,

$$a_n = a + (n-1)\,d$$

Since 17^{th} term of an AP is 5 more than twice of its 8^{th} term, thus

$$a + (17-1)\,d = 5 + 2\,[a + (8-1)\,d]$$
$$a + 16d = 5 + 2\,(a + 7d)$$
$$a + 16d = 5 + 2a + 14d$$
$$2d - a = 5 \quad ...(1)$$

e301

Since 11^{th} term of AP is 43,

$$a + (11-1)\,d = 43$$
$$a + 10d = 43 \quad ...(2)$$

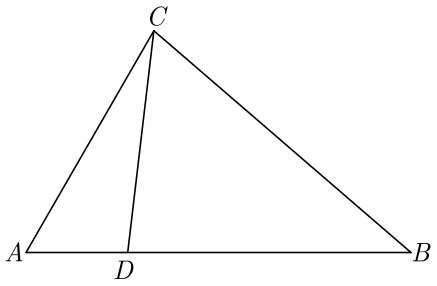
Solving equation (1) and (2), we have

$$a = 3 \text{ and } d = 4$$

Hence, n^{th} term would be

$$a_n = 3 + (n-1)4 = 4n - 1$$

29. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



Ans : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$\angle ACB = \angle CDA$ [given]

$\angle CAB = \angle CAD$ [common]

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$

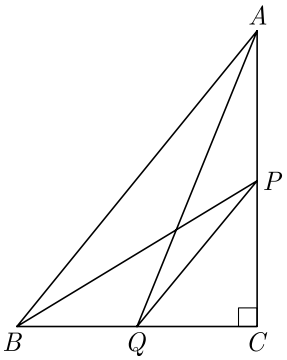
f245

Now $\frac{AB}{AC} = \frac{AC}{AD}$

$$AC^2 = AB \times AD$$
$$6^2 = AB \times 3$$
$$AB = \frac{36}{3} = 12 \text{ cm}$$

or

If P and Q are the points on side CA and CB respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



f246

Ans : [Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \quad ...(1)$$

and $$BP^2 = PC^2 + CB^2 \quad ...(2)$$

Adding eq (1) and eq (2), we get

$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$
$$= (AC^2 + CB^2) + (PC^2 + CQ^2)$$

Thus $AQ^2 + BP^2 = AB^2 + PQ^2$ Hence Proved

30. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or ½.

Ans : [Board 2020 OD Standard]

We have, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$
$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$
$$\text{cosec}^2\theta + 1 = 3\cot\theta$$
$$1 + \cot^2\theta + 1 = 3\cot\theta$$
$$\cot^2\theta - 3\cot\theta + 2 = 0$$
$$\cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$$
$$\cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$
$$(\cot\theta - 2)(\cot\theta - 1) = 0$$
$$\cot\theta = 1 \text{ or } 2$$
$$\tan\theta = 1 \text{ or } ½.$$

h291

or

Prove that

$(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

Ans : [Board 2019 Delhi Standard]

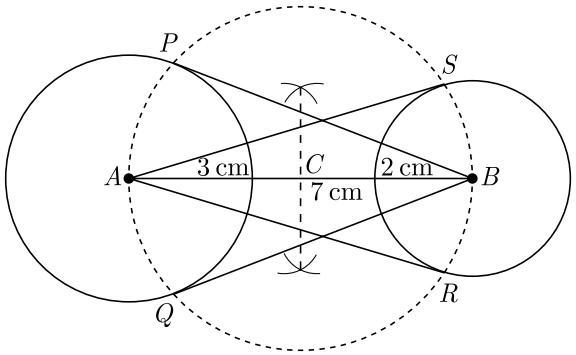
$$\text{LHS} = (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$$
$$= (\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta) +$$
$$+(\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$$
$$= (\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta + \sec^2\theta)$$
$$+2\sin\theta \times \frac{1}{\sin\theta} + 2\cos\theta \times \frac{1}{\cos\theta}$$
$$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2$$
$$= 7 + \tan^2\theta + \cot^2\theta = \text{RHS}$$

31. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as center, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

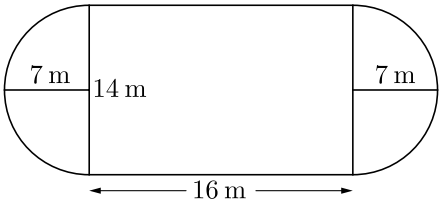
Ans : [Board Term-2 Delhi 2015]

Steps of Construction :

1. Draw a line segment AB of 7 cm.
2. Taking A and B as centre draw two circle of 3 cm and 2 cm radius respectively.
3. Bisect the line AB. Let mid-point of AB be C.
4. Taking C as centre draw a circle of radius AC with intersects the two circles at point P, Q, R and S.
5. Join BP, BQ, AS and AR. BP, BQ and AR, AS are the required tangents.



32. Find the area of the adjoining diagram.



Ans : [Board Term-2, 2014]

The given figure is combination of one rectangle and two semicircle of same radii .

Required area,

= area of two semi-circles + area of rectangle

= area of one circle + area of rectangle

$= \pi r^2 + (l \times b)$

(where r is radius of circle and l and b are length and breadth of rectangle)

$= \frac{22}{7} \times 7^2 + (16 \times 14)$

$= \frac{22}{7} \times 7 \times 7 + (16 \times 14)$

$= 154 + 224 = 378$ m^2

33. The mean of the following distribution is 53. Find the missing frequency p :

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	p	13

Ans : [Board Term-1 2011]

Class	x_i **(Class marks)**	f_i	$f_i x_i$
0-20	10	12	120
20-40	30	15	450
40-60	50	32	1600
60-80	70	p	$70p$
80-100	90	13	1170
	Total	$\sum f_i = 72 + p$	$\sum f_i x_i = 3340 + 70p$

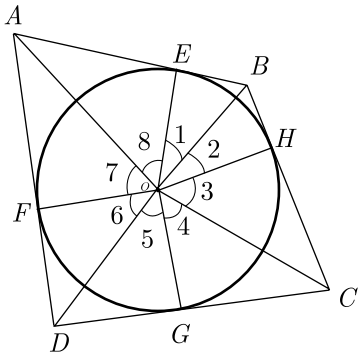
Mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$$53 = \frac{3340 + 70p}{72 + p}$$
$$3340 + 70p = 53(72 + p)$$
$$3340 + 70p = 3816 + 53p$$
$$70p - 53p = 3816 - 3340$$
$$17p = 476$$
$$p = \frac{476}{17} = 28$$

34. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans : [Board Term-2 Foreign 2017, OD 2014]

A circle centre O is inscribed in a quadrilateral $ABCD$ as shown in figure given below.



Since OE and OF are radius of circle,

$$OE = OF$$

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle OEA = \angle OFA = 90^\circ$

Now in ΔAEO and ΔAFO,

$OE = OF$

$\angle OEA = \angle OFA = 90^\circ$

$OA = OA$ (Common side)

Thus $\Delta AEO \cong \Delta AFO$ (SAS congruency)

$\angle 7 = \angle 8$

Similarly, $\angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$

Since angle around a point is 360°,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360^\circ$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$

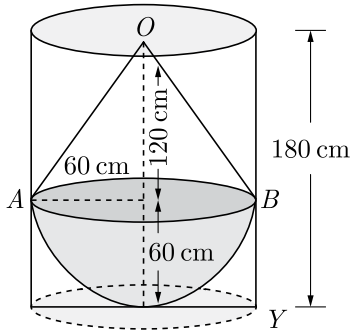
$\angle AOB + \angle COD = 180^\circ$ Hence Proved.

35. A solid is consisting of a right circular cone of height 120 cm and radius 60 cm standing on hemisphere of radius 60 cm. It is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Ans : [Board Term-2, 2015]

As per question the figure is shown below.

m218



Height of cone, $h = 120$ cm,

Radius of cone, $r = 60$ cm

Radius of hemisphere, $r = 60$ cm.

Height of cylinder, $H = 180$ cm,

Radius of cylinder, $R = 60$ cm

Radius of cone, hemisphere and cylinder is equal to $r = 60$ cm

Volume of solid,

$$V_{\text{solid}} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}(h + 2r)$$

$$= \frac{\pi r^2}{3} \times 240 = 80\pi r^2$$

Volume of water in the cylinder is equal to the volume of cylinder.

$$V_{\text{cylinder}} = \pi r^2 h$$

$$= \pi \times r^2 \times 180 = 180\pi r^2$$

Water left in the cylinder is equal to the difference of the volume of water in cylinder and volume of solid.

Water left in the cylinder,

$$= V_{\text{cylinder}} - V_{\text{solid}}$$

$$= 180\pi r^2 - 80\pi r^2$$

$$= 100\pi r^2$$

$$= 100 \times \frac{22}{7} \times (60)^2$$

$$= \frac{100 \times 22 \times 60 \times 60}{7}$$

$$= 1131428 \text{ cm}^3$$

36. A bag contains 20 balls out of which x balls are red.

(i) If one ball is drawn at random from the bag, find the probability that it is not red.

(ii) If 4 more red balls are out into the bag, the probability of drawing a red ball will be $\frac{5}{4}$ times the probability of drawing a red ball in the first case. Find the value of x.

Ans : [Board Term-2 Foreign 2015]

o262

Total ball, $n(S) = 20$

Red ball $n(R) = x$

(i) not red

$P(\text{red ball}), \quad P(R) = \frac{n(R)}{n(S)} = \frac{x}{20}$

$P(\text{no red ball}),$

$$P(\overline{R}) = 1 - \frac{x}{20} = \frac{20 - x}{20} \quad \ldots(1)$$

(ii) Now two more red balls are added.

Total ball $n'(S) = 20 + 4 = 24$

There are total $x + 4$ red ball.

$$n'(R) = x + 4$$

$P(\text{red balls}), \quad P'(R) = \frac{n'(R)}{n'(S)} = \frac{x+4}{24}$

Now, according to the question,

$$\frac{x+4}{24} = \frac{5}{4} \times \frac{x}{20}$$

$$\frac{x+4}{24} = \frac{x}{16}$$

$$16x + 64 = 24x$$

$$64 = 8x \Rightarrow x = 8$$

For first case, substituting $x = 8$ in equation (1) we have

$$P(\overline{R}) = \frac{20 - 8}{20} = \frac{12}{20} = \frac{3}{5}$$